The challenging concept of time in quantum mechanics

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Abstract

Time plays a peculiar role in quantum mechanics. What makes this concept so interesting is the question "what can quantum mechanics tell us the about the nature and role of time?" and conversely, "what can time tell us about the structure of quantum theory?" In standard quantum mechanics probabilities are assigned to measure the outcomes of a given observable at a given moment of time. Time enters the Schrodinger equation as an external parameter, and not a dynamical variable. It is not a standard quantum mechanical observable. But in concepts such as the time of arrival , the time-energy uncertainty relation , tunneling time , and time in quantum gravity , time can no longer be viewed as a mere parameter. This survey explores various attempts made in order to treat time as a dynamical variable (observable) and hence measure it.

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I. Introduction

What is time? This is a very old philosophical question. Even Einstein had a hard time answering this question, but in spite of that, we can measure time more accurately than any other quantity. Atomic clocks are the most accurate timepieces ever made, and are essential for such features of modern life as synchronization of high speed communication and the operation of the Global Positioning System (GPS) that guides aircraft, cars and so on."

The role of time is a source of confusion and controversy in quantum mechanics [1]. In the Schrodinger equation time represents a classical external parameter, not a dynamical variable. The time measured in experiments, however, does not correspond to an external parameter; it is actually an intrinsic property of the system under consideration, which represents the duration of a physical process; the life time of unstable particles is a well-known example. Quantum mechanics was initially formulated as a theory of quantum micro-systems interacting with classical macro systems [2]. Quantum mechanics allows the calculation of dynamical variables of systems at specified instants in time using the Schrodinger equation [3]. The theory also deals with probability distributions of measurable quantities at definite instants in time [4]. The time of an event does not correspond to a standard observable in quantum mechanics [5].

Asking the question of when a given situation occurs, time is no longer an external parameter. Time, in such a situation, becomes dynamical. However, such a time observable does not have the properties of a "standard" quantum mechanical observable. This research is dedicated to exploring various attempts made in order to treat time as a dynamical variable (observable). All attempts use essentially one of two approaches, namely those of direct and indirect the measurement of time. Direct approaches use theoretical toy model experiments while indirect approaches are of mathematical nature. The controversy of time arises in the time of arrival concept, the search for a time operator, the time-energy uncertainty relation and the tunneling time. A very important issue as well is the role of time in the context of quantum gravity. The problem of time in quantum gravity also opens the door to the ever-lasting question: what is time?

To determine the time of arrival or the tunneling time, the measurement of the required quantities must always be done, directly or indirectly. The notion of measurement emerges from interpretations of quantum mechanics, however the time problem arises in all of them. The interaction of a quantum Microsystems with a classical macro system is described in terms of quantum measurements [2]. As time is treated as an external parameter in standard quantum theory, quantum observation theory talks about observations made at given instants in time [3]. The system in standard quantum theory interacts with a measuring device through the time dependent interaction Hamiltonian. Quantum mechanics is actually designed to answer the question "where is a particle at time t?" In standard quantum mechanics, the probability corresponds to a measurement result of a particle being at a given location at one specific time. The above mentioned micro-system is taken to be in a superposition of states of its variables. Suppose the macro-system interacts with one of the micro-system's variables, then the macro-system only sees one of the many possible values of the variable [2]. The interaction itself projects the state of the micro-system into a state with the given value. In terms of wave-functions, the interaction (act of measurement) causes the wave-function of the Microsystems (a superposition of states) to "collapse" into one state with a specific value (eigenvalue). Dirac mentioned that the superposition is one of two most important concepts in quantum mechanics; the other one is Schrodinger equation [6]. Even though several alternative interpretations have been devised (Bohm, many- worlds, etc.), they all have one problem in common: how can the exact time at which a measurement occurs be determined? Rovelli [2] illustrates how the problem of time arises in each interpretation. If a system is viewed as having a wave-function which collapses during a measurement, is the collapse immediate? If a system is viewed in terms of values of its dynamical variables which become definite when observed, how to determine exactly when this occurs in an experiment? If a system's wave-function is taken a branch, when does this occur? If a wave-function does not branch and the observer selects one of its components and sticks with the choice, when does the selection occur? If there exists probability for sequences of events to happen, when does such an event occur? The above questions indicates the universality and challenging concept of time. In section II, The concept of time of arrival, in the context of quantum mechanics is discussed. The issue of time arises also in Heisenberg's time-energy uncertainty relation (section III). This relation has direct consequences to defining a time operator. Another important concept is tunneling time (section IV). It is purely quantum mechanical phenomenon with no classical analogue. Many of the ideas president in the context of the time arrival can be carried over to tunneling time. In section V, we review some attempts to set up the time operators. Last but not least the, problem of time in quantum gravity is outlined, where time, if it is a fundamental variable, must also be a dynamical variable. Quantum gravity has the interesting feature that the philosophical question of what time actually is raised (section VI). If one would know what time is fundamentally, then perhaps the problems encounter in determining time in quantum mechanics could be solved, as one would then know what one is actually looking for. In what follow, we only highlight the subjects, and to understand more each part needs to be explored in details. Another approach to the time problem is the decoherent histories approach to quantum mechanics [7, 8, 9, 10]. This formalism makes use of the fact that what one considers to be a closed quantum system, is never completely closed, as there always is an interaction present with the environment. The Brownian motion model is the main idea presented in this context [7].

II. Time of Arrival (TOA)

II. 1. Time of Arrival in Classical Mechanics

The basic question to answer is, can the exact time be measured at which a particle arrives at a specified point? Taking a beam of free particles, a measurement needs to be done to find the time of arrival at the specified point $x = x_1$. An experiment can be constructed which involves a clock positioned at the point $x = x_1$ This clock will turn itself off when a particle reaches x_1. In classical mechanics the time of arrival can be measured in this way with extreme accuracy, as the non-vanishing interaction between the particle and the clock is very small. The time of arrival can also be measured indirectly in classical mechanics. The equation of motion of the particle is inverted to get time as a function of location x and momentum p:

$$T_1 (x (t) , p (t) , x_1) \qquad (1)$$

This can be evaluated at any time *t* by measuring p(t) and x(t) simultaneously. Classically direct and indirect measurements are completely equivalent. Both methods give exactly the same result [11]. Muga et al [12] give an example of a particle moving in one dimension with position q and momentum p. The particle's trajectory might cross a given point X only once if no reflection mechanism is present. If a potential barrier is introduced, it can reflect the particle's trajectory and cause it to cross the point X more than once. The first passage time is defined as the first crossing of the trajectory of the point X. Considering ensembles of non-interacting particles, Muga et al state that a distribution of times is associated with the nth passage given by the nth crossing. A phase space distribution F(q,p,t) can be used to describe an ensemble of free moving, non-interacting particles. The distribution is normalised to one and is defined such that it only considers particles moving towards the right: $F (q,\ p \leq 0) = 0$. The particle trajectories cross the point X only once and a current density J(X, t) at X and time t (probability flux) gives the distribution of the first passage arrival times. Let J(X, t) dt be the fraction of particles which cross X between t and t+dt. Defining

$$J(X,t) = \int_{-\infty}^{\infty} F\ (X,p,t)\frac{p}{m}\,dp \qquad (2)$$

and the trajectory equation $q(t) = q_0 + \frac{pt}{m}$, the average time for free motion is given by

$$\int j(X,t)tdt = \iint F(q_0,p,0)\,\frac{(X-q_0)m}{\rho}\,dq_0dp$$

The integral is well-defined if F cancels the singularity.

II.2. Time of Arrival in Quantum Mechanics

The time of arrival problem in quantum mechanics arises by turning around the question: "at what time is the particle at a specified location?" Attempts to answer this question raise several problems which lead to ambiguous answers. The best way to illustrate this is through a simple example [5]:

Consider a N-particle ensemble. The aim is to measure the time at which a particle is located at the point x. A simple way of doing this would be to consider a detection process where the detector is switched on by each particle only at a given time t=T. Then this process is repeated on a second ensemble at $t = T^1$ and so on. The probability of finding the particle

$$\left|\psi(x,t=T)\right|^2 \text{ and } \left|\psi(x.T\right|^2 N = n_T \qquad (4)$$

is the average number of particles found at position x at time t =T. Unfortunately, (4) does not represent a probability as it is not normalised properly. To overcome this problem consider

$$\frac{\left|\psi(x.t\right|^2}{\int\left|\psi(x.t^1\right|^2 dt^1} \qquad (5)$$

However, to be able to use this equation, the state

ψ(x,t) must be known at all times in the past and in the future. The reason for the problem is that the particle could be at the point x at several times. For example, if the particle is found at point x at time t_l with probability one, it is not possible to say that the particle was not at x at all other times.
To try to overcome the above mentioned difficulty the measurement of the time of arrival of a particle at a given point seems a good candidate, as a particle can arrive only once at a given location. To measure the time of arrival of a particle, it must be possible to be able to detect it at a given point, as well as knowing that is was not there before the measurement takes place. This requires a continuous monitoring of the point of arrival. Now the problem arises that the probability of detecting a particle at a time t = t_l is not independent of detecting it at t = t_2. In mathematical terms the projections onto the arrival position x, denoted by the projection operator P_x, at given times t_l and t_2 will not commute:

$$[P_{\mathrm{x}}(\mathrm{t}_1), P_{\mathrm{x}}(\mathrm{t}_2)] \neq 0$$

This means that the measurements that are done at different times do not commute and disturb one another. This also means that (5) is not a probability distribution in time.

III. Uncertainty Principles

III.1. Introduction

In trying to change time, as the classical external parameter, into an observable, one cannot deduce the time-energy uncertainty relation:

$$\Delta t \Delta E \geq \frac{\hbar}{2}; \qquad (7)$$

where t = time , E = energy from kinematical point of view, as time does not belong to the algebra of observables [12]. In spite of this, (7) is generally regarded as being true. The relation (7), unlike other canonical pairs, is not the consequence of fundamental quantum in-complementarity of two canonical variables. The time-energy uncertainty relation is very different to the standard quantum uncertainty relation, such as the position momentum one. The precise meaning of the time-energy relation is still not exactly known. The problem lies in the fact that one cannot give the precise meaning to the quantity Δt . This is because time is not a standard quantum mechanical observable associated with an Hermitian operator. If such an operator canonically conjugate to the Hamiltonian did exist, then, t could be defined conventionally and the uncertainty principle could be applied to the physical quantity corresponding to the time operator.

III.2. Quantum Mechanical Uncertainty

In classical mechanics any quantity can be measured to an arbitrary precision In quantum mechanics the same is possible by preparing a quantum system in a well defined state of position and hence perform a measurement which reveals where the particle is located very accurately. The difference from classical mechanics arises when the values of two different observables are desired to be determined. In classical mechanics there is no reason why two quantities cannot be measured with high precision. In quantum mechanics only compatible (commuting) observables can be measured simultaneously. In general the uncertainties in measurements of two observables obey the uncertainty relation, which creates a lower bound on the product of the individual uncertainties, which is not equal to zero. For any two observables $\hat{A}$ and $\hat{B}$, their uncertainties

$$\Delta\hat{X} = \left(\langle\hat{X}^2\rangle - \langle\hat{X}\rangle^2\right)^{1/2}$$

are used to derive the uncertainty relation [13]:

$$\Delta\hat{A}\Delta\hat{B} \geq \frac{|\langle[\hat{A},\hat{B}]\rangle|}{2} \qquad (8)$$

An important fact that should be noted is that the uncertainty. $\Delta\hat{X}$ of an observable $\hat{X}$ is an intrinsic property of any quantum state .

III.3. Heisenberg's Uncertainty Principle

The uncertainty principle expresses the physical content of quantum theory in a qualitative way

[13]. The uncertainty principle was first proposed by Heisenberg in 1927. It basically states that is not possible to specify exactly and simultaneously the values of both members of a pair of physical variables which describe the behavior of an atomic system. In a sense the principle can also be seen as a type of constraint. The members of a pair are canonically conjugate to each other in a Hamiltonian way. The most well known example is the coordinate x of a particle (position in one dimension) and its corresponding momentum component P_x:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \qquad (9)$$

Another example is the angular momentum component J_z of a particle and the angular position ϕ in the perpendicular (x,y) plane:

$$\Delta\phi\Delta J_z \geq \hbar \qquad (10)$$

In classical mechanics these extreme situations complement each other and both variables can be determined simultaneously. Both variables are needed to fully describe the system under consideration. In quantum theory, Eqn. (9) states that one cannot precisely determine a component of momentum of a particle without loosing all information of the corresponding position component at a specific time. If the in- between extremes case is considered, the product of the uncertainty in position and the uncertainty in the corresponding momentum must numerically be equal to, at least, $\hbar/2$

To understand the physical meaning of the uncertainty principle, Bohr in 1928 stated the complementary principle. This principle shows the fundamental limits on the classical concept that a system's behaviour can be described independently of the observation procedure. The complementary principle states that "atomic phenomena cannot be described with the completeness demanded by classical dynamics" [13]. Basically the principle states that experimental apparatus cannot be used to determine a measurement more precisely than the limit given by the uncertainty principle. In a sense when a measurement is done to determine the value of one of a pair of canonically conjugate variables, the second variable experiences a shift in value. This shift cannot be calculated exactly without interfering with the measurement of the first variable

III.4. The Relation of the Uncertainty Principle to a Time Operator

Bohr also realized that the two uncertainty principles (9) and (7) can be interpreted in two different ways; the first is as limitations on the accuracy of a measurement and the second is as statistical laws referring to a large sequence of measurements. The difficulty in giving meaning to the relation (7) is due to the quantity Δt. In the way it is interpreted above, the uncertainty relation (7) implies the existence of a self-adjoint operator, canonically conjugate to the Hamiltonian $\hat{H}$, which itself is self-adjoint. If this time operator $\hat{T}$ exists, then the quantity Dt can be interpreted in the same way as Dx or Dp_x and the uncertainty principle can be applied to the physical observable corresponding to T. To obtain the uncertainty relation for energy and time, the commutator of the Hamiltonian and the time operator is assumed to be of the form:

$$[\hat{H},\hat{T}] = i\hbar \qquad (11)$$

The form of (11) is such that $\hat{T}$ and $\hat{H}$ are canonically conjugate to each other. It also implies that both operators have a continuous spectrum. This in turn means that neither of the two can be a Hamiltonian, as such an operator is defined to have a semi- bounded spectrum. From this line of reasoning the supposed time operator $\hat{T}$ cannot exist. This problem is encountered when one uses (11) to derive the uncertainty relation (7) in the same way as (9) is derived from $[\hat{x},\hat{p}] = i\hbar$ [14].

IV. Tunneling Time

In quantum tunneling, a part of a particle's wav-function has a significant probability of being transmitted a potential barrier, even if its energy is less than the energy of the top of the barrier [15]. This is not true classically, hence tunneling is a purely quantum phenomenon where a particle has the probability of moving through classically forbidden regions. The transmission probability of a particle's wav eigenfunction is calculated from the time-independent Schrodinger equation. The problems arise when time dependence is required. The root of the problem of time dependence lies in the uncertainty relation. However, the fundamental problem is that in quantum mechanics, there exist no real physical paths along which a particle moves. This problem seems a logical basis to employ the method of Feynman path integrals, which are virtual paths in configuration space [16]. The path integral approach to the tunneling time , yielding a complex time, is due to Sokolovski and Baskin [17] .

The question "how long does a particle spend a potential barrier?" has been controversial for many years [18] (for an extensive review see [19]. Part of the controversy lies in the fact that in tunneling processes only a particles tunnel, so one cannot discuss the entire ensemble. Over the years there have been many different approaches to calculate tunneling time; among them are the path integral approach, physical clock gedanken experiments, and consistent (decoherent) histories approach. There have also been attempts to use interactions of wave-packets with the barrier [15]. Yamada [16] studies the tunneling time problem using the decoherent histories approach to quantum mechanics to define the probabilities for histories. To minimize the interference, such that probabilities can be assigned to histories, Yamada uses the weak de coherence condition, where only the real part of the decoherence functional is required to vanish. Along with the decoherence condition, he [20] imposes that the initial condition satisfies the decoherernce condition as well.

Attempts to construct a Time Operator

Standard quantum theory, as proposed by Pauli [21], requires that measurable quantities (observables) are represented by self-adjoint operators, which act on the Hilbert space of physical states [4]. The probability distribution of the measurement outcomes of an observable are obtained as "an orthogonal spectral decomposition of the corresponding self-adjoint operator" [4]. The indirect measurement of time basically is the quest of finding a self-adjoint operator whose eigenstates are orthogonal. As the time operator is one of the canonically conjugate pair of time and energy, the time operator must be defined in such a way as to preserve the semi bounded spectrum of the Hamiltonian. Pauli pointed out [2 L] that the existence of a self-adjoint time operator is incompatible with the semi-bounded character of the Hamiltonian spectrum. By using a different argument based on the time-translation property of the arrival time concept, Allcock has found the same negative conclusion [22-24]. The negative conclusion can also be traced back to the semi-infinite nature of the Hamiltonian spectrum.

Kijowski [25] tried different approaches to find a time operator. He chose to (interpret the uncertainty relation (11) in a statistical way.

Grot, Rovelli and Tate [26] construct a time of arrival operator as the solution to the problem of calculating the probability for the TOA of a particle at a given point. They argue, using the principle of superposition, that a time operator T can be defined, whose probability density can be calculated from the spectral decomposition of the wave-function l. y(x) into eigenstates of $\hat{T}$ (in the usual way). They found an uncertainty relation which approaches (11) to arbitrary accuracy. Oppenheim, Reznik and Unruh [27] follow the method by Grot at al. They used coherent states to create a positive operator valued measure (POVM). The standard method to find an operator is by using the correspondence principle, which states that the corresponding classical equations are quantized using specific quantization rules. Taking the Hamiltonian of a classical system H(p,q) where p and q are canonical variables (H,T), where H is the

Hamiltonian and T is its conjugate variable. These variable satisfy Hamilton's equation:

$$\frac{dT}{dt} = \{H, T\} = 1 \qquad (12)$$

where T is the interval of time and the curly brackets denote a Poisson bracket. This relation can be translated to quantum mechanics through canonical quantization. This is a procedure where classical expressions remain valid in the quantum picture by effectively substituting Poisson brackets by commutators:

$$\{H, T\} = \frac{1}{i\hbar}\,[\hat{H}, \hat{T}\,] \qquad (13)$$

In the Heisenberg picture H and T are hence interpreted as self-adjoint operators. Further it seems natural to require that the time operator satisfies an eigenvalue equation in the usual way:

$$\hat{T}(t)|t_A\rangle = t_A|t_A\rangle \qquad (14)$$

In all of physics, except in General Relativity, physical systems are supposed to be situated in a three-dimensional Euclidean space. The points of this space will be given by cartesian coordinated **r** = (x, y, z). Together with the time parameter t, they from the coordinates of a continuous space - time background. The (3 + 1) dimensional space-time must be distinguished from the 2N-dimensional phase space of the system, and space-time coordinates (r, t) must be distinguished from the dynamical variables ($q_{k,}$ P_k) characterizing material systems in space-time.

A point particle is a material system having a mass, a velocity and acceleration, while r is the coordinate of a fixed point of empty space. It is assumed that three dimensional space is isotropic (rotation symmetric) and homogeneous (translation symmetric) and that there is translation symmetry in time. In special relativity the space-time symmetry is enlarged by Lorentz transformations which mix **x** and t, transforming them as the components of a four-vector.
The generators of translation in space and time are the total momentum P and the total energy H, respectively. The generator of space rotations is the total angular momentum **J**.

It is worth noting that the universal time coordinate t should not be mixed with dynamical position variables. The important question to ask is: Do physical systems exist that have a dynamical variable which resembles the time coordinate t in the same - way as the position variable q of a point particle resembles the space coordinate **x**? The answer is yes! Such systems are clocks. A clock stands, ideally, in the same simple relation to the universal time coordinate t as a point particle stands to the universal space coordinate **x**. We may generally define an ideal clock as a physical system which has a dynamical variable which behaves under time translations in the same way as the time coordinate t. Such a variable, which we shall call a "clockvariable" or, more generally, a "time-variable", may be a pointer position or an angle or even a momentum. Just as a position-variable indicates the position of a system in space, a clock-variable indicates the 'position' of a system in time t. In quantum mechanics the situation is essentially not different. The theory supposes a fixed, unquantized space-time background, the points of which are given by c-number coordinates **x**, t. The space time symmetry transformations are expressed in terms of these coordinates.

Dynamical variables of physical systems, on the other hand, are quantized: they are replaced by self-adjoint operators on Hilbert space. All formulas of the preceding section remain valid if the poisson-brackets are replaced by commulators according to

$$\{,\} \rightarrow (i\hbar)^{-1}\,[\,,].$$

So, the idea, that t can be seen as the canonical variable conjugate to the Hamiltonian, leads one to expect t to obey the canonical commutation relation [t,H]= $i\hbar$. But if t is the universal time operator it should have continuous eigenvalues running from - ∞ to + ∞ and, from this, the same would follow for the eigenvalues of any H. But we know that discrete eigenvalues of H may occur.

From this Pauli concluded [21]: ... that the introduction of an operator t is basically forbidden and the time must necessary be considered as an ordinary number ("c-number"). Thus, the 'unsolvable' problem of time in quantum mechanics has arisen. Note that it is crucial for this argument that t is supposed to be a universal operator, valid for all systems: according to Pauli the introduction of such an operator is basically forbidden because some systems have discrete energy eigenvalues. From our previous discussion it should be clear that the universal time coordinate t is the partner of the space coordinates x. Neither the space coordinates nor is the time coordinate quantized in standard quantum mechanics. So, the above problem simply doesn't exist! If one is to look for a time operator in quantum mechanics one should not try to quantize the universal time coordinate but consider time-like (in the literal sense) dynamical variable of specific physical system, i.e. clocks. Since a clock-variable is an ordinary dynamical variable quantization should not, in principle, be especially problematic. One must, however, be prepared to encounter the well-known quantum effects mentioned above: a dynamical system may have a continuous time-variable, or a discrete one or no time-variable at all. Recently, some efforts have been advanced to overcome Pauli's argument [28]. The proposed time operator is canonically conjugate to $i\hbar\partial$ rather than to H, therefore Pauli's theorem no longer applies. It is argued that "the reasons for choosing time as a parameter lie not so much in ontology as in methodology and epistemology. The time operator idea needs to be more explored in an accurate way.

VI. The problem of Time in Quantum Gravity

VI. l. The basic problem

The problem of time is a fundamental concept that needs to be considered in quest for a consistent theory of quantum gravity [29]. The main issue contributing to the problem of time is the invariance of classical general relativity under space-time diffiomorphism. This means that the invariance contradicts the Newtonian image of a fixed, absolute time parameter. This situation is encountered in all theories, which have a classically invariant, reparametrization of time [30]. This leads to time disappearing when quantizing the theory. This situation comes out to the question of what to make of the classical Hamiltonian constraint in the quantum version of the theory. Basically, if time is a fundamental concept, then it must be a dynamical variable of the theory.

As stated above, when quantizing general relativity in a canonical way, time seems to have no fundamental notion. Isham [29] summarizes the key points in four statements: 1. How should time re-enter quantum gravity theory? 2. Should time be defined classically before quantization? Or 3. Should it be defined after quantization? 4. If time is not the fundamental quantity, which it is said to be, how relevant is quantum mechanics when dealing with time? The definition of time also has a direct effect in quantum cosmology, The main problem is the Newtonian concept of time, which is replaced by the concept of an internal time in many approaches of problem of time in quantum gravity.

The problem of time in quantum gravity arises when one wants to quantize general relativity. The canonical quantization method involves expressing general relativity in Hamiltonian from, to then apply a quantization scheme. General relativity is a theory with constraints, which generate asymmetry. Such theories are invariant under the reparametrization of time. The action of such a system is invariant under canonical transformations.

One can proceed to quantize general relativity in Hamiltonian from using Dirac's proposal and functional Schrodinger quantization to find all the information contained in the constraints. Using the metric representation of the wave-function, one obtains the Wheeler- DeWitt equation. This equation shows the absence of time. One way to interpret this is to consider that as general relativity is a parametrized theory, physical time is already contained amongst the dynamical variables. All such theories have H = 0. Alternatively, there exists a geometric interpretation.

VII. Conclusion and further comments

This survey explores various ways of defining time in standard quantum mechanics and some different ways of measuring it. The approaches of measuring time yield a whole spectrum of results along with a range with a range of difficulties encountered. All methods yield results which have a strict limit on their accuracy and generality. This reflects the quantum nature of the problem.

The main difficulty in defining a quantum time operator lies in non-existence, in general, of a self-adjoint operator conjugate to the Hamiltonian, a problem which can be traced back to the semi-bounded nature of the energy spectrum. In turn, the lack of a self-adjoint time operator implies the lack of a properly and unambiguously defined probability distribution of arrival time. There are two possibilities to overcome the problem. If one decides that any proper time operator must be strictly conjugate to the Hamiltonian, then one has to perform the search for a self-adjoint operator. If, on the contrary, one imposes self-adjoint property as a desirable requirement for any observable, then one necessarily has to give up the requirement that such an operator be conjugate to the Hamiltonian. The two main equations of motion, the Schrödinger and wheeler- Dewitt equation, reflect two different presupposed natures of time: in the schrödinger equation, time corresponds to an external parameter and in the Wheeler-deWitt equation there is no time. This research explores the concept of trying to tum a time parameter into an observable, a dynamical variable. Why was time in quantum mechanics represented by a parameter in the first place? A possible answer is that it is due to our perception. It is meaningful, for us, to talk about events happening at a certain time. This lets us put events into a chronological order in our minds. We do not think about an event happening to us. Another question is, why does one want time to be an observable? One major reason is our notion of change: we seem to perceive that time changes. Another motivation for the study of time in quantum mechanics is the problem of time in quantum gravity. Quantum cosmology represents an analogy to closed quantum systems, as both cosmology and closed quantum systems are describing the same type of situation, the difference being the size scale. . Saunders states: "quantum cosmology is the most clear-cut and important failing of the Copenhagen interpretation" [31]. Perhaps the lack of understanding of time in quantum gravity is due to a fundamental reason, based on the two quantum gravity components: quantum mechanics and general relativity. The problem does not lie in general relativity however, so it must be rooted in the formulation of quantum mechanics.

Quantum theory of measurement is based on measurements occurring at given instants of time. A measurement corresponds to a classical event. Dirac said "the aim of quantum mechanic was to account for the observables: behavior in the simplest possible ways" [6]. Kant [32] held Newtonian absolute space and space- time for an "idea of reason". Saunders states" In particular, we need a global time coordinate' which enters in to the fundamental equations; it is no good if this involves ad hoc or ill-defined approximations, available at only certain length scales or cosmological epochs" [33]. His idea of a universal definition of time sounds very appealing. Does this universal concept of time require the reformulation of quantum mechanics? Tunneling time might .also be a candidate to shed some light on to the mystery of time. Quantum mechanical tunneling is "one of the most mysterious phenomena of quantum mechanics" and at the same time it is one of the basic and important processes in Nature, party responsible for our existence [34]. The question of the duration of a tunneling process is an open problem. Experiments to record the tunneling time were motivated by the many different theories trying to describe this phenomenon. Questions arise such as, "is tunneling instantaneous?","is it subluminal or superluminal (faster than the speed of light)?" Chiao published a paper with experimental evidence that tunneling is superluminal [34]. If this is true, what implications does superluminal tunneling have on our understanding of the nature of . time? What does it mean to say that something happens faster than instantaneously?

Does time undergo a change in nature when it "enters" a classical forbidden region? If so, what is it and what does it change to?

In quantum gravity, the evolution of the gravitational field does not correspond to evolution in physical time. The internal time on a manifold i~ not ':l:~ absolute quantity. Barbour [35] claims that an instant in time corresponds to a configuration and Deutsch, in his interpretation of quantum mechanics, claims that a change in time corresponds to a change in his interpretation of quantum mechanics, claims that a change in time corresponds to a change in the number of Deutsch worlds [37]. Is it possible that the notion of absolute time be a hint towards timelessness? If time does not exist then the various different formulations of the nature of time only appear through our perception and we cannot follow these back to a universal truth. Perhaps there does exist a universal concept of time, which is far too abstract to grasp. Whatever time may be, the time discussed in this overview raise various questions, which perhaps are trying to point us into .a certain direction. Trying to answer questions about the concepts of the time of arrival, the time-energy uncertainty relation, tunneling time and time in quantum gravity show us that perhaps nothing is more important than to first of all understand the basic building block-time-without which no structure can be perfectly built. The problem of time still stands to be resolved, the quest for this research still continues.

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