The RC circuit experiment

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Abstract

The RC circuit experiment reveals critical aspects about the systematic perturbation introduced in a circuit by an instrument. A pedagogical value experiment is suggested.

1. Introduction

The experiment on the charging and discharging of a capacitor through a resistor has been, to date, been presented by the author using a square wave generator and an oscilloscope. The Analog oscilloscopes employed and the low time constant of the circuits did not pose particular critical aspects. A typical setup was an RC circuit whereby R = 100 Ω and C = 1 μF, a square wave generator with a frequency of 500 Hz and time base 0.2 ms/cm on the oscilloscope. It was only towards the end of his teaching career, that the author discovered and appreciated the versatility of dataloggers. In re-proposing the RC experiment the role of the meter resistance becomes relevant, as highlighted in Literature [1-4].

This paper describes a critical presentation of the experiment in order to concretely introduce the concept that every instrument inserted in a circuit can introduce systematic perturbations on the measurement.

2. The experiment.

Two RC circuits with the same *time constant* τ =RC are mounted on two Plexiglas sheets visually illustrating the schematics and components to the students. One of the two circuits is made using a 1 MΩ resistor and a 1 μF polyester capacitor and the other, using a 100 Ω resistor and a 10000 μF electrolytic capacitor. A switch connects the RC series to the battery (charging phase) or shorting C on R (discharging phase). A packet of four AA rechargeable batteries in series simplifies the setting up of the experiment. In both cases the time constant is one second, which is quite a long time when compared with the experiment on the oscilloscope. The experiment can be presented starting with the first circuit and a datalogger. Using a DS1M12 datalogger, the charge and the discharge of the capacitor are easily shown. The Datalogger DS1M12 has two input channels and its software allows the use of one or both channels, as an oscilloscope or as datalogger. In this experiment, a single channel is used. The output of the datalogger enters the USB port of a Notebook containing Software and the drivers to the OS system used. In our case, Windows 7 and a Notebook with a well visible screen was used for a demonstration experiment. The Instructor can show the voltage across the capacitor or the resistor. A screen snapshot can then be printed, as a record for the students, another advantage of dataloggers over the use of an ordinary CRO for this experiment. Indeed, the use of a datalogger for a classroom experiment means that the instructor need only choose the sample interval and the interval on the *x* axis in order to have the charge and discharge process in a single window. The Instructor may call student's attention on the fact that any instrument in a circuit introduces a systematic perturbation (the real circuit is shown in Fig. 1). With our datalogger a sample interval of 200 ms and a *x* width of 20 s (or more) gives satisfactory results as shown in Figs. 2 and 3 and the above parameters appear in the figures. Another feature of the software is that the *x* and *y* parameters can be changed even after file has been saved. An attentive student can recognize that the voltage drop across the capacitor in the first circuit (R = 100 Ω, C = 10000 μF) is lower than the value V_o measured across the battery (V_o = 4.85 V) as shown in Fig. 4. Ask your students why this happens. The experiment also shows how a capacitor does not have a perfect electrical isolation. Using the second circuit (R = 1 MΩ and C = 1μF) the data logger shows a voltage drop at the ends of C which is approximately *half* of the voltage across the battery. There are two reasons. The ohmic resistance across the 1μF capacitor *is not negligible* and measured with an Ohm-meter gives a value around 1 MΩ while in our 10000 μF capacitor, the resistance is greater than 20 MΩ. It should not be difficult for the student to explain why the voltage across the capacitor is about half that across the battery. This analysis must take into account the internal resistance of the voltmeter used i.e. the input impedance of our datalogger. Following these qualitative observations the true circuit equation can be written. Using Kirchhoff's laws applied to the equivalent circuit in Fig. 1, the resistance of our voltmeter R_v (datalogging input) is taken into account [4].

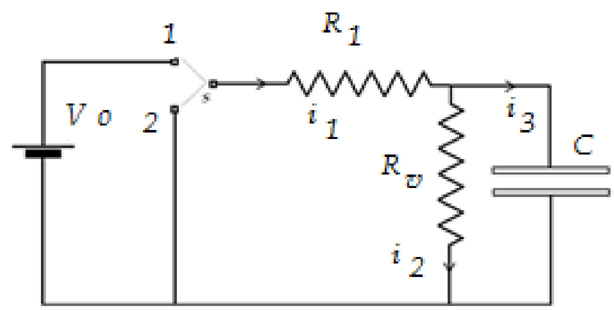

Fig. 1. The experimental setup. A little notebook and a low cost DS1M12 oscilloscope/datalogger is used. As D. C. source, four AA 1.2 V rechargeable batteries in series were used. The only practical caution is the use of a good quality switch.

Still applying Kirchhoff laws and referring to the notations in Fig. 2 we have

$$\begin{cases} i_1 = i_2 + i_3 \\ V_o - R_1 i_1 - R_v i_2 = 0 \\ R_v i_2 - \dfrac{1}{C}\displaystyle\int_0^t i_3 dt = 0 \end{cases} \qquad (1)$$

Substituting the first equation in the second and in the third equation we find the relation

$$\frac{R_v}{R_1+R_v}V_o-\frac{R_1R_v}{R_1+R_v}i_3=\frac{1}{C}\int_0^t i_3dt \qquad (2)$$

Differentiating this equation it follows a linear equation

$$\frac{R_1R_v}{R_1+R_v}\frac{di_3}{dt}=\frac{1}{C}i_3 \qquad (3)$$

Whose solution

$$i_3=A\exp\left(-\frac{t}{\tau}\right) \qquad (4)$$

where A is a constant to be determined from the condition at the time t = 0 and

$$\tau=\frac{R_1R_v}{R_1+R_v}C \qquad (5)$$

In the discharging process $A = V_o/R_p$ where R_p is

$$R_p=\frac{R_1R_v}{R_1+R_v}$$

In the charging process a particular integral must be added so that, in the charging process the equation is, as follows:

$$i_3=\frac{V_o}{R_p}\left[1-A\exp\left(-\frac{t}{\tau}\right)\right] \qquad (6)$$

Aside from the analytical treatment, an interesting feature of a datalogger is an experimental estimate of the characteristic time. This can be made printing the screen after an opportune choice of *x* and *y* parameters has been made. A young student can have fun with DS1M12 software and its features, an old teacher, such as the Author, can print the diagram on paper and evaluate the characteristic time with a pencil, having previously established an accurate scale factor on the print. The estimated measurements are: First circuit: data in Fig. 2. R =1 MΩ rand C = 1 μF

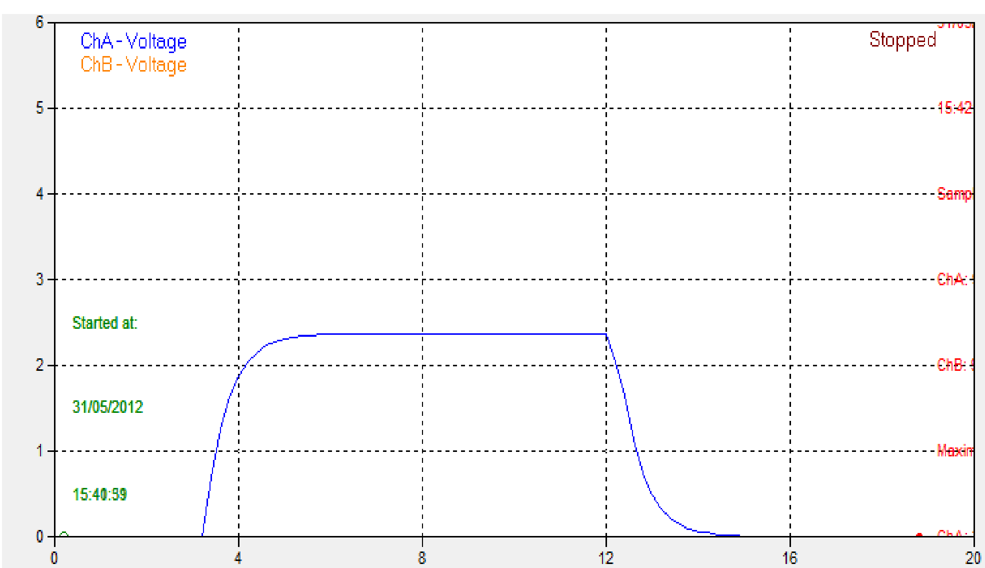

Fig. 2. Charging and discharging of the circuit having R =1 MΩ rand C = 1μF.

where the uncertainty on τ was estimated using "1 mm resolution" on the rule used in printed diagram. Supposing that R and C are without uncertainties, Equation 5 gives $R_v \approx 1.5$ MΩ in the correct magnitude order with a measurement of the capacitor *ohmic loss* through an Ohmeter (R = 1 MΩ ± 0.05 MΩ).

Second circuit: data in Fig. 3. R =100 Ω and C = 10000 μF: τ ≈ 1.1 s ± 0.1 s

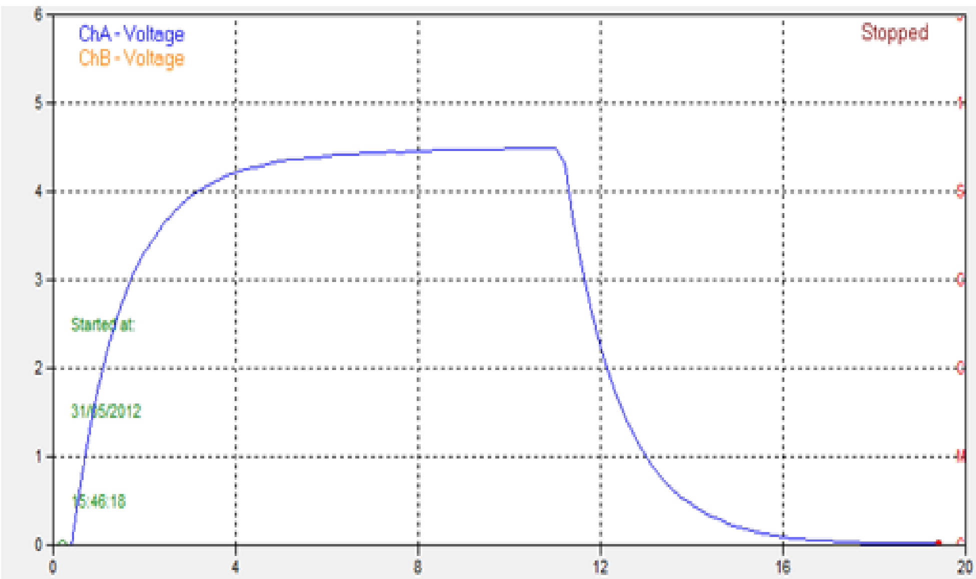

Fig. 3. Charging and discharging of the circuit having R =1 MΩ rand C = 1μF.

Within uncertainties evaluated, no perturbations appeared in this measurement. No ohmic loss minor of 20 MΩ is detected by a good ohmmeter. Student can use Eq. (5) in order to evaluate R_v finding a negative value if this slightly over 1 s value is inserted in Eq. (5). We leave to an attentive student the following matter: "why equilibrium voltage at the ends of the capacitor in Fig. 3 is minor than voltage across the battery measured by the datalogger and shown in Fig. 4?

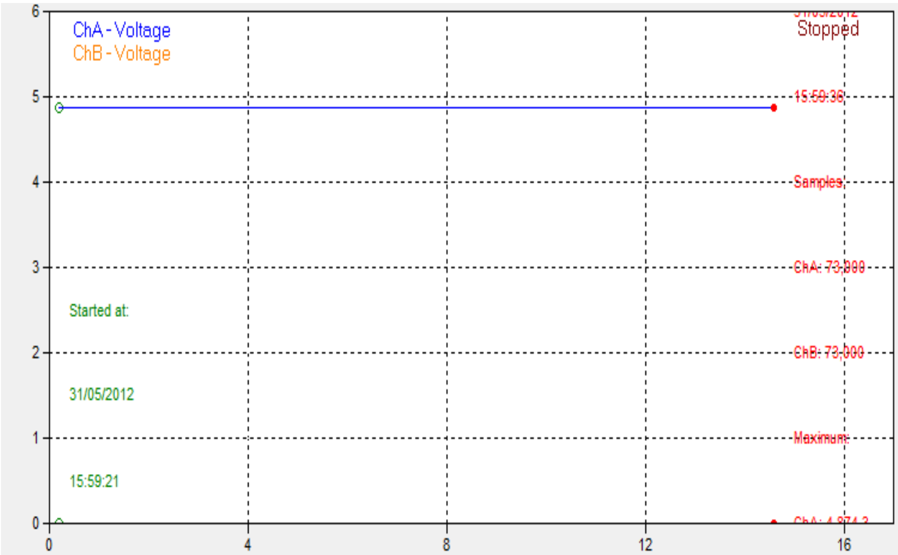

Fig. 4. Measured voltage across the battery. Datalogger and a good voltmeter gives values in agreement within the uncertainty of the voltmeter used (a good electronic instrument)

Can we *estimate* the impedance of our datalogger at work? Or in the diagram in Fig. 3 a long time were required for a *complete* charge?

3. Conclusion.

A matter usually emphasized as a good use of the mathematical language in General Physics shows surprising matter of discussion if a teacher shows it in real time with the classroom lesson. As shown, the experiment appears more interesting because quantitative estimate of time constant becomes possible. Nowadays students are more experienced with notebooks and Lab software than with classic Lab instruments, a good link between an old experienced teacher and a young attentive student.

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