Simulation of three-level lasing system using Cuboctahedron

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Abstract

Einstein's constants and its importance in understanding the principles of laser technology are best understood in this play-learn mode. Dice throwing is a pedagogical tool for learning probabilities. The aim of the experiment is

- to determine Einstein's constants for a lasing system.
- to find life time of metastable state and excited state of an atom.
- to show why 3-level lasing system is preferred to two level lasing systems.

Keywords: Einstein's constants, Population inversion, optical pumping, lifetime, two level lasing system, dice throwing, cuboctahedron, three level lasing system, metastable state.

1. Introduction

Dice rolling is being used as a pedagogical tool in schools as well as undergraduate studies in physics, mathematics, statistics and computer science curriculum. Students learn while they play. Todd W. Neller et al [1] has used dice rolling in a dice game Pig for undergraduate research in machine learning. Arthur Murray et al [2] mention that "the 'radioactive dice' experiment is a commonly used classroom analogue to model the decay of radioactive nuclei". Simple dice rolling can unfold important concepts elegantly, in fact, this experiment explains all the keywords mentioned above.

2. Dimensional Analysis

The Einstein constant A is defined as the *probability per unit time*. And the product of the Einstein constant B and the energy density per unit volume u of the stimulant is the *probability per unit time* (refer to Eqs. (3) and (1) respectively)

Rolling the dice and watching for a particular "face up" gives the *probability per throw*, and therefore can represent the Einstein's constants A and Bu.

Thus, simulation of dice with atoms, time with throw and three faces of the dice with the three processes help in understanding the lasing operation.

3.Theory

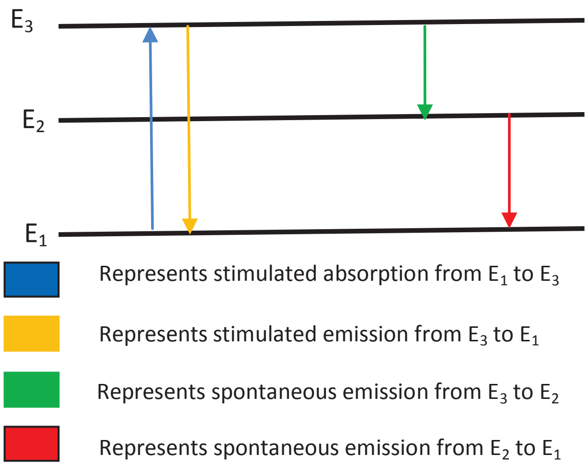


Fig **1. Three level system**

For a three level system with population N_1, N_2 and N_3 in the energy levels E_1, E_2 and E_3 respectively and a stimulant of energy density $u(\omega)$ where $\omega = \frac{E_3 - E_1}{\hbar}$,

the number of stimulated absorptions per unit volume per unit time from level 1 to 3 is

$$\frac{dN_3}{dt} = -\frac{dN_1}{dt} = N_1 B_{13} u(\omega) \qquad (1)$$

The number of stimulated emissions per unit volume per unit time from level 3 to 1 is

$$\frac{dN_3}{dt} = -N_3 B_{31} u(\omega) \qquad (2)$$

The rate of spontaneous transitions (per unit volume) per unit time from level 3 to 2 is proportional to N_3 and is given by

$$\frac{dN_3}{dt} = A_{32} N_3 = -\frac{N_3}{t_{sp3}} \qquad \ldots\ldots\ldots\ldots(3)$$

The rate of spontaneous transitions (per unit volume) per unit time from level 2 to 1 is proportional to N_2 and given by

$$\frac{dN_2}{dt} = -A_{21} N_2 = -\frac{N_2}{t_{sp2}} \qquad (4)$$

The **life time** of the level 2 is

$$t_{sp2} = \frac{1}{A_{21}}$$

and that of th

e level 3 is

$$t_{sp3} = \frac{1}{A_{32}}$$

The quantities **B_{31}, B_{13}, A_{32}** and **A_{21}** are known as **Einstein's constants** and depends on the atomic system [3].

Thus, the rate equation for level 3 is,

$$\frac{dN_3}{dt} = -B_{13}(N_3 - N_1) - N_3 A_{32} \qquad (5)$$

the rate equation for level 2 is,

$$\frac{dN_2}{dt} = N_3 A_{32} - N_2 A_{21} \qquad (6)$$

the rate equation for level 1 is,

$$\frac{dN_1}{dt} = N_3 B_{31} + N_2 A_{21} - N_1 B_{13} \qquad (7)$$

For a system where $A_{21} < B_{13} = B_{31} < A_{32}$ (or $t_{sp3} < t_{sp2}$), at equilibrium,

$$\frac{dN_3}{dt}=0 \;\; therefore \;\; N_3=\frac{B_{13}N_1}{A_{32}+B_{31}}\approx\frac{B_{13}N_1}{A_{32}} \qquad (8)$$

$$\text{and} \;\; \frac{dN_2}{dt}=0; \; therefore \; N_2=\frac{A_{32}N_3}{A_{21}} \qquad (9)$$

Or using Eq(8) we get

$$N_2\approx\frac{B_{31}N_1}{A_{21}} \qquad (10)$$

Since $B_{13} > A_{12}$, N_2 should be greater than N_1

4. Experimental procedure

4.1 Einstein constants

The cuboctahedron (dice) used is shown in Fig 2. Yellow face 'up' represents spontaneous emission from 2 $\rightarrow$ 1, Red face up for stimulated absorption/emission from $3\leftrightarrow1$ and the uncoloured face up for spontaneous emission from 3 $\rightarrow$ 2. The probability of spontaneous emission from the level 2 to 1 is obtained .by throwing (about) 100 dice and removing the ones with yellow-face-up

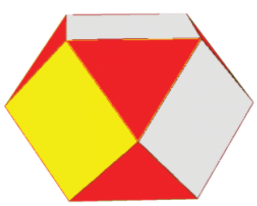
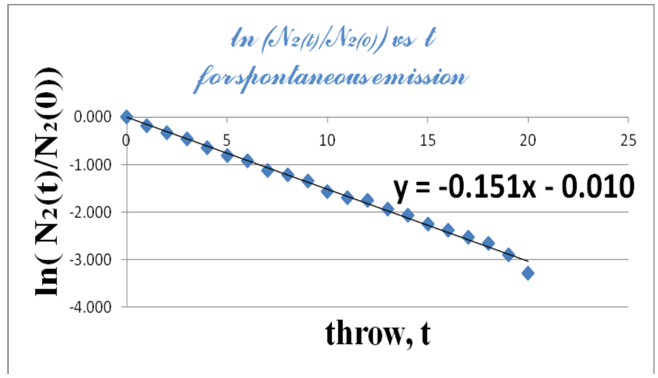


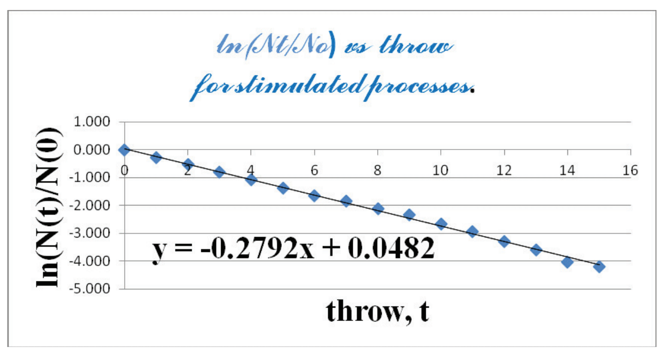
Fig 2. Cuboctahedron with three different faces: yellow, red and colourless.

(representing the atom that has undergone spontaneous emission from level 2 to 1). The number left $N_2(t)$ at each throw, t is played repeatedly till the die left is approximately 10% of the original. The plot of $\ln {}^{N_2(t)}/_{N_2(0)}$ *versus* t gives A_{21} as shown in Graph 1.[4,5]



Graph 1. Einstein constant A_{21} for spontaneous transition from 2 $\rightarrow$ 1 is (0.151±0.003) per throw and t_{sp2} =(6.67±0.013) throw.

Assuming that the Einstein's constants B_{13} is equal to B_{31}, the above procedure is repeated with the red-face-up to give $B_{13}\, u(\omega)$. $B_{13}\, u(\omega)$ is obtained from Graph 2.[4,5]



Graph 2: Einstein constant B_{13} multiplied with energy density $u(\omega)$ for the stimulated transitions $3\leftrightarrow1$ is (0.279 ± 0.014) per throw.

Therefore, the probability of the uncoloured-face-up representing A_{32} is given by Fig **2:**

$$A_{32} = 1 - A_{21} - B_{13}$$

$$=1-(0.151\pm0.003)-(0.279\pm0.014)$$

$$A_{32}=0.570\pm0.017 \qquad (11)$$

4.2 Simulation of four processes in the 3-level lasing system with cuboctahedrons

In the cuboctahedron the probability of the yellow face up is 0.151, the red face up is 0.279 and the colorless face up is 0.570 as determined in the first part of the experiment and is used for representing A_{21}, $B_{13}u(\omega)$ and A_{32} respectively.

1. In the first throw, of the $N_3(0)$ dice (atom) from level 3, let $dN_{3st}(t)$ and $dN_{3sp}(t)$ be the red and white-faces-up respectively. They represent the stimulated and spontaneous emission from level $3 \rightarrow 1$ and $3 \rightarrow 2$ respectively. These dN_{3st} (t) and dN_{3sp} (t) dice are removed, to be left with $N_3(t)$ dice .
2. In the first throw, of the $N_2(0)$ dice in level 2, $dN_2(t)$ represents dice with yellow face up which have undergone spontaneous emission from level $2 \rightarrow 1$. This $dN_2(t)$ dice are removed, to be left with $N_2(t)$ dice
3. Similarly, of the $N_1(0)$ dice of level 1; $dN_1(t)$ with the red-face-up, represents the *stimulated absorption* from level $1 \rightarrow 3$. These $dN_1(t)$ dice are removed, to be left with $N_1(t)$ dice .

In the (t+1) throw, the **i**nitial number of atoms (dice) in level 3 is given by

$$N_{3i}(t+1) = [N_3(t)+dN_1(t)] \qquad (12)$$

And the number of atom in the level 1 is

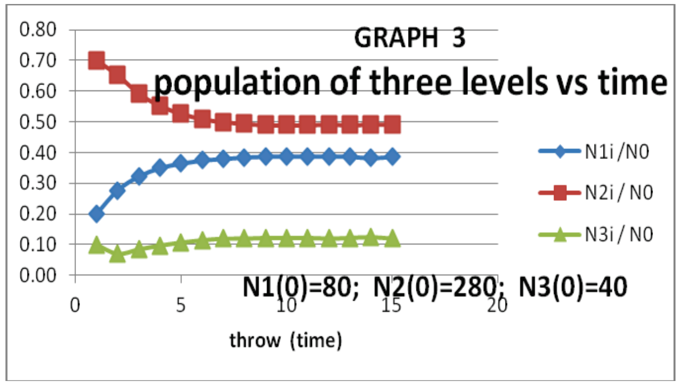
$$N_{1i}(t+1)=[N_1(t)+dN_2(t)+dN_{3st}(t)] \qquad (13)$$

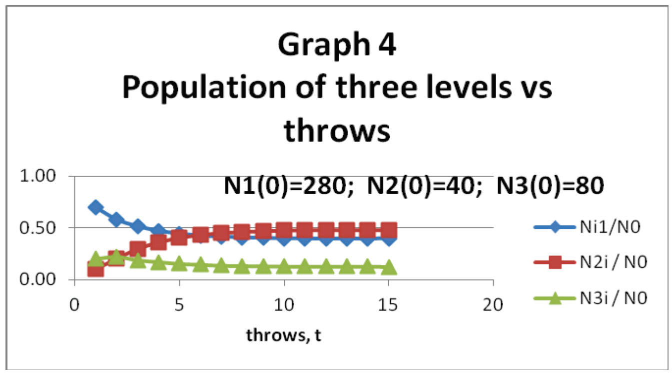
Similarly, in the (t+1) throw atoms at level 2 is

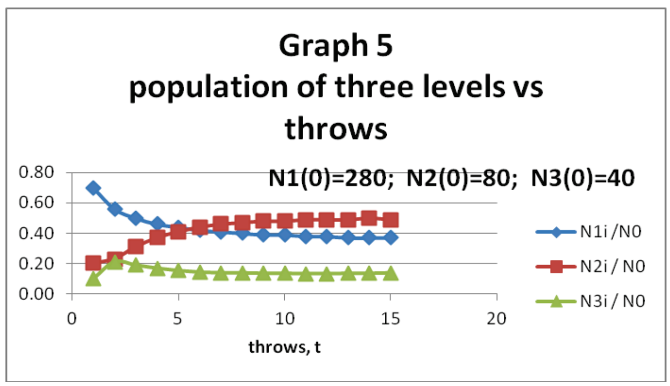
$$N_{2i}(t+1)=[N_2(t)+dN_{3sp}(t)] \qquad (14)$$

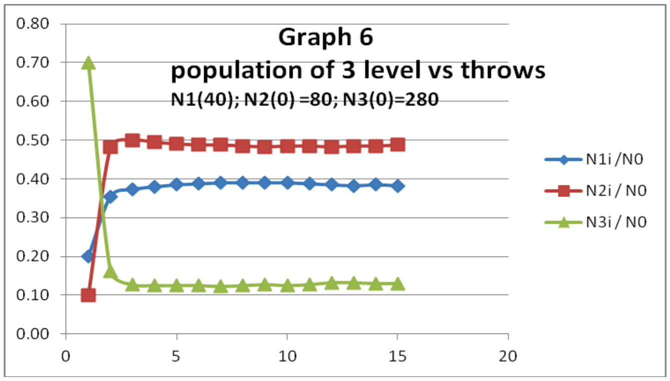
The above processes are repeated till the number of atoms in each level is stabilized.

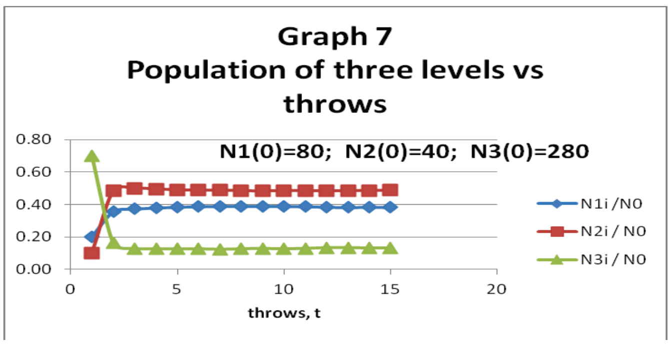
1. The plot of N_{1i}/N_o; N_{2i}/N_o; N_{3i}/N_o *vs t* is shown in Graphs 3, 4, 5, 6, 7, 8 for different initial populations.

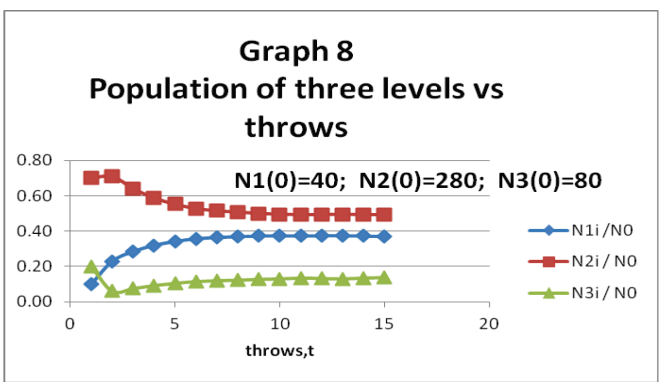












5. Results

1. From the graphs 3,4,5,6,7 and 8 we get

Graph no.	(N_{1i}/N_0)	(N_{2i}/N_0)	(N_{3i}/N_0)
3	0.39	0.49	0.12
4	0.40	0.48	0.12
5	0.38	0.49	0.13
6	0.39	0.48	0.13
7	0.39	0.48	0.13
8	0.39	0.48	0.13
Average—	**0.390**	**0.483**	**0.127**

2. From Graph 1, A_{21}= 0.151±0.003 per throw
3. From Graph 2, B_{13}=0.279±0.014 per throw
4. From Eq. (8)

$$N_3 = \frac{B_{13}N_1}{A_{32}+B_{13}}$$

Therefore,

$$A_{32} = \frac{B_{13}(N_1 - N_3)}{N_3}$$

$$= 0.578 \pm 0.013 \quad \ldots\ldots\ldots..(15)$$

5. From Eq. (9)

$$N_2 = \frac{A_{32}N_3}{A_{21}}$$

Therefore,

$$A_{32} = \frac{N_2 A_{21}}{N_3} = 0.574 \pm 0.012 \quad \ldots.(16)$$

Thus, **A_{32} is 0.576±0.012 per throw.**

6. Inferences

1. The probability per unit throw of the spontaneous emission from level 2 to 1 is A_{21}=**0.151±0.003** per throw

2. The probability per unit throw of the spontaneous emission from level 3 to 2 is A_{32}=**0.576±0.012** per throw. This matches well with the assumed value of Eq. (12)

3 .The life-time of the atoms in the level 2 is t_{sp2} = (**6.67±0.013**) throw and level 3 is t_{sp3} = (**1.75±0.105**) throw. Hence $t_{3sp} < t_{2sp}$ .This represents level 2 of the system has a large life time as in a ***metastable*** state.

4. The product of the probability per unit throw of the stimulated absorption from 1 to 3 or stimulated emission from level 3 to 1 and the energy density is

$$B_{13}\, u(\omega) = B_{31}\, u(\omega) = \mathbf{(0.279 \pm 0.014)}$$

5 . The Population of the level 2 at equilibrium is always ***greater*** than that of level 1, when $N_2 > N_3 > N_1$, $N_2 > N_1 > N_3$, $N_3 > N_1 > N_2$, $N_3 > N_2 > N_1$, $N_1 > N_3 > N_2$, $N_1 > N_2 > N_3$ at the starting point.

Hence, amplification is possible and lasing action can take place.

6. A 3-level lasing system is preferred to two level since 2-level system[1] 2-level lasing systems are not good lasing material as there is no retention of population inversion for long, hence no amplification and therefore, no lasing action is possible

7. Conclusion

Three or more levels, with at least one metastable state, can produce amplification, hence lasing.

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