S. Chandrasekhar : White Dwarfs, H^- ion,.., Black holes, Gravitational waves

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This is a concise review of S. Chandrasekhar's research contributions to astrophysics, ranging from his early studies on white dwarfs using relativistic quantum statistics to topics as diverse as dynamical friction, negative hydrogen ion, fluid dynamical instabilities, black holes and gravitational waves. The exposition is based on simple physical explanations in the context of observational astronomy, addressed primarily to the undergraduate students. Black holes and their role as central engines of active, compact, high energy sources have been discussed in some details.

I INTRODUCTION

The impactful research journey of Subrahmanyan Chandrasekhar began on July 31, 1930, from Bombay port on a ship. The 19 year old Chandra was on his way to England for higher studies. Armed with his understanding of Fowler′s work on white dwarfs [1], Chandra was immersed in the mathematical equations describing these dense objects, during that voyage. He had realized that Fowler's theory needed modification, since for sufficiently massive white dwarfs, particle number densities could be so high that a large fraction of electrons would be occupying very high energy levels, moving with relativistic velocities.

At this point, a quick summary of stellar evolution theory is in store. In main sequence stars (like Sun), nuclear fusion of hydrogen to helium supplies the required thermal energy to stall gravitational contraction of a star, enabling it to attain a quasi-hydrostatic equilibrium. As the star advances in age, a further sequence of nuclear fusion reactions gets activated in its core - helium burning to carbon and oxygen, carbon burning to sodium and magnesium and so on, if the star is massive enough, till the formation of iron-rich core. Iron nucleus being the most stable one, subsequent nuclear burning cease to take place. As the core cools, it collapses under its own weight, till the electron density becomes so high that electron degeneracy pressure prevents further contraction.

Degeneracy pressure is a consequence of quantum statistics in extremely dense matter. Pauli exclusion principle (PEP) states that no two identical fermions can have the same state. Electrons, protons, neutrons, neutrinos, etc., being spin half particles, are fermions. According to PEP, in a gravitationally bound system like the iron-rich core of an evolved star, all the electrons cannot occupy the lowest energy level (unlike, what happens to identical bosons in Bose-Einstein condensates, e.g. He-4 superfluid). So, the energy levels are filled up with two electrons (one with spin up state and the other with spin down) per orbital, as demanded by the PEP. Hence, more the density of electrons, higher is the energy level that gets to be occupied.

Gravitational shrinking of such a dense core leads to an increase in electron density, thereby facing a resistance since the contraction implies putting electrons at higher energy levels. Therefore, in such a degenerate system, gravitational collapse instead of lowering the energy of the star tends to increase it. The resulting pressure against shrinking, arising out of PEP in such electron-rich dense matter is called electron degeneracy pressure (EDP). A white dwarf is a star that is in hydrostatic equilibrium not because of thermal pressure but due to the EDP that counteracts gravitational contraction. Fowler had assumed that electrons are moving non-relativistically inside the core and had shown that the EDP of a white dwarf is proportional to $\rho^{5/3}$, where ρ is the density of the core[1].

II CHANDRASEKHAR LIMIT AND COMPACT OBJECTS

In his investigations, Chandra incorporated special relativity in the analysis of white dwarfs, and found that the EDP is proportional to $\rho^{4/3}$ instead, demonstrating that the relativistic degeneracy pressure does not increase as rapidly as in Fowler's case. Performing an accurate study of the relativistic problem of a dense star ruled by a polytropic equation of state, in which gravity is countered by the EDP, he arrived at the celebrated Chandrasekhar mass limit [2],

$$M_{Ch} = \frac{0.2}{(m_p \mu_e)^2}\left(\frac{\hbar c}{G}\right)^{3/2} \quad , \tag{1}$$

where $\hbar$, G, c, m_p and μ_e are the reduced Planck′s constant, Newton′s gravitational constant, speed of light, mass of a proton and mean molecular weight per electron, respectively. It is remarkable that such a significant result concerning stars should be expressible in terms of fundamental quantities (except for μ_e). In white dwarfs, the value of μ_e is about 2, so that from eq.(1) one finds the limit to be $M_{Ch} \approx 1.4\ M_\odot$, where $M_\odot = 2 \times 10^{30}$ kg is the Sun′s mass.

Chandra was unaware initially that Anderson in 1929 and Stoner in 1930 had independently applied special relativity to obtain mass limits for a degenerate, dense star of uniform density without taking into account the condition of hydrostatic equilibrium [3,4,7]. Fowler pointed this out to him when Chandra reached Cambridge, and he added these references to his papers on relativistic degeneracy in white dwarf stars [5]. Landau too had arrived at a mass limit independently in 1931, which appeared in print one year later [6].

The Chandrasekhar mass limit implies that no white dwarf with mass greater than this limit can hold out against gravitational collapse. So far, all the white dwarfs discovered (e.g. Sirius B, the companion star to Sirius) in the cosmos, have mass less than M_{Ch}. For masses beyond this limit, two prescient ideas were put forward independently, that played important roles later - one of Landau [6], before the discovery of neutrons by Chadwick in 1932 and the other by Baade and Zwicky [8,9], after the discovery. Landau had speculated that for stellar cores whose mass exceeded M_{Ch}, the density would become so large due to shrinking that the atomic nuclei in the core would come in contact with each other - the whole core turning into a giant nucleus [6]. Baade and Zwicky, while attributing the origin of cosmic rays to stellar explosions called supernovae, correctly identified the energy liberated due to sudden decrease in the gravitational potential energy (as the core collapses rapidly to form a neutron star of radius $\sim$ 10 km) as the one that powers supernova explosion [8,9]. A core with mass M_c, shrinking from a large size to a radius R_c, has to give up an energy,

$$E_{exp} \sim \frac{GM_c^2}{R_c} \quad , \tag{2}$$

since its gravitational potential energy decreases to $\sim -E_{exp}$. For a 1.4 $M_\odot$ core collapsing to form a neutron star of radius $R_c \approx 10$ km, the energy E_{exp} available for explosion is as high as $\sim 10^{53}$ ergs.

Why does the core become neutron-rich? As the core shrinks, its density rises till it reaches nucleonic values $\sim 10^{12}$ - 10^{14} gm/cm^3, when protons in the core transform into neutrons by capturing electrons and emitting neutrinos [10]. Neutrinos, being weakly interacting particles, escape from the core. While in the neutron-rich core, the neutron degeneracy pressure (arising from PEP, as neutrons too are spin half particles) prevents further gravitational contraction, resulting in the formation of a neutron star.

With the detection of periodic emission of radio-pulses from a source by Jocelyn Bell and Anthony Hewish in 1967, existence of neutron stars as pulsars was established. Pulsars are rapidly spinning neutron stars with rotation period ranging from about few milli-seconds to few seconds. The observed pulses are due to electromagnetic radiation from accelerated charge particles moving along strong magnetic field lines inclined with respect to the rotation axis (The polar magnetic field strengths vary from $\sim 10^{10}$ to $\sim 10^{14}$ gauss). Recently, a milli-second pulsar was found to have a mass of ≈ 2 $M_\odot$, determined using a general relativistic effect called Shapiro delay in which radiation grazing past a compact, massive object, arrives at the observer with a time lag because of the strongly curved space-time geometry it encounters near the massive star [11].

As long as the core is lighter than about $2 - 3$ $M_\odot$, it can survive as a neutron star (The mass limit in this case is uncertain as it depends crucially on the equation of state for nuclear matter which, for such huge densities existing inside neutron stars, is unknown [11,12]). The released neutrinos, after travelling long distances, eventually lose their energy to the stellar envelope, causing the latter to be blown apart, giving rise to a Type II supernova. Measurements concerning detected neutrinos from the supernova SN 1987A indicate that these ultra-light, weakly interacting particles carry away 99% of the gravitational binding energy released from the collapsing core, lending credence to the neutrino driven explosion models [10].

The observed masses for neutron stars do not appear to exceed ~ 3 $M_\odot$ [11,12], suggesting that a massive star whose core is heaver than this limit, would certainly collapse to form a black hole. The long duration gamma ray burst sources that exhibit prompt gamma emissions with photons having energy predominantly in 0.1 - 1 MeV range, and lasting for about 2 - 1000 s are likely to be collapsing massive cores [13]. Eddington had found the idea of a star shrinking gravitationally to a point absurd [14]. Three decades later, Penrose and Hawking, employing Raychaudhuri equation, proved the remarkable singularity theorems, according to which gravitational collapse of normal matter generically lead to formation of point singularities, namely, the black holes [15–17].

III DYNAMICAL FRICTION

Chandra played a significant role in the research area of stellar dynamics from 1939 to 1944 that culminated in the publication of his celebrated papers on dynamical friction[18,19]. Cosmos is filled with gravitationally bound systems of massive objects like globular clusters, galaxies, clusters of galaxies, etc. Objects that make up these bound systems, apart from moving in gravitational potential wells, also suffer two-body gravitational encounters, resulting in exchange of energy and momentum. It was Chandra who showed for the first time that a massive body in motion, surrounded by a swarm of other less massive objects, suffers deceleration that is proportional to its mass [18].

Dynamical friction arises out of cumulative gravitational encounters that the massive body experiences due to the presence of other objects in the background. The physical origin of dynamical friction can be intuitively understood by going to the reference frame in which the body is at rest. In this frame, the swarm of background objects while moving past the massive body get gravitationally focused behind the body, forming a wake of higher mass density. Now, switching back to the frame in which the massive body is moving, we find that the mass density of the wake behind is greater than the density ahead. Consequently, because of a greater gravitational pull from behind, the massive body suffers a gravitational drag force whose magnitude is proportional to the square of its mass and inversely proportional to the square of its speed [20,21].

Observational consequences of dynamical friction include sinking of globular clusters towards the central regions of galaxies and galactic cannibalism in which the orbit of a satellite galaxy decays, leading eventually to its merger with the bigger galaxy [21,22].

IV NEGATIVE HYDROGEN ION

Around the same time, Chandra was also involved with the quantum theory of negative hydrogen ion. Can a proton capture two electrons to form a charged bound state? How is it relevant to astrophysics? The first issue had been settled by Bethe in 1929 who showed that quantum mechanics indeed predicts formation of H^- ions [23]. As to the second question, it has been found over the years that H^- is a weakly bound system with a binding energy of ≈ 0.75 eV. Since it takes only about 0.75 eV to knock off the extra electron from H^-, its life-time under terrestrial conditions is small but in thin and tenuous plasma where the collision frequency is low, one expects negative hydrogen ions to survive for longer duration.

Early on, Wildt had foreseen that because of the presence of hydrogen atoms and electrons, in large numbers, in the upper atmosphere of Sun, H^- would form. He had also realized that photo-detachment of H^- would contribute greatly to solar opacity, since radiation from Sun would be attenuated as they photo-ionize H^- ions on their way out[24–26].

At this juncture, Chandra and his collaborators played an important role in calculating H^- photo-absorption matrix element, so crucial for estimating the quantum probability (and, therefore, the cross-section) of photo-ionization of H^- [27–33]. The opacity or the optical depth is proportional to the photo-absorption cross-section σ as well as n, the number density of H^-. This is because, the number of photo-ionizations per photon per unit time is $c\ n\ \sigma$, so that the mean free path length for photons is simply,

$$l = \frac{1}{n\ \sigma} \quad .$$

The optical depth essentially is the ratio of the geometrical path length traversed by the radiation to mean free path length l (i.e., it is the number of absorptions suffered by the photons on an average).

The negative hydrogen ion has only the ground state as a bound state, with no singly excited states. As a result, photons with energy above 0.75 eV, executing random walks out of Sun due to multiple scatterings, would be absorbed by H^- ions after detaching their extra electrons to the continuum. This is the dominant cause for solar opacity in the infra-red to visible range of the electromagnetic spectrum.

In 1943, Chandrasekhar and Krogdahl drew attention to the fact that dominant contribution to this matrix element comes from the wavefunction at large distances (several times the Bohr radius), and therefore an accurate knowledge of electronic wavefunction of H^- was required [27].

Chandra and his collaborators made seminal contributions towards calculating the continuous absorption coefficient κ_λ of H^- as a function of the photon wavelength λ, incorporating dipole-length and dipole-velocity formulae, that provided a solid theoretical foundation for the characteristic κ_λ - λ plot which exhibits a rise in the range 4000 to 9000 angstroms and then drops to a minimum at 16000 angstroms, with a subsequent rise [34].

The charged hydrogen ion has also played an important role in cyclotrons and particle accelerators [35]. The advantages in making use of H^- arise out of the possibility of accelerating them by applying electric fields and obtaining hot neutral beams in Tokamaks (like in ITER)[36]. This is because of the relative ease in detaching its extra electron when H^- ion is present in the gas cells.

V MAGNETOHYDRODYNAMICS

Astrophysical entities are usually permeated with magnetic fields, be it planets like earth, Jupiter, etc., Sun, sunspots, stars, flares, spiral arms of Milky Way, galaxies, and so on. Magnetic field in a conducting medium like metal or plasma decays due to Ohmic dissipation. So, how does terrestrial magnetic field, generated by the electric currents flowing in the molten, conducting and rotating core of Earth, prevent itself from Ohmic decay?

Dynamo theories involving differential rotation and convection in conducting fluids are invoked to solve this conundrum. However, Cowling had proved that magnetohydrodynamical flows with axisymmetric geometry will always entail a decaying magnetic field [37]. About two decades later, Backus and Chandra generalized Cowling's theorem [38]. In this context, Chandra studied the possibility of lengthening the decay duration so that an axisymmetric dynamo provides a feasible explanation for geomagnetism [39]. It was immediately followed by a paper in which Backus showed that the increase was not large enough to be of geophysical interest [40]. Chandra studied several fluid dynamical stability problems employing variational methods that have interesting consequences [41,42].

An evolved binary system, consisting of a Roche lobe[20] filling star, spewing out gaseous matter, and a massive compact object (MCO) like a neutron star or a black hole (BH), both going around the common centre of mass, very often acts as a luminous source of high energy photons. In such a binary system, gas leaking out from the bloated star cannot radially fall on the MCO as it has angular momentum. Instead, it spirals inwards, forming an accretion disc around the MCO so that each tiny gaseous volume element of the disc moves along a circular Keplerian orbit [43].

For a thin disc with a total mass much less than the mass M of the MCO, the Keplerian speed $v(r)$ of a fluid element at a distance r is given by,

$$v(r) = \sqrt{\frac{GM}{r}} \quad , \tag{3}$$

Eq.(3) implies that the fluid elements of the accretion disc rotate differentially. Farther the element from the MCO, lower is its circular speed. Differential rotation leads to viscous rubbing of neighbouring fluid elements at varying distances, causing the accretion disc to heat up. If the disc is sufficiently hot, it emits copious amount of electromagnetic radiation with a spectrum ranging from visible wavelengths to UV photons and X-rays.

There are strong observational evidences that the rapidly time varying, intense X-ray sources, like Cygnus X-1, are accreting black holes (see section VII). Essentially, the gravitational potential energy of the gas spiralling in, gets converted into radiative energy at the rate corresponding to a luminosity of,

$$L = \epsilon \frac{GM\dot{m}}{r_{min}} \quad , \tag{4}$$

where $\dot{m}$, r_{min} and ϵ are the rate of mass accretion, minimum distance reached by the infalling gas and an efficiency factor for the conversion of gravitational energy to radiation, respectively. The importance of accretion on to compact objects is evident from eq.(4), since source luminosity is larger for smaller values of r_{min}. Similarly, a luminous source requires larger rates of accretion and higher conversion efficiency.

For the efficiency factor ϵ to be large, the accretion disc is required to have a high viscosity. The physics of the mechanism responsible for large viscosities in the disc is an active area of research. Interestingly, as shown by Balbus and Hawley in 1991, the Chandrasekhar instability might be the key to the origin of accretion disc viscosity [44]. Chandra had pointed out that a differentially rotating, conducting and magnetized incompressible fluid in a cylindrical configuration, is unstable with respect to oscillating axisymmetric perturbations [41].

While investigating Rayleigh-Benard convection in conducting and viscous fluids threaded with magnetic field, Chandra studied the onset of convection and its dependence on a dimensionless number Q, representing the square of the ratio of magnetic force to viscous force [41]. Today, this number Q is referred to as Chandrasekhar number (or, also as the square of Hartmann number). Chandra made several other contributions in the field of plasma physics and magnetohydrodynamics that had far reaching consequences [45].

VI CHANDRASEKHAR-FRIEDMAN-SCHUTZ INSTABILITY

While studying self-gravitating and rotating fluid configurations, Chandra showed that a uniformly dense and uniformly rotating incompressible spheroid is unstable because of non-radial perturbations, causing emission of gravitational radiation [46]. According to Einstein's general relativity, the curvature of space-time geometry manifests as gravitational force. Gravitational radiations are wave-like perturbations in the space-time geometry that propagate with speed of light, general relativity being a relativistic theory of gravitation. Gravitional waves are radiated whenever the quadrupole moment of the mass distribution in a source changes with time. Friedman and Schutz extended Chandra's findings in 1978, and demonstrated the existence of gravitational wave driven instability in the general case of rotating and self-gravitating stars made of perfect fluid [47].

A physically intuitive way to understand this Chandrasekhar-Friedman-Schutz (CFS) instability is to look at a perturbation mode in a rotating star that is retrograde, i.e. moving in the backward sense relative to the fluid element going around. According to general relativity, the space-time geometry around a rotating body is such that inertial frames are dragged along the direction of rotation (This has been recently verified by the Gravity Probe B satellite-borne experiment [48]). The frame dragging, therefore, would make the retrograde mode appear prograde to an inertial observer far away from the star. Gravitational waves emitted by this mode will carry positive angular momentum (i.e. having the same sense as the angular momentum of the fluid element) as measured in the distant inertial frame. Since, the total angular momentum is conserved, gravitational radiation carrying away positive angular momentum from the mode, would make the retrograde mode go around more rapidly in the opposite direction, leading to an instability.

Andersson in 1998 showed that a class of toroidal perturbations (the so called r-modes) in a rotating star are generically unstable because of the gravitational wave driven CFS instability [49]. Close on heels, it was demonstrated that the r-mode instability would put brakes on the rotation of a newly born and rapidly spinning neutron star [50,51]. Consequently, as the neutron star spins down, a substantial amount of its rotational energy is radiated away as gravitational waves, making it a likely candidate for future detection by the laser interferometric gravitational wave detectors, namely, the LIGOs [52,53]. The CFS instability may soon be put to experimental tests.

VII BLACK HOLES AND GRAVITATIONAL WAVES

In his book on black holes (BHs), Chandra called the astrophysical BHs the most perfect macroscopic objects [54]. Things macroscopic - like chairs, books, computers, etc. around us, require an astronomically large number of characteristics each for their description. For instance, just to specify a suger cube would need not only its mass, density, temperature, but also amount and nature of trace compounds present, the manner in which sugar molecules are stacked, porosity, surface granularities, etc . On the other hand, a BH is characterized by just three physical quantities - its mass, charge and angular momentum.

Schwarzschild BHs do not possess charge or angular momentum, while Kerr BHs rotate but have no charge. On the other hand, Reissner-Nordstrom BHs have charge but do not rotate. Kerr-Newman BHs are theoretically the most general ones, as they possess non-zero mass, charge and angular momentum. Astrophysical black holes are all likely to be Kerr BHs since charge of a BH would get neutralized by the capture of oppositely charged particles present in the cosmic rays and other ambient matter, and since most cosmic objects possess angular momentum. Chandra was particularly fascinated by the stationary, axisymmetric vacuum solutions of Einstein equations that described the Kerr BHs.

BHs are characterized by a fictitious spherical surface called the event horizon centred around the point singularity created by the collapse of matter. Nothing can escape from regions enclosed within the event horizon. For a Schwarzschild BH of mass M, the radius of the event horizon is given by the Schwarzschild radius,

$$R_s = \frac{2GM}{c^2} = 3\times 10^6 \left(\frac{M}{10^6\ M_\odot}\right)\ \mathrm{km}\ . \quad (5)$$

But do BHs exist? Classical BHs by themselves do not radiate. Hawking radiation, which is of quantum mechanical origin, from astrophysical BHs, is too miniscule in amount to be of any observational significance [55]. So, how does one find BHs in nature? In conventional astronomy, their detection relies on the presence of gas or stars in their vicinity and the ensuing stellar or dissipative gas dynamics around an accreting MCO. As discussed in section V, if the MCO has an accretion disc around it like in galactic X-ray sources, quasars, blazars or radio-galaxies, the swirling and inward spiralling gas gets heated up, emitting radio, optical, UV and X-ray photons, often accompanied by large scale jets [56].

If gas can spiral down to a distance $r_{min} = \alpha\ R_s$ from the central BH, then according to eqs. (4) and (5) the radiation luminosity is given by,

$$L = \frac{0.5\epsilon}{\alpha}\dot{m}c^2\ . \quad (6)$$

The real parameter α quantifies the proximity to the central BH. Eq.(6) tells us that accretion taking place close to the event-horizon can convert an appreciable fraction of rest energy mc^2 of the inflowing gas. Higher the accretion rate $\dot{m}$, larger is the luminosity L. (Provided that fluid viscosities in the disc are large enough, as discussed in section V.)

The central engine for a quasar or a blazar is, in all likelihood, an accreting supermassive BH with M lying in the range 10^7- 10^9 $M_\odot$ [56]. Invoking eq.(6) with sufficiently large accretion rates for blazars, one can theoretically explain high luminosities (at times, exceeding 10^{48} erg/s) observed in these sources.

Quasars and blazars also exhibit fluctuating X-ray luminosities on time scales of only few hours. One can derive an upper limit for the size of the central engine from causality arguments. If the observed time scale over which the luminosity varies accreciably is Δt, the size of the source participating in emission of photons cannot be larger than $c\Delta t$. This is because, firstly, every part of the entire region must be causally connected to each other and, secondly, special relativity tells us that parts of the region can physically communicate with each other (to remain in causal touch) only with speeds $\leq c$. X-ray variability on time scales of an hour corresponds to a causal size $\leq 10^9$ km. Now, from eq.(5), a BH of mass $3\times 10^8\ \ M_\odot$ has a Schwarzschild radius of about 10^9 km. Short time fluctuations and central engines involving gas dynamics close to the event horizon of BHs, fit together neatly.

Observational evidence for accreting super-massive BHs comes not only from short time variability of X-ray fluxes but also from the details of the continuum spectra (e.g. presence of the big blue bump in quasar spectra) observed in these active sources. Hence, quasars, blazars and powerful radio-galaxies are most probably distant galaxies housing acccreting supermassive BHs with mass in excess of $10^6\ M_\odot$ in their central regions [56].

Similarly, by monitoring stellar dynamics around the central region of Milky Way for decades, one infers that the Galactic nucleus contains a heavy and compact object, most likely to be a supermassive BH with a mass of about $4\times 10^6\ M_\odot$, within a radius of 10^{13} km from the Galactic Centre [57]. It is interesting to note that the Chandra X-ray observatory (launched on July 23, 1999, and named after S. Chandrasekhar) revealed the presence of a X-ray source as well as hot gas with high pressure and strong magnetic field in the vicinity of the Galactic Centre.

However, these are indirect detections, implying strictly speaking the presence of a very compact, massive central object. Inference of an astrophysical BH, although very likely, relies on theoretical interpretation. What happens when a BH is perturbed by incident gravitational waves or electromagnetic radiation or Dirac waves describing electrons or neutrinos? Does a perturbed BH have a signature emission like a 'ringing′, analogous to the case of a struck bell? To answer such questions, Chandra devoted himself to studying BH perturbations from 1970s onwards [54,60–67].

When a BH is perturbed, the curved space-time geometry around the BH will be subjected to metric fluctuations. For sufficiently small perturbations, a linear analysis of the metric fluctuations can be carried out in terms of normal modes except that dissipation due to both emission of gravitational waves as well as their absorption by the BH make the mode frequencies complex, with the decay reflected in the imaginary parts. In the case of a perturbed BH, such quasi-normal modes (QNMs) correspond to a characteristic ringing that eventually decays due to dissipation.

QNMs were discovered by Vishveshwara [58] and Press [59] while studying gravitational wave perturbations of BHs. Chandra and Detweiler suggested for the first time numerical methods for calculating the QNM frequencies [62] . Such investigations throw light on methods for direct detection of BHs. For example, matter falling into a Schwarzschild BH would lead to excitation of QNMs, resulting in emission of gravitational waves with a characteristic frequency that is inversely proportional to the BH mass.

One can understand this dependence from simple dimensional analysis. QNMs would involve perturbations of the event horizon characterized by the Schwarzscild radius R_s (eq.(5)). So, the oscillation wavelengths would be typically of a size proportional to R_s, making the frequencies depend inversely on the BH mass. A supermassive BH with mass $10^6\ M_\odot$ would ring with a frequency of about 10^{-2} Hz. Because of seismic noise, LIGOs cannot detect gravitational waves having such low frequencies. Only a space-based gravitational wave detector like LISA (Laser Interferometer Space Antenna) can pick up such low frequency signals from supermassive BHs [53,70].

Chandra developed innovative techniques to study BH perturbations, and showed that radial and angular variables could be decoupled to obtain separable solutions for Dirac equation in Kerr background, corresponding to a massive particle (like an electron) [62]. Using similar techniques, Don Page extended the separation of variables for massive Dirac equation to the Kerr-Newman case [68]. In 1973, Teukolsky had separated the Dirac equation for two component massless neutrinos in the Kerr background [69]. It will be interesting to investigate if Chandra′s technique can succeed in separating the Dirac equation for massive neutrinos (with flavour mixing and massive right-handed components included) in the Kerr or Kerr-Newman background.

Kerr BHs possess ergosphere, a region surrounding the event-horizon where test particles with negative angular momenta (i.e. with reverse sense of rotation relative to BH rotation) can have negative energy (as measured by a distant inertial observer) orbits. Penrose, in 1969, had shown an ingenious way to extract rotation energy of a Kerr BH that involved sending an object that breaks up into two in the ergosphere, with one of the parts going into a negative energy trajectory, while the other escaping with an energy greater than the initial energy (since energy is conserved) [71].

The wave analogue of Penrose process is superradiance wherein impinging scalar, electromagnetic or gravitational waves emerge out with greater energy after scattering off Kerr BHs. Zel′dovich was the first to show the existence of superradiance in 1970 [72]. Chandra and Detweiler undertook a thorough investigation of scattering of electromagnetic, gravitational and neutrino waves in the Kerr background, and showed that neutrinos do not exhibit superradiance [73]. Absence of neutrino superradiance is most likely due to PEP [73–76].

Exact solutions of two plane gravitational waves colliding with each other were obtained for the first time by Szekeres [77] as well as Khan and Penrose [78]. Their work showed that due to mutual gravitational focusing, the collision leads to curvature singularity where gravity becomes infinite. Chandra, along with Valeria Ferrari and Xanthopoulos, showed that the mathematical theory of colliding gravitational waves can be cast in the form of mathematical theory of BHs, and that the formation of curvature singularity due to gravitational focusing is generic [79–82].

In the later years, Chandra and Valeria Ferrari studied non-radial oscillations of rotating stars taking into account general relativistic effects [83–85]. They showed that the oscillations could be described in terms of pure metric perturbations, reducing the problem to scattering of gravitational waves in curved space-time geometry. For strongly gravitating objects like neutron stars, such gravitational waves may get trapped inside due to deep gravitational potential well, leading to trapped modes that survive for long durations.

In 1983, Chandra was awarded the Nobel prize in Physics. His method of studying diverse astrophysical topics involved applying physical theories that had been corroborated experimentally, and then subjecting the relevant equations to rigorous and innovative mathematical analysis. No wonder that most of the new results he obtained were later confirmed by observations.

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