Increasing student understanding of Spectroscopy and Hertzsprung-Russell Diagrams

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Abstract

In this article the role of spectroscopy in understanding Hertzsprung-Russell diagrams is discussed. Intuitive methods of calculating equivalent and line widths of spectral lines are first presented. This is followed by a discussion justifying the use of line widths in temperature-luminosity diagrams.

1. Introduction

Among the more crucial topics in introductory astronomy are spectroscopy and Hertzsprung-Russell Diagrams (HRDs). Spectroscopy, the study of the light after passing through a prism or grating, reveals a wealth of information about the source including its temperature and composition; knowledge about some of these can only be obtained from spectroscopy. Introductory astronomy texts generally list spectroscopy in the beginning after a discussion of light, but its true application is revealed when studying properties of stars, including our sun. HRDs also come up when studying stellar properties, more specifically during a discussion of stellar life cycles. A HRD is a plot of, typically, luminosity and temperature of stars. Information about sizes and spectral classes of stars are some of the information obtained by analyzing HRDs. The aim of this article is twofold, firstly to introduce the concepts of equivalent width (EW) or the strength of a spectral line (including an easy measurement method using MS Excel) and line width (LW). Spectral width measurements are important because, for one, they allow us to determine the physical conditions in the atmospheres of stars. Moreover they let us quantify spectral lines and are rarely discussed in introductory texts. Simple algebraic calculations are employed to calculate the area under a curve and provide an intuitive way to calculate the EW. The article concludes with a discussion about the relationship between spectral features (i.e. EW & LW) and the HRD: that the HRD is actually a representation of the spectral features[1, 2, 3]. It emphasizes that the spectral features are actually dependent on the temperature and luminosity of a star-a critical concept that forms the physical basis for understanding HRDs.

2. Calculation of Line Widths

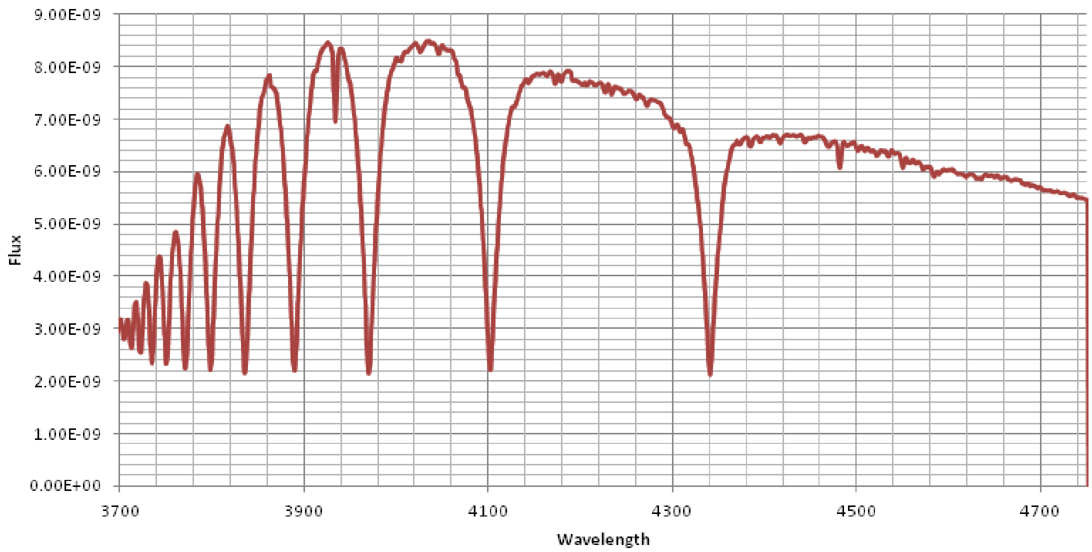

Figure 1: Sample spectra data obtained from STELIB and plotted using MS Excel showing the absorption features. Flux is expressed in solar luminosities per Angstrom.

Fig. 1 obtained using STELIB[4], shows a sample spectrum with flux plotted along the vertical axis and visible spectrum wavelengths along the horizontal. The dips in the plot are the absorption lines with the number of photons absorbed at a certain wavelength governing its shape.

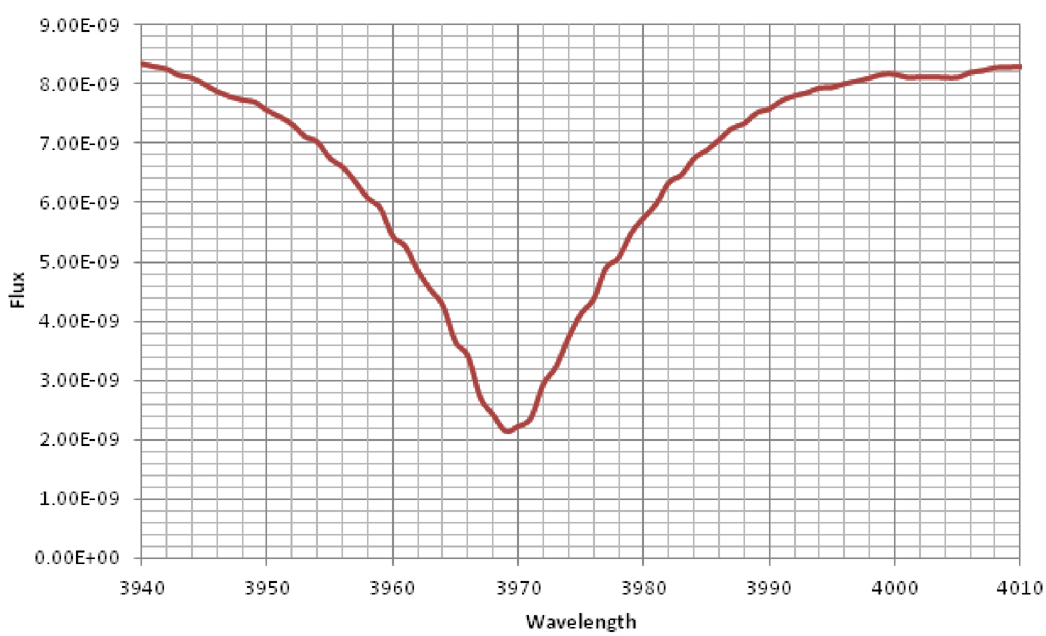

Figure 2: Same spectra as Fig. 1 but zoomed into the 3940-4010 A wavelength range. The equivalent width is calculated centered on 3969 A.

Fig. 2 is a snapshot of Fig. 1 in the wavelength range 3940A to 4010A. The choice of wavelengths is arbitrary and is just for the purpose of measuring the equivalent width of this particular absorption feature. Note that the peak wavelength occurs at about 3969A around which the EW would be measured. EW is defined as the width of the rectangle extending from the continuum to zero flux that has the same area as that enclosed by the spectra between the absorption and continuum lines. There are different ways of achieving this (apart from using calculus) and the methods mentioned herein offer a good approximation. More importantly, it gets the measurement idea across without employing any complicated mathematics. One way is to print the plot and count the number of boxes between the absorption and continuum line. A rectangle comprising of the same number of boxes could then be drawn around the peak wavelength extending from zero flux to the continuum, whose width would be the EW of the spectral line. A second method involves approximating the area in question by a triangle and calculating its area. Both of these methods are a visual way of calculating the EW and is therefore very appealing.

A third and more accurate method involves dividing the area under the curve into trapeziums[5]. The sum of the areas of the individual trapeziums is the area in question. This is illustrated in Fig. 3. For example, the area of the trapezium labeled 1 is calculated using the formula,

$$\text{Area of a trapezium} = \frac{1}{2} \text{ X sum of parallel sides X distance between the parallel sides.}$$

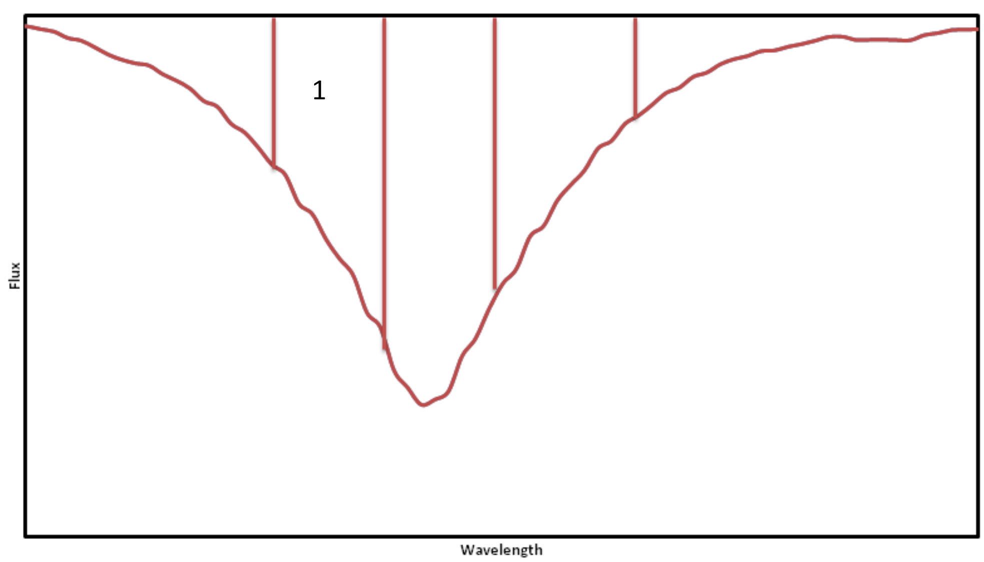

Figure 3: Calculating the area under a curve by dividing it into a number of trapeziums.

As an example, in Fig. 4, the area of the trapezium in the wavelength range 3950A-3952A is 0.22A. This process is automated using Excel until all trapeziums have been taken into account. Next, a rectangle centered on the peak wavelength is drawn to match the area calculated previously. Measuring the width of this rectangle is the EW of the line, as shown in Fig. 4. In this example, the

EW is approximately 15.5A around 3969A (as shown by the double headed arrow).

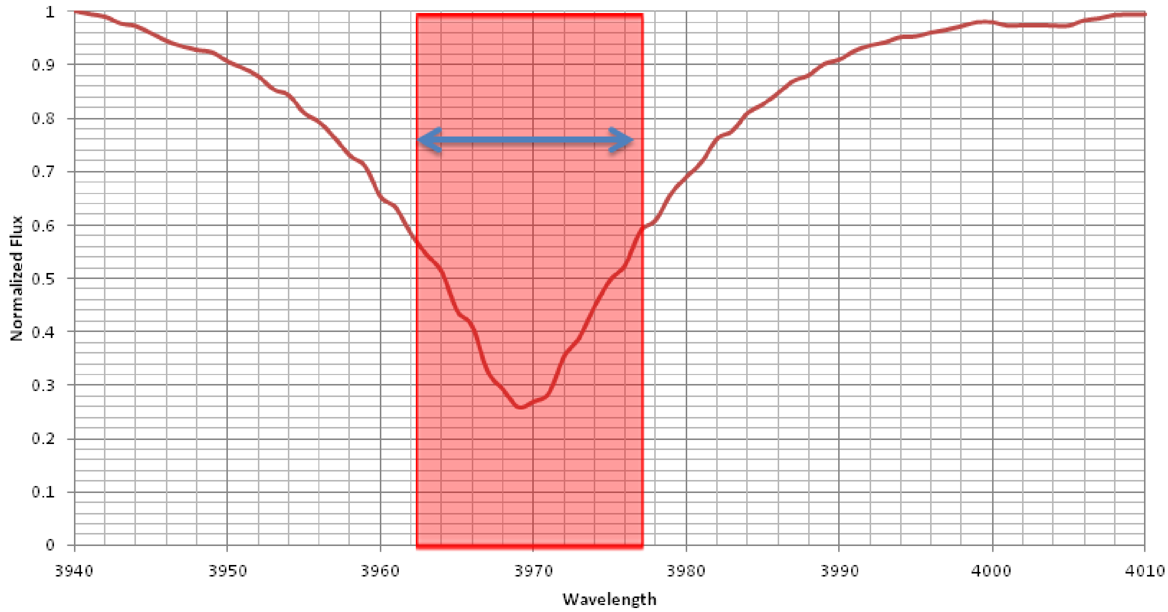

Figure 4: Shaded rectangle represents the equivalent width of the absorption line centered on 3969A. Note the vertical axis represents normalized flux.

Another way of quantitatively expressing the properties of a spectral line used by astronomers is LW. This is best outlined using an example. The solid and dashed spectral lines in Fig. 5 could have the same EW (i.e. same area under the each curve), but differ in the thickness around the central region. More precisely the width is measured at half the maximum vertical extent of the plot. This is a measure of the LW and it typically denoted by $\Delta\lambda$ in advanced literature. For a more technical definition of LW refer any astrophysics text[2].

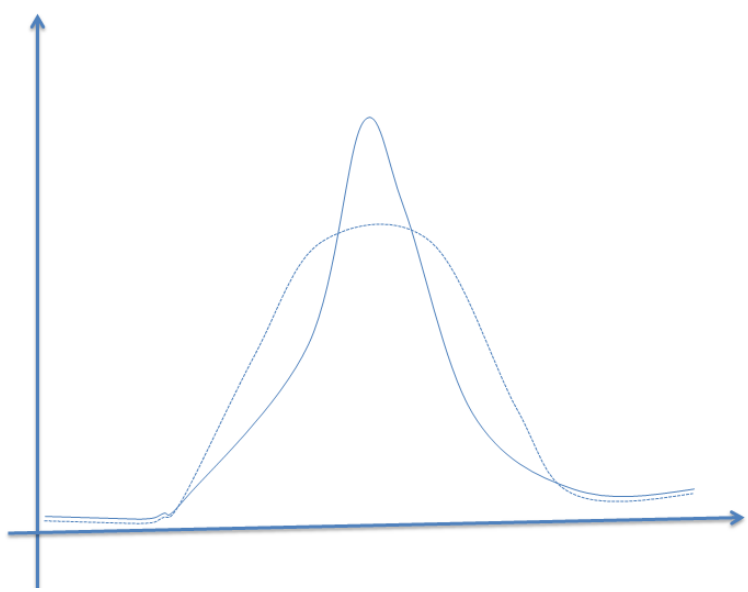

Figure 5: Solid and dashed spectral lines assumed to have the same equivalent width differ in line widths. Flux (wavelength) is along the vertical (horizontal) axis.

3. Relating Line Widths to HR Diagrams

We now proceed to relate spectral features (EW and LW) to our two dimensional HRD. The aim here is to reason how the spectral features are dependent on the temperature and luminosity. It starts with a simple question, how can we justify a plot of luminosity vs. temperature? The role of temperature in spectral line formation is well understood and appreciated by students. The line strength is dependent on the number of atoms making a certain atomic transition. The equations governing which atoms occupy a certain level and the number of ions formed are due to Boltzmann and Saha[2,3]. Together they provide an accurate description of the variation of line strength with temperature. The strength of the hydrogen Balmer lines, for example, peaks at 10^4K and decreases with either an increase or decrease in temperature. A lower temperature suggests not enough of the species available for making transitions and a rise in temperature favors ion formation, both of which are detrimental to formation of Balmer lines. Thus, the temperature dependence of the spectral features is established.

The other, not so much touted, factor that affects spectral line formation is the density of atoms, which other than being temperature dependent is also pressure dependent. It is also dependent on the composition of the star but we could ignore that for our purposes (by considering; for example, only Population I or II stars). Spectral lines vary in width with natural and Doppler broadening being the most important factors. Another type of broadening is known as pressure broadening. Spectral lines of two stars having the same temperature might not have the same width. A main sequence, giant or dwarf star of the same spectral type does indeed have different line widths. This is because a giant has a much less denser atmosphere (larger size) than a main sequence star. Low density in the photosphere implies less collisions with atoms (or ions) leading to less perturbations in the energy levels and hence less broadening of the spectral lines. As a result, spectral lines for the main sequence star would be broader than the corresponding giant.

4. Conclusions

Thus, other than the temperature dependence of spectral lines, the above discussion also suggests that stars with large surface area (and therefore brighter) have different widths than main sequence stars with the same temperature. Thus, the spectral features are also dependent on luminosity. Therefore, measurements of spectral features illustrate the role of spectroscopy in revealing the temperature and luminosity of stars, and their evolutionary status on an HRD.

References:

[1]. An Introduction to the Sun and Stars. Edited by Simon F. Green & Mark H. Jones. (Cambridge University Press).

[2]. Bradley W. Carroll & Dale A. Ostlie, An Introduction to Modern Astrophysics, 2nd Edition (Benjamin Cummings).

[3]. Stephen A. Gregory & Michael Zeilik, Introductory Astronomy and Astrophysics (Brooks Cole).

[4]. STELIB website, http://www.ast.obs-mip.fr/article181.html

[5]. A Simple Tool for Integration and Differentiation of Tabular Values in Microsoft Excel by Ole Anton Haugland, The Physics Teacher, December 2011, Volume 49, Issue 9, pp. 580.