Set Theoretic Approach to Resistor Networks

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Abstract

We study the properties of the sets of equivalent resistances arising in the resistor networks constructed from identical resistors. This enables us to obtain the bounds of the set of n equal resistors combined in series and parallel.

1. Introduction

The net resistance of n resistors having the values R_1, R_2, $\cdots$, R_n connected in series and parallel is given by the well-known relations

$$R_{series} = \sum_1^n R_i = R_1 + R_2 + \cdots + R_n .$$

$$R_{parallel} = \frac{1}{\sum_1^n \frac{1}{R_i}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}} .$$

The net resistance R_{series} is greater than the largest resistance among the resistances R_1, R_2, $\cdots$, R_n. The net resistance $R_{parallel}$ is less than the smallest resistance among the resistances R_1, R_2, $\cdots$, R_n. The net resistance of an arbitrary circuit (using any conceivable combination series, parallel, bridge or non-planar) must therefore lie between $R_{parallel}$ and R_{series} [1]. We shall use the symbols S and P to denote the series and parallel connections respectively. For two equal resistors R_0, there are two configurations R_0SR_0 and R_0PR_0 whose equivalent resistances are $2R_0$ and $(1/2)R_0$ respectively, giving rise to the set $\{(1/2)R_0,\ 2R_0\}$. We can omit R_0 without any loss of generality and write it as $\{1/2,\ 2\}$. For three resistors, there are 4 configurations, $(1S1)S1$, $(1S1)P1$, $(1P1)S1$ and $(1P1)P1$ giving the set $\{1/3, 2/3, 3/2, 3\}$. Continuing the exercise as in [2-3], using the series and parallel connections we obtain

$$A(1) = \{1\},$$

$$A(2) = \left\{\frac{1}{2},\ 2\right\},$$

$$A(3)=\left\{\frac{1}{3},\frac{2}{3},\frac{3}{2},\ 3\right\},$$

$$A(4)=\left\{\frac{1}{4},\frac{2}{5},\frac{3}{5},\frac{3}{4},\ 1,\frac{4}{3},\frac{5}{3},\frac{5}{2},\ 4\right\},$$

$$A(5)=\left\{\frac{1}{5},\frac{2}{7},\frac{3}{8},\frac{3}{7},\frac{1}{2},\frac{4}{7},\frac{5}{8},\frac{5}{7},\frac{4}{5},\frac{5}{6},\frac{6}{7},\right.$$
$$\left.\frac{7}{6},\frac{6}{5},\frac{5}{4},\frac{7}{5},\frac{8}{5},\frac{7}{4},\ 2,\frac{7}{3},\frac{8}{3},\frac{7}{2},\ 5\right\},$$

$$A(6)=\left\{\frac{1}{6},\frac{2}{9},\frac{3}{11},\frac{3}{10},\frac{1}{3},\frac{4}{11},\frac{5}{13},\frac{5}{12},\frac{4}{9},\right.$$
$$\frac{5}{11},\frac{6}{13},\frac{7}{13},\frac{6}{11},\frac{5}{9},\frac{7}{12},\frac{8}{13},\frac{7}{11},$$
$$\frac{2}{3},\frac{7}{10},\frac{8}{11},\frac{3}{4},\frac{7}{9},\frac{4}{5},\frac{5}{6},\frac{9}{10},\frac{10}{11},$$
$$1,\frac{11}{10},\frac{10}{9},\frac{6}{5},\frac{5}{4},\frac{9}{7},\frac{4}{3},\frac{11}{8},\frac{10}{7},\frac{3}{2},$$
$$\frac{11}{7},\frac{13}{8},\frac{12}{7},\frac{9}{5},\frac{11}{6},\frac{13}{7},\frac{13}{6},\frac{11}{5},\frac{9}{4},$$
$$\left.\frac{12}{5},\frac{13}{5},\frac{11}{4},\ 3,\frac{10}{3},\frac{11}{3},\frac{9}{2},\ 6\right\}.$$

The sets of higher-order need not contain the elements from a set of lower-order. For example, 1/3 is in $A(3)$ but not in $A(4)$ and $A(5)$. The element 1 is not present in $A(2)$, $A(3)$ and $A(5)$. These three sets have even cardinality. We shall shortly prove that these are the only three exceptional sets, which do not have 1 as its element and have an even cardinality. The resistor networks do have other sets. For instance, if we use at most three resistors, then the set $A(3)$ is replaced by the larger set $C(3)=\{1/3,1/2,2/3,1,3/2,2,3\}$, which is a union of the three sets $A(1)$, $A(2)$ and $A(3)$. The $C(n)$ of higher order contains all the $C(n)$ of lower orders. For five or more resistors, it is possible to include the bridge connections, giving rise to the sets $B(n)$. The order of the sets $A(n)$ (denoted by $|A(n)|$) grows for $n=1, 2, 3,\cdots$, as 1, 2, 4, 9, 22, 53, $131,\cdots$, and this sequence is known by the unique identity A048211 in *The On-Line Encyclopedia of Integer Sequences* (OEIS), created and maintained by Neil Sloane [4]. The $|C(n)|$ grow as 1, 3, 7, 15, 35, $77,\cdots$ [A153588]. The $|B(n)|$ grows as 1, 2, 4, 9, 23, $57,\cdots$ [A174283]. The problem for $n\le 16$ has been addressed computationally in [2] and suggests that $|A(n)|\sim 2.53^n$. How does $|A(n)|$ behave for large n? Does the base 2.53 increase or decrease? In this article, we shall address these questions through the properties of the set $A(n)$ and arrive at the bounds $(0.25)(2.41)^n<|A(n)|<2.73^n$. The number of configurations [A000084] is much larger than the number of equivalent resistances [A048211] and in this article we are not concerned about it. The computer memory has restricted the numerical studies of resistor networks to $n=23$. Hence the analytical studies are of extra significance.

2. Set Theoretic Properties

We shall derive some properties of $A(n)$ and the other sets arising in the resistor networks.

The Scaling Property:
If a/b is a member of $A(m)$, then we can construct the resistances $k(a/b)$ and $(1/k)(a/b)$ using k such blocks in series and parallel respectively, using km number of unit resistors. Hence, $kA(m)\in A(km)$ and $(1/k)A(m)\in A(km)$.

A block of i equal resistors in series has an equivalent resistance i. If i such blocks are combined in parallel we get back the unit resistance. From this we conclude that $1 \in A(i^2)$. The same result can equivalently be obtained by taking i blocks in series, each containing i unit resistors in parallel. Once the unit resistor has been obtained, using i^2 resistors (or much less as we shall soon see), we can use it to construct other equivalent resistances. Every set $A(m)$ is made from m unit resistors. The same set can be replicated by using m number of unit resistors constructed with i^2 resistors. So, $A(m) \subset A(i^2 m)$. Whenever 1 belongs to some set $A(i)$, we label it as 1_i to indicate that it has been constructed from i number of basic unit resistors, R_0.

The Translation Property:
It is the statement that $1 \in A(i)$ implies $1 \in A(i+3)$. This can be seen by taking either of the following two combinations of 1_i with 3 basic unit resistors: $(1S1)P(1S1_i) = 2P2 = 1$, or $(1P1)S(1P1_i) = (1/2)S(1/2) = 1$. Note, that we have consumed $(i+3)$ resistors. So whenever, $1 \in A(i)$, it follows that $1 \in A(i+3)$. We shall use this translation property to prove the theorem, that 1 belongs to all $A(n)$ barring three exceptions.

Theorem-1:
We have $1 \in A(n)$, $n \neq 2$, $n \neq 3$, *and* $n \neq 5$.

From an exhaustive search (or otherwise) we know that 1 belongs to $A(6)$, $A(7)$ and $A(8)$. Using the translational property, 1 also belongs to $A(9)$, $A(10)$ and $A(11)$; $A(12)$, $A(13)$ and $A(14)$; and so on. Thus we conclude that 1 belongs to all $A(n)$ for $n \geq 6$. As for the lower $A(i)$, 1 belongs to $A(1)$ and $A(4)$; and 1 does not belong to $A(2)$, $A(3)$ and $A(5)$. Hence, the theorem is proved. In passing, we note, that an exhaustive search is not always required. Two resistors in parallel lead to1/2; and two such blocks in series lead to 1 and hence, $1 \in A(4)$. The set $A(3)$ contains 1/3 and 2/3; combining these two blocks in series gives 1, implying $1 \in A(6)$. Combining the 1/2 present in $A(2)$ with the 1/2 in $A(5)$ in series, we conclude that $1 \in A(7)$. Similarly 1/4 and 3/4 are present in $A(4)$ and lead to $1 \in A(8)$. From references [5-7], we know that all elements in $A(n)$ have a reciprocal pair (a/b and b/a) and 1 is its own partner; presence of element 1 implies that the order of $A(n)$ is always odd with the exception of $|A(2)| = 2$, $|A(3)| = 4$, and $|A(5)| = 22$.

Corollary-1:
We have $1/2 \in A(n)$, $n \neq 1$, $n \neq 3$, $n \neq 4$ *and* $n \neq 6$.

The parallel combination of 1 basic unit resistor, R_0 with 1_i ($i = 4$ and $i \geq 6$) results in an equivalent resistance of 1/2, (since, $1P1_i \equiv (1 \times 1_i)/(1 + 1_i) = 1/2$), which implies that $1/2 \in A(i+1)$, for $i = 4$ and $i \geq 6$. The corollary is proved for $n = 5$ and for all $n \geq 7$. Resorting to the exhaustive search, we note, that 1/2 belongs to $A(2)$; the four exceptional sets are $A(1)$, $A(3)$, $A(4)$ and $A(6)$, which do not contain the element 1/2.

We constructed 1_i from i basic unit resistors ($i = 4$ and $i \geq 6$). Any set $A(m)$ can be constructed from m number of 1_i, using mi number of resistors, consequently $A(m) \subset A(mi)$ for $i = 1$, $i = 4$ and $i \geq 6$. The above statement is silent about $i = 2$, 3 and 5. The argument, mi is multiplicative, giving no information about the near or immediate neighbours of $A(m)$. Additive statements have the arguments of the type $(m+i)$ and when they exist, they provide information about the neighbours of

$A(m)$. In the present context the additive statements are more informative and override the multiplicative statements. We examined the occurrence of 1 in $A(i)$, since 1 is the basic unit and all other resistances can be constructed from it. The element 1/2 was examined as a special case. Remaining elements (infinite in number) shall be discussed collectively using the modular property.

Theorem-2 (Modular Theorem):
We have $A(m) \subset A(m+3)$ and $A(m) \subset A(m+i)$ for $i \geq 5$.

Every set $A(m)$ is constructed from m basic unit resistors R_0. If we replace any one of these basic unit resistors with 1_i ($i = 4$ and $i \geq 6$), we will reproduce the complete set $A(m)$ using $(m+i-1)$ resistors. Consequently, $A(m) \subset A(m+i-1)$ for $i = 4$ and $i \geq 6$. Thus the modular theorem is proved stating that every set $A(m)$ is completely contained in all the subsequent and larger sets, $A(m+3)$ along with the infinite and complete sequence of sets $A(m+5)$, $A(m+6)$, $A(m+7)\cdots$, and so on. However, it is very curious to note, that the infinite range theorem is silent about the three important sets: the *nearest neighbour*, $A(m+1)$; *next-nearest neighbour*, $A(m+2)$ and the *near-neighbour* $A(m+4)$. From the modular relation, $A(m) \subset A(m+i)$ for $i \geq 5$, we conclude that $A(n-5) \subset A(n) \cap A(n+1)$ for $n \geq 6$. This is the closest we can get to know the overlap between $A(n)$ and its *nearest neighbour* $A(n+1)$. An immediate consequence of the modular theorem is on the sets $C(n)$, obtained by taking the union of $A(i)$

$$\begin{aligned} C(n) &= \bigcup_{i=1}^{n} A(i) = \bigcup_{i=n-2}^{n} A(i) \\ &= A(n-2) \cup A(n-1) \cup A(n). \end{aligned}$$

It suffices to consider only the last three sets $A(n-2)$, $A(n-1)$ and $A(n)$ in the union. Hence, it is not surprising that the ratios $|C(n)|/|A(n)|$ are close to 1.

Decomposition of $A(n)$:
The set $A(n)$ can be constructed by adding the $n-th$ resistor to the set $A(n-1)$. This addition can be done in three distinct ways and results in three basic subsets of A(n). Treating the elements of $A(n-1)$ as single blocks, the $n-th$ resistor can be added either in series or in parallel. We call these two sets as *series set* and *parallel set* and denote them by $1SA(n-1)$ and $1PA(n-1)$ respectively. The $n-th$ resistor can also be added somewhere within the $A(n-1)$ blocks, and we call this set as the *cross set* and denote it by $1 \otimes A(n)$. The set $A(n)$ is the union of the three sets formed by different ways of adding the $n-th$ resistor. The decomposition $A(n) \equiv 1PA(n-1) \cup 1SA(n-1) \cup 1 \otimes A(n-1)$, is very illustrative, and enables us to understand some of the properties of $A(n)$. All the elements of the parallel set are strictly less than 1 (since $1P(a/b) = a/(a+b) < 1$) and that of the series set are strictly greater than 1 (since $1S(a/b) = (a+b)/b > 1$). So, $1PA(n-1) \cap 1SA(n-1) = \varnothing$ and the element 1 necessarily belongs to the cross set alone.

The series and the parallel sets each have exactly $|A(n-1)|$ number of configurations and the same number of equivalent resistances. Let c/d and d/c be any reciprocal pair (ensured by the reciprocal theorem in [5-7]) in $A(n-1)$, then it is seen that $1P(c/d) = c/(c+d)$ and $1P(d/c) = d/(c+d)$ belong to the set $1PA(n-1)$; and $1S(c/d) = (c+d)/d$ and

$1S(d/c) = (c+d)/c$ belong to the set $1SA(n-1)$. Thus all the reciprocal partners of $1PA(n-1)$ always belong to $1SA(n-1)$ and vice versa. These two disjoint sets contribute $2|A(n-1)|$ number of elements to $A(n)$. The order of the cross set, $1 \otimes A(n)$ is $(|A(n+1)| - 2|A(n)|)$ and results in the sequence, 0, 0, 0, 1, 4, 9, 25, 75,$\cdots$ [A176497]. For $n \geq 7$, all the three basic sets have odd number of elements, since $A(n)$ is odd for $n \geq 6$.

The cross set is not straightforward, as it is generated by placing the $n-th$ resistor anywhere within the blocks of $A(n-1)$. It is the source of all the extra configurations, which do not always result in new equivalent resistances. For, $n > 6$, the cross set has at least $|A(n-2)|$ elements, since $A(n-1)$ has $|A(n-2)|$ connections corresponding to $1 \otimes A(n-2)$; this leads to the recurrence relation $|A(n+1)| > 2|A(n)| + |A(n-1)|$, for $n \geq 6$. Similar, arguments lead to the relation $|A(n+1)| < 2|A(n)| + 2|A(n-1)|$. From the decomposition, we note, that the element 1 can belong only to the cross set and not the other two (since all elements of $1PA(n-1)$ are less than 1 and all the elements of $1SA(n-1)$ are greater than 1). We noted that the two disjoint sets, $1PA(n-1)$ and $1SA(n-1)$ are reciprocal to each other. Consequently all elements in $1 \otimes A(n-1)$, have their reciprocal partners in $1 \otimes A(n-1)$ itself; 1 is its own partner. The cross set is expected to be dense near 1 with few of its elements below half (recall that 1/2 is contained in $1PA(n)$ for $n \geq 6$, and not a member of the cross sets). This is reflected in the fact that the cross sets up to $1 \otimes A(7)$ do not have a single element below half. The successive cross sets have, 1, 6, 9, 24, 58, 124,$\cdots$ elements respectively [A176498], a small percentage compared to the size of the cross sets, 195, 475, 1265, 3125,$\cdots$ [A176497].

It is straightforward to carry over the set theoretic relations to the bridge circuits sets; since, $A(n) \subset B(n)$. Unlike the sets $A(i)$, the sets $B(i)$ have the additional feature $1 \in B(5)$. So, the various statements must be modified accordingly. In particular, we have

$$1 \in B(n) \text{ for } n \neq 2 \text{ and } n \neq 3,$$

$$\frac{1}{2} \in B(n) \text{ for } n \neq 1,\ n \neq 3 \text{ and } n \neq 4,$$

$$B(m) \subset B(mi) \text{ for } i = 1 \text{ and } i \geq 4,$$

$$B(m) \subset B(m+i) \text{ for } i \geq 3,$$

$$B(n-3) \subset B(n) \cap B(n+1) \text{ for } n \geq 4.$$

Complementary Property:

It is the statement that every set $A(n)$ with $n \geq 3$ has some complementary pair such that their sum is equal to 1. As an example, in $A(3)$ we have the pair $(1/3, 2/3)$; in $A(4)$ we have two pairs $(1/4, 3/4)$ and $(2/5, 3/5)$; and so on. By virtue of the Corollary-1, we see that 1/2 can be treated as its own complementary partner. We shall soon conclude that each element of the set $1PA(n-1)$ has a complementary partner in $1PA(n-1)$ itself. By the reciprocal theorem, the elements c/d and d/c occur as reciprocal pairs in every $A(n-1)$ (see [2-3, 5-7] for details and proofs). So in $1PA(n-1)$ we have

$$\left(1P\frac{c}{d}\right)+\left(1P\frac{d}{c}\right)=\frac{c}{c+d}+\frac{d}{c+d}=1.$$

Thus all the elements (except the element 1/2) of $1PA(n-1)$ have a complementary partner in $1PA(n-1)$ itself. For $n \geq 7$, the number of such pairs in the set $1PA(n-1)$ is $(|A(n-1)|-1)/2$, since $|A(n-1)|$ is odd for $n \geq 7$ and $1/2 \in A(n)$ for $n \geq 7$. It is obvious that the set $1SA(n-1)$ does not have complementary pairs.

3. Bounds of $|A(n)|$:

The decomposition of $A(n)$ enabled us to obtain the relations, $|A(n+1)| > 2|A(n)| + |A(n-1)|$, for $n \geq 6$ and $|A(n+1)| < 2|A(n)| + 2|A(n-1)|$. The solution of these two relations provides the strict bounds

$$(0.25)(1+\sqrt{2})^n < |A(n)| < (1+\sqrt{3})^n.$$

The numerically obtained result $|A(n)| \sim (2.53)^n$ in [2], for $n \leq \mathbf{16}$. is consistent with the strict bounds presented here.

4. Concluding Remarks

Several set theoretic relations among the sets $A(n)$ and $B(n)$ were derived using simple arguments. The decomposition of the set $A(n)$ into three basic subsets derived from $A(n-1)$ leads to the strict lower and upper bounds analytically. The set theoretic relations point to the complexity of estimating the upper bound of the order of $A(n)$ and other sets using combinatorial arguments. The Haros-Farey sequence approach presented in [3, 5-8], is another method to estimate the upper bounds of the various sets occurring in the resistor networks. The strict upper bound $|A(n)| < 0.318(2.618)^n$, obtained using the Haros-Farey sequence is also valid for the sets $B(n)$, $C(n)$ and other sets (using any conceivable combination series, parallel, bridge or non-planar). A comprehensive account of the resistor networks along with the computer programs using the symbolic package MATHEMATICA [9] is available in [5-8]. The program to generate the order of the set $A(n)$, using the symbolic package MATHEMATICA is presented in the Appendix.

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Appendix

Computer Program in MATHEMATICA

The problem of resistor networks is intrinsically a computational problem. The following programs have been written using the symbolic package MATHEMATICA [9]. They just need to be run on a faster computer. The results obtained can be shared at *The On-Line Encyclopedia of Integer Sequences* (OEIS) [4].

```
(* n Equal Resistors connected in Series and/or Parallel*)
NumberResistors = 4;
ClearAll[CirclePlus, CircleTimes];
SetAttributes[{CirclePlus, CircleTimes}, {Flat,Orderless}];
SeriesCircuit[a_, b_]:= a⊕b;
ParallelCircuit[a_, b_]:= a⊗b;
F[a_,b_]:= Flatten[Outer[SeriesCircuit, a, b]⋃Outer[ParallelCircuit, a, b], 2];
S={{R}, {R⊕R, R⊗R}};
Do[SX = F[S[[1]], S[[i-1]]];
    Do[SX = Flatten[SX⋃F[S[[k]], S[[i-k]]], 2]; , {k, 2, i/2}];
    S = S⋃{SX}; , {i, 3, NumberResistors}];
S[[NumberResistors]] (*This line displays the Full Set of Configurations*)
Print[StringForm["NumberResistors = ``, NumberConfigurations = ``", NumberResistors,
Dimensions[S[[NumberResistors]]]]]
SetAttributes[{CirclePlus, CircleTimes},{NumericFunction, OneIdentity}];
a_⊕b_:= a + b;
a_⊗b_:= a*b/(a+b);
CirclePlus[x_]:= x;
CircleTimes[x_]:= x;
(*Print[S[[NumberResistors]]/.R→1]*) (*This line displays the Set of Equivalent Resistances
corresponding to the Set of Configurations*)
(*Print[Union[S[[NumberResistors]]/.R→1]]*) (*This line displays the Full Set of Equivalent
Resistances*)
Print[StringForm["NumberResistors = ``, NumberConfigurations = ``,  NumberEquivalentResistances =
``, CPU time in seconds = ``", NumberResistors, Dimensions[S[[NumberResistors]]],
Dimensions[Union[S[[NumberResistors]]]], TimeUsed[%]]]
{R⊕R⊗(R⊕R), R⊕R⊗R⊗R, R⊗R⊕R⊗R, R⊕R⊕R⊗R, R⊕R⊕R⊕R,
 R⊗(R⊕R⊗R), R⊗(R⊕R⊕R), (R⊕R)⊗(R⊕R), R⊗R⊗(R⊕R), R⊗R⊗R⊗R}
NumberResistors = 4, NumberConfigurations = {10},
NumberEquivalentResistances = {9}, CPU time in seconds = 0.031`
```

This Program in MATHEMATICA is designed to compute the “Set of Configurations” [A00084] and the “Set of Equivalent Resistances” [A048211] of n Equal Resistors connected in Series and/or Parallel. The output is shown for $n = 4$.