Cosmological Special Relativity: Fundamentals and Applications

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Abstract

The theory of Special Relativity has been a centre of interest for the scientific community for more than a century now. In recent times, people have worked on various aspects of the subjects, besides attempting to disprove some of the postulates of Special Relativity and the emergence of ideas such as that of *Doubly Special Relativity*, one effort is especially noteworthy: Dr. Moshe Carmeli's attempt at extending the concept of Special Relativity to the larger picture - our Cosmos. In his famous paper '*Cosmological Special Relativity: The Large-Scale Structure of Space, Time and Velocity*' published in 1997, Dr.Carmeli has laid the framework for Cosmological Special Relativity. This paper is an attempt at understanding the nuances of the subject and trying to work on certain key ideas put forth by the theory. I have focussed on the explanations provided by Cosmological Special Relativity for the accelerated expansion of the universe and the existence of Dark Matter, as per Galaxy Rotation Curves.

Keywords: Cosmological Special Relativity, Galaxy Rotation Curves, Hubble's Law

Introduction

Cosmological Special Relativity (CSR) is an off-shoot of Einstein's Special Relativity, which associates the idea of Hubble Expansion of the Universe with a four-dimensional *space-velocity*, which can be used as a tool for describing the state of our cosmos.[1] Though not a widely followed paradigm, the theory is an interesting theory and a mathematical formulation that tries to explain certain aspects of Cosmology.

To begin with, one needs to define the mathematical space and relevant transformations for CSR. One needs a coordinate for the velocity of the object, which is independent of the spatial coordinates and is analogous to the time coordinate used in special relativity in general, besides the spatial dimensions. This unified space-velocity is used to describe our expanding universe. For every point in space-velocity, one has three spatial coordinates and a velocity coordinate.

The line element in this space is given by

$$ds^2 = \tau^2 dv^2 - (dx^2 + dy^2 + dz^2)$$

τ – Hubble Carmeli Constant

As will be mentioned later, τ is actually the inverse of Hubble's constant in empty space. Physically, it gives us the age of the Universe. If our present time is taken as t=0, then the time, moving backwards, when the Big Bang occurred would be given by τ.

1. Hubble's Law

Since ages, man has been awed by the universe. So magnanimous does it seem to us that the concept of an expanding universe was hard to accept, when it was put forth by Edwin Hubble. Building on George Lemaitre's idea of an expanding universe, Hubble came up with the idea that the universe is expanding at a constant rate: the Hubble's constant (H_O). All celestial objects in intergalactic space are found to have a Doppler Shift in the relative velocity, when observed from the Earth, and also relative to each other. Moreover, this velocity is found to be directly proportional to the distance of the object from the observing point (be it on the Earth or on some other celestial object).

Mathematically,

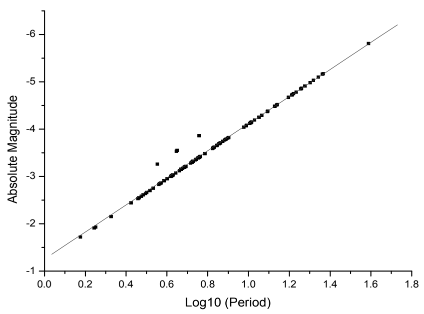
$$v = H_O D$$

where v = velocity of recession of the object, D = Proper Distance, as per *Cosmological Special Relativity*.

Hubble studied the *Radial Velocity vs. Distance* relations, derived with the help of the period-luminosity relation of Cepheid variables, as shown below for a set of 100 Cepheids. For calculating the distance from the PL relationship, one uses the formula

$$m - M = 5\log\left(\frac{Distance}{10\,pc.}\right)$$

where m is the apparent magnitude and M is the absolute magnitude of the pulsating star.



Graph: *Period-Luminosity Relation* of 100 Galactic Classical Cepheids with absolute magnitudes ranging between -1.72 to -5.81. *Period is in days.*
Courtesy: David Dunlap Observatory

$$Distance = (10\,pc.) \times 10^{\frac{m-M}{5}}$$

One can then plot the radial velocity vs. Distance curve. Given below is the plot for values obtained by Hubble-Humason and published in their 1931 paper

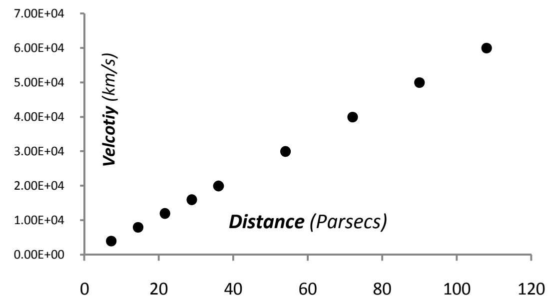


Figure 1: Courtesy: Edwin Hubble, Milton L. Humason. *The Velocity-Distance Relation among Extra-Galactic Nebulae*, Astrophysical Journal, vol. 74, p.43 (1931)

Various Models and Cosmological Special Relativity

Friedmann-Robertson-Walker metric

In the Friedmann-Robertson-Walker metric, one has the line element

$$ds^2 = -c^2dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$$

Here a(t) serves as a scale factor to describe points in four-dimensional space-time for an expanding universe. In this model, the Hubble's constant is defined as

$$H = \frac{1}{a}\frac{da}{dt}$$

The solution to Einstein's field equations, in Friedmann-Robertson-Walker model is found to be

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \Lambda\frac{c^2}{3}$$

In this equation, one finds that the pressure and density terms contribute to the decrease in the expansion rate of the universe, whereas the last term leads to the increase in rate of expansion of the universe. Physically, this term is associated with the negative density energy or the *dark energy.*

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Accelerated Expansion of the Universe

Recent astronomical studies on type Ia Supernovae have shown that the receding velocities of celestial bodies are not linearly dependent on the distance of the object from the Earth. This gives rise to the idea that our Universe may be expanding in an accelerated manner. (Fig. 3)

The Supernova Cosmology Project (SCP) and the High-Z Supernova Search team (HST) showed that the universe is expanding and the results were sensitive to combinations $a\Omega_M + b\Omega_\Lambda$ [2], where $ab < 0, |a| > |b|$. Here Ω_M represents the matter density and Ω_Λ represents the vacuum energy densities. The vacuum energy contributes to the expansion of the Universe whereas matter reduces it. This can be understood by considering the attractive gravitational force of matter, against which the vacuum energy tends to contribute to the expansion. This vacuum energy is also called *dark energy*. Eventually the contribution of the dark energy is more than that of matter, and as is well-established now, we have an accelerated expansion of the universe.

One pertinent question that has been addressed by Kaya[3] is regarding the idea of expansion of the Universe at a speed greater than speed of light. Kaya[3] has mentioned about the idea that the expansion in an inertial frame may not have a physical significance, and may give rise to faster-than-light expansion. Rather when one considers the other objects with respect to the rest frame of one, then using Relativistic Addition of velocities, one can see that one does not have faster-than-light expansion.

However, the idea of faster-than-light expansion is addressed by the CSR model in a different way. One can see that the coordinates themselves are in space-velocity. For two objects moving in relative cosmic time t,

$$\tau^2 v^2 - x^2 - y^2 - z^2 = \tau^2 v'^2 - x'^2 - y'^2 - z'^2$$

Taking one-dimensional case,

$$\tau^2 v^2 - x^2 = \tau^2 v'^2 - x'^2$$

If we take this in one frame of reference, say in the un-primed frame of reference with v = 0 and x = 0,

$$\tau^2 v'^2 = x'^2$$
$$\tau v' = x'$$
... (VX)
$$\tau \frac{dv'}{dt} = \frac{dx'}{dt}$$

$$\tau \times Acceleration = v'_{rel}$$

Now, using observations, one sees that with increase in cosmic time, the acceleration of expansion of the universe is increasing.

For
$v'_{rel} > c$, and Age of Universe[4] $= 13.798 \times 10^9$ years.

$$\tau \times Acceleration > c$$

$$Acceleration > \frac{3 \times 10^8 m/s}{13.798 \times 10^9 \times 3.16 \times 10^7 s} = 6.88 \times 10^{-10} m/s^2$$

Thus, using CSR, one can define no upper bound for the velocity v until one can definitely define the acceleration of the universe. Only if the acceleration is above the calculated value, as per CSR, can one see faster-than-light expansion.

One can also see that for celestial bodies to move away from each other at a speed faster than light,

$$\frac{x'}{\tau} = v > c$$

Using (VX)
$$x' > c\tau = 130.8 \times 10^{24} m$$
$$= 42.39 \times 10^5 kpc$$

$$x' > 42.39 \times 10^2 mpc \approx 4240\ mpc$$

Thus, the bodies need to be at a distance of 4240 mpc to have faster than light expansion. That means light has to travel roughly 13.826 billion years to travel this distance!

2. Postulates of Cosmological Special Relativity

The postulates put forth by Dr.Carmeli, for Cosmological General Relativity[1], in his seminal paper were:

1. *Principle of Constancy of Expansion of the Universe* at all cosmic times.
2. *Principle of Cosmological Relativity*: Principles of Physics are the same at all cosmic times.

One can find an analogy between these postulates and those of Special Relativity in general. I have tried to sum up the analogy in the given table.

Velocity is given fundamental importance in this theory over time since one is more interested with the velocity at which celestial bodies are receding rather than the time of the movement. However, just as in the case of inertial frames in Special Relativity in general, one has a Cosmic Time for any particular event. This is the time an event has taken place, with respect to a particular origin-time. Usually our present time is given the value t=0 and all other events are assigned cosmic time based on this assumption.

The fundamental reason for taking velocity as a more fundamental variable is because of the dimensionality of the most significant constant in the theory: the Hubble-Carmeli Constant. In Special Relativity, c has the dimensions of length/time, and hence one has Space-Time to describe the displacement of an object in terms of the time taken, subject to the constraint of an upper velocity bound.

$$[\tau] = Distance/Velocity$$

Hence we have the idea of Space-Velocity.

Table 1: Analogies	
Special Relativity	Cosmological Special Relativity
Inertial Frames	Cosmic Times
Space-Time	Space-Velocity
Speed of Light	Hubble-Carmeli Constant

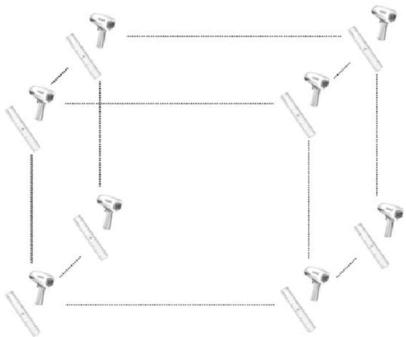


Figure 2: Hypothetical Lattice Unit for describing points in Space-Velocity, as per Cosmological Special Relativity. Just as observers describing an event in Space-Time would require scales and stop-clocks at each point to describe events in Space-Time, one could hypothetically think of scales and velocity trackers at each point in Space-Velocity for doing so in Space-Velocity. One could think of a lattice of points in Space-Velocity, each equipped with a scale and a velocity tracker.

3. Cosmological Transformation

Just as in the case of the Lorentz transformation one uses to convert space-time coordinates of an entity in one coordinate system to the space-time coordinates in another inertial system, one has Cosmological Transformation. Just as the corner-stone of the Lorentz transformation was the constant velocity of light, for Cosmological transformation, the constant expansion described by Hubble serves the purpose. When one is converting space-velocity coordinates of an event from one cosmic-time frame to another, one needs to keep the following quantity constant:

$$\tau^2 v^2 - x^2 - y^2 - z^2$$

For systems where the coordinates in one frame is given by x, y, z, v and in another is given by x', y', z', v' at a relative cosmic time t with respect to the first frame, the transformation is such that it should satisfy

$$\tau^2 v^2 - x^2 - y^2 - z^2 = \tau^2 v'^2 - x'^2 - y'^2 - z'^2$$

Let us take the simplest case of one dimensional motion.

$$\tau^2 v^2 - x^2 = \tau^2 v'^2 - x'^2 \text{... (1)}$$
$$y = y' \text{and} z = z'$$

I have tried to derive the transformation using first principles.

Let us have a transformation of the form,

$$x' = ax + b\tau v$$
$$\tau v' = cx + d\tau v$$

For x' = 0

$$ax = -b\tau v$$

$$\frac{b}{a} = -\frac{x}{v\tau} = -\frac{t}{\tau}$$

$$x' = a\left(x + \frac{b}{a}\tau v\right) = a(x - tv) \text{...(2)}$$

Using (2) in (1),

$$\tau^2 v^2 - x^2 = (cx + d\tau v)^2 - a^2(x - tv)^2$$

$$\tau^2 v^2 - x^2 - (cx)^2 - 2(cx)(d\tau v) - (d\tau v)^2 + a^2 x^2 + a^2(tv)^2 - 2a^2(x)(tv) = 0$$

Evaluating coefficients of x^2, v^2 and xv

$$a^2 - c^2 = 1, \tau^2(d^2 - 1) = a^2 t^2, cd\tau + a^2 t = 0$$

Simplifying,

$$\frac{t}{\tau} = -\left(\frac{cd}{a^2}\right) = \pm\sqrt{\frac{d^2-1}{a^2}},\ a^2 = 1 + c^2,$$

$$d = \pm\sqrt{\frac{a^2}{a^2 - c^2}} = \pm\sqrt{1 + c^2}$$

$$\frac{t}{\tau} = \pm\sqrt{\frac{c^2}{(1+c^2)}},\ 1 - \left(\frac{t}{\tau}\right)^2 = \frac{1}{1+c^2}$$

So

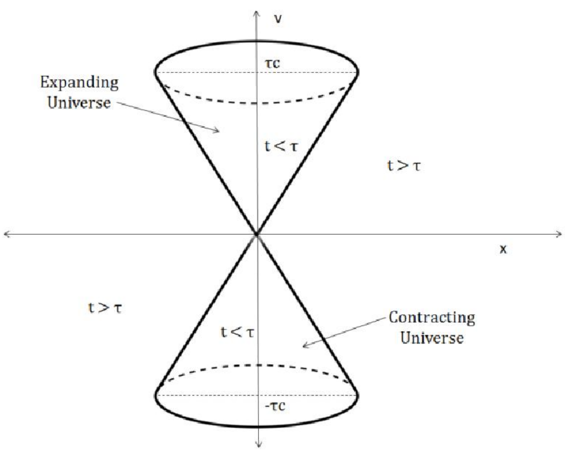
$$d = \pm\frac{1}{\sqrt{1-(t/\tau)^2}},\ c = \pm\sqrt{\frac{(t/\tau)^2}{1-(t/\tau)^2}}$$

$$a = \pm\frac{1}{\sqrt{1-(t/\tau)^2}},\ b = \mp\frac{t}{\tau}\frac{1}{\sqrt{1-(t/\tau)^2}}$$

Going by signs assigned as per the relations

$$x' = \frac{x-tv}{\sqrt{1-(t/\tau)^2}},\ v' = \frac{v-\frac{xt}{\tau^2}}{\sqrt{1-(t/\tau)^2}}$$

$$x = \frac{x'+tv'}{\sqrt{1-(t/\tau)^2}},\ v = \frac{v'+\frac{x't}{\tau^2}}{\sqrt{1-(t/\tau)^2}}$$



This gives us the set of equations that define the Cosmological Transformations. One cannot help but observe that this is similar to the Lorentz Transformation but with $\beta = \frac{t}{\tau}$ and not $\beta = \frac{v}{c}$.

One can define a transformation matrix for transformation between frames separated with a relative cosmic time. The transformation matrix is given by

$$\begin{pmatrix} {}^{1}\!/\!\sqrt{1-({}^{t}\!/\!_{\tau})^2} & {}^{-t}\!/\!\sqrt{1-({}^{t}\!/\!_{\tau})^2} & 0 & 0 \\ {}^{-1}\!/\!_{\tau^2}\Big/\sqrt{1-({}^{t}\!/\!_{\tau})^2} & {}^{1}\!/\!\sqrt{1-({}^{t}\!/\!_{\tau})^2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

4. Space-Velocity Diagram

Now we can represent any entity on the space-velocity diagram.

Just as in Space-Time Diagrams for Special Relativity, one can construct Space-Velocity Diagrams, as shown. In such diagrams, one can define what is known as a *Galaxy Cone*. This is the cone defined by the lines t = τ, followed by the galaxies in our universe. Any space-velocity line outside the cone is not found physically in the universe, and I am using the term *vel-like* for such lines, which satisfy $t > \tau$ i.e. $\tau^2\Delta v^2 > \Delta r^2$ and there is a relative velocity between two events which allows it to recede in a time period greater than the age of the Universe, with respect to each other. One also has cases for which $\tau^2\Delta v^2 < \Delta r^2$.

1. *Length and Velocity Contraction*

   We know that

$$x = \frac{x' + tv'}{\sqrt{1 - (t/\tau)^2}}$$

So,

$$\Delta x = x_1 - x_2 = \frac{x'_1 - x'_2}{\sqrt{1-(t/\tau)^2}}$$
$$= \frac{\Delta x'}{\sqrt{1-(t/\tau)^2}}$$

Also,

$$v = \frac{v' + \frac{x't}{\tau^2}}{\sqrt{1-(t/\tau)^2}}$$

For x’ = 0

$$v = \frac{v'}{\sqrt{1-(t/\tau)^2}}$$

Hence, in cosmological space-velocity, the length and velocity of an object moving in a cosmic time frame with cosmic time t has lesser value than the values obtained in our cosmic- time frame (t=0).

This raises a fundamental question to the idea of an accelerating universe. What if the universe is not accelerating, but it is only because of velocity contraction that we feel that the Universe is undergoing accelerated expansion?

Law of Addition of Cosmic Times:

$$\frac{x}{v} = \frac{x' + v't_1}{v' + \frac{x't_1}{\tau^2}} = \frac{\frac{x'}{v'} + t_1}{1 + \frac{x't_1}{v'\tau^2}}$$

Using coordinate transformation for frames moving away with a relative cosmic time t_1.

For $t = \frac{x}{v}$, $t_2 = \frac{x\prime}{v\prime} t = \frac{t_1 + t_2}{1 - \frac{t_1 t_2}{\tau^2}}$

One can never reach the valueτ, even with large values of t_1 and t_2, which would have yielded a number larger than τ using simple addition.

2. *Cosmological Red-Shift*

Applying the idea of Doppler Effect for Cosmological Special Relativity, one can write the wavelength of light emitted by a source in the cosmic time frame with relative cosmic time t, with respect to us (t = 0), as

$$\frac{\lambda}{\lambda_O} = \sqrt{\frac{1+\beta}{1-\beta}}, \beta = \frac{t}{\tau}$$

For$\frac{t}{\tau} \ll 1$,

$$z = \frac{\lambda}{\lambda_O} - 1 \approx \frac{t}{\tau} + \frac{1}{2}\left(\frac{t}{\tau}\right)^2$$

Thus, sources further back in time are more red-shifted.Also,

$$1 + z = \sqrt{\frac{1 + t/\tau}{1 - t/\tau}}$$

$$\beta = \frac{t}{\tau} = \frac{(1+z)^2 - 1}{(1+z)^2 + 1}$$

5. Rotation Curves

Rotation curves are the plots between the rotational velocities of components of galactic systems and the distance of the component from the centre of the galaxy.

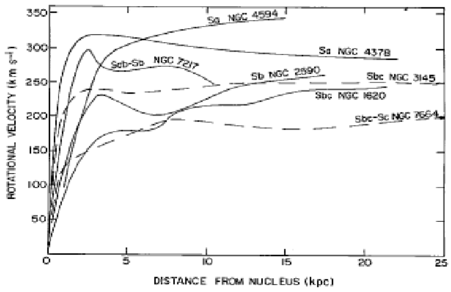
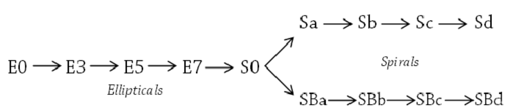


Figure 4: A series of rotation curves for spiral galaxies.
Figure from Rubin, Ford, and Thonnard (1978), Ap. J. Lett., 225, L107

One of the most significant results obtained by the study of the galaxy rotation curves has been the conjecture of the existence of dark matter. Given the luminous matter in galaxies such as the Sc 1 Spiral Galaxies, and considering the mass distribution of the galaxy to be equal to the mass of only luminous matter (*Rubin et al. 1985; Corradi& Capaccioli1990; Persic et al. 1996 have studied the relation between luminous properties and the rotation curves of galaxies*) one would expect the galaxy rotation curves to follow a curve similar to that of celestial bodies orbiting around a star or a planet: to decline at large distances from the central body. However, what one observes is that the velocity tends to become nearly constant and does not decline, as expected. The most commonly accepted idea for this is given by the conjecture that one has non-luminous massive entities that comprise *Dark Matter*, which contributes to the gravitational effect on the orbiting bodies. It is conjectured that the Dark Matter forms a halo around the luminous galactic matter.

To understand the relation between luminous matter and the rotation curves, *R. A. Swaterset al*[5] have used the rotation-curves of low-luminosity galaxies and dwarf-galaxies, as part of the *Westerbork H1 Survey of Spiral and Irregular Galaxies Project* (WHISP). They focussed on galaxies of Hubble type later than *Sd*, and used a refinement technique that involved initial rotation and final rotation curve estimates, using *H1* distribution rather than the conventional *Hα* distribution. They established some interesting characteristic relations for galaxies, with emphasis on Dwarf Galaxies.



HUBBLE SEQUENCE

- *Rubin et al.*, *Persic&Salucci*and*Broeils* showed that rotation curves rise more rapidly towards highly luminosities as compared to lower luminosities. To this *R. A. Swaters et al.* added that galaxies with luminosity magnitude range = -20 to -16, the log-slopes of the outer rotation curves *do not* depend on the luminosity. It is found that the correlation between surface brightness and logarithmic slope of the rotation curve is weak and clustered in an intermediate surface brightness range.

- It is also seen that galaxies of the type Sc, Sc-Irregular and Irregular are most observed, with a luminosity magnitude range = -16 to -19.5. Also MR = -17 to -14 are seen to be irregular galaxies.

- For galaxies for which the last measured point lay beyond 3 disk scale lengths and had inclinations between 39^{O} and 80^{O}, the *logarithmic slopes* of the rotation curves were found to be more in the interval 0 to 0.42, for the maximum velocity range of 20 to 140 km/s.

MOND Theory

Another school of thought puts forth the idea that Newtonian Mechanics breaks down on the galactic scale and the law of gravitational attraction becomes proportional to 1/r (as given by Modified Newtonian Dynamics or MOND theory).[6]

In 1983, MordehaiMilgrom hypothesized a modification of Newtonian Dynamics for cases that involved accelerations below the typical centripetal acceleration of a star in a galaxy

$$a_O \approx 10^{-10} m/s^2$$

This theory is appropriate for minor accelerations in a galactic system. For $g \gg a_O$, we get the classical Newtonian case ($a_O \rightarrow 0$): $g = g_N$.

For $a_O \rightarrow \infty$ (i.e. for the small acceleration case), we have the modification

$$Small\ Acceleration: g = \sqrt{g_N a_O} \ldots \text{(SA)}$$

g_N denotes the Newtonian acceleration

Now using the relation for centripetal acceleration of an object carrying out circular motion around a central body (mass: M) at a distance of R,

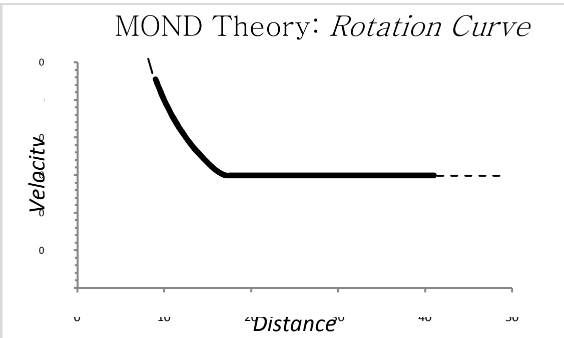
$$\frac{v^2}{R} = g_N = \frac{GM}{R^2}$$

For small values of acceleration, using relation (SA),

$$g = \sqrt{g_N a_O} = \sqrt{\frac{GMa_o}{R^2}}$$

$$\frac{v^2}{R} = \sqrt{\frac{GMa_o}{R^2}} \Rightarrow v^4 = GMa_o$$

As can be seen, this velocity function is independent of distance from the centre, and is directly proportional to the mass of the central object.[7] One can take the simplest normalized case with G = 1, M = 1 and find that the profile of the curve should look something like this far from the centre:



To ensure a smooth transition between the cases for which $g \ll a_o$ and $g \gg a_o$, Milgrom's Law can be written as:

$$\mu\left(\frac{g}{a_o}\right) g = g_N$$

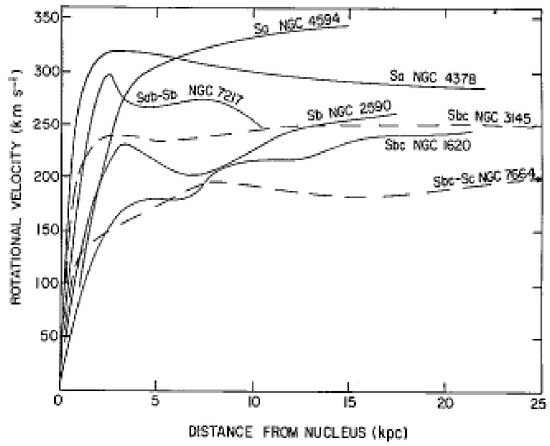
where$\mu(x)$ is an interpolating function with the property:

$$\mu(x) \to 1, x \gg 1$$

$$\mu(x) \to x, x \ll 1$$

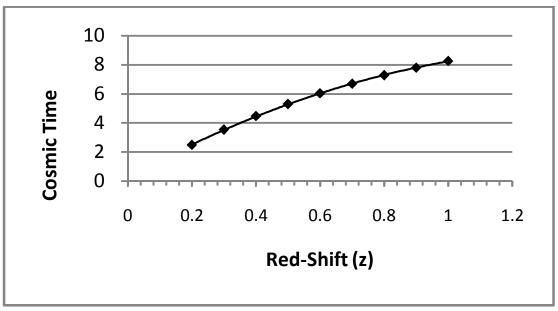
Recently, certain results have brought the attention of the scientific community back to the idea of Dark Matter. One of them is the discovery of ring-like 'Dark Matter' structure in the core of the galaxy cluster C1 0024+17.[8] However, I have tried to focus on CSR and identify how the theory addresses the issues relating to the expansion of the Universe and Galaxy Rotation Curves.

Results



Red-Shift Study and Comparison with Friedmann Model

Experimentally, one of the most useful ways of studying this theory is using Astrometry, especially the study of Red-Shifts of the galaxies. Here, I have tried to compare the plot of *Redshift* vs*Cosmic Time* given by this model, and that given by a popular model: the Friedmann Model.

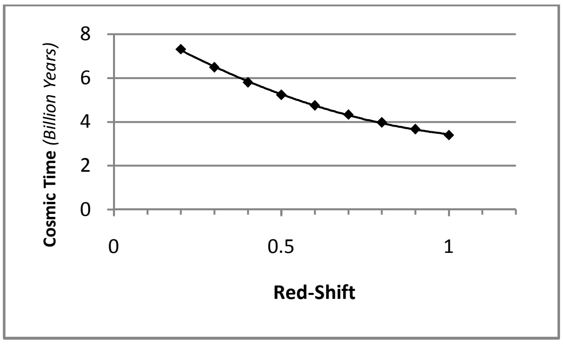


Graph 1: Cosmic Time vs. Red-Shift, as per Cosmological Special Relativistic Model

Now Considering Total Density Parameter (Ω_O) to be ~1.02 for the universe and following the Friedman World Model, we have[9, 10]

$$t = \frac{\Omega_O}{H_O(\Omega_O - 1)^{3/2}}[\sin^{-1} y^{1/2} - y^{1/2}(1-y)^{1/2}]$$

with $y = \frac{\Omega_O - 1}{\Omega_O(1+z)}$ and H_O- Hubble's Constant



Graph 2: Cosmic Time vs. Red-Shift, as per Friedmann Model

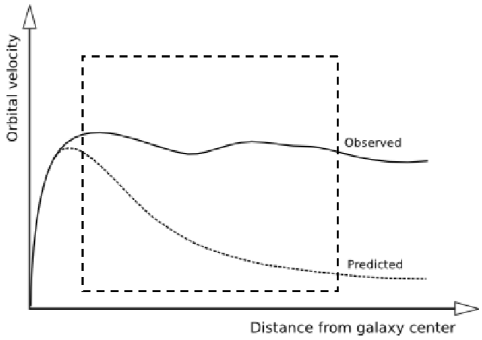
In this case, we have considered zero curvature for the *Friedmann Model*, thereby modelling a flat universe. According to the model, as evident, the model predicts that the universe will expand forever, but at a rate that eventually approaches zero. The zero-curvature assumption is valid for *special* relativity as gravitational effects are neglected.

However, by Cosmological Special Relativity, the acceleration is *increasing*. This is a well-accepted fact today, given the seminal papers by Nobel Laureates Saul Perlmutter, Adam G. Reiss and Brian P. Schmidt [11, 12], based on their observations on Type Ia Supernovae.

Radial Velocity Distributions and Dark Matter

The existence of Dark Matter has been a much debated topic. A primary basis for justifying the existence of Dark Matter has been the discrepancy of theoretically expected values, given assumptions of mass-to-luminosity ratios, and the observed values in Rotation Curves. Rotation curve of a galaxy is the curve that represents the relation between the rotational velocity of the visible entities in the galaxy and their radial distance from the centre of the galaxy.

Vera Cooper Rubin was one of the pioneers of the Gravity Rotation problem. She worked on the Rotation Curves of the Sc I and Sc II galaxies.



A series of rotation curves for spiral galaxies.

Edited on Figure from www.citizendium.org

Now, the velocity function for a galaxy, in terms of the distance from its centre is given by

$$v(r) = \sqrt{\frac{G_O M}{r}} \left(\frac{r}{r_c + r}\right)^{\left(\frac{3}{2}\right)\beta} \times$$
$$\times \sqrt{1 + \sqrt{\frac{M_O}{M}}\left(1 - e^{-\frac{r}{r_O}}\left(1 + \frac{r}{r_O}\right)\right)}$$

Let us concentrate on $r_c \ll r$ and $r < r_O$, and for HSB Galaxies, β = 1. For a given mass M,

$$v(r) \propto \sqrt{\frac{1}{r}}\sqrt{1+\sqrt{\frac{M_O}{M}}\left(1-e^{-\frac{r}{r_O}}\left(1+\frac{r}{r_O}\right)\right)}$$

where k is a constant dependent on the mass of the celestial body, r_c- the radius of inner core and parameter r_O.[13]

Taking the Milky Way as a test example,

$$M = 5.8\times 10^{11} M_{\odot}, M_O = 9.60\times 10^{11} M_{\odot}$$

$$r_O = 13.92\ kpc$$

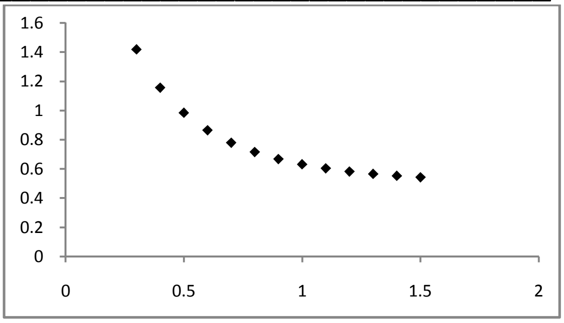
We can now introduce r in *kpc* and with constant K having dimensions of [length$^{3/2}$/time].

$$v(r) = K\sqrt{\frac{1}{r}}\sqrt{2.29-1.29\times\left(e^{-\frac{r}{13.92}}\left(1+\frac{r}{13.92}\right)\right)}$$

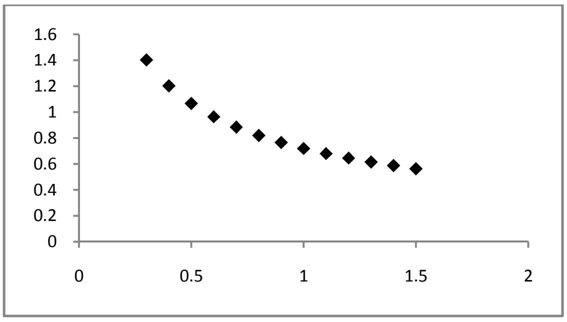
$$v(r) \approx K\times 1.513 \times\sqrt{\frac{1}{r}}\,\left[1 - 0.282\left(e^{-\frac{r}{13.92}}\left(1 + \frac{r}{13.92}\right)\right)\right]$$

Now, we know that the curve for the function

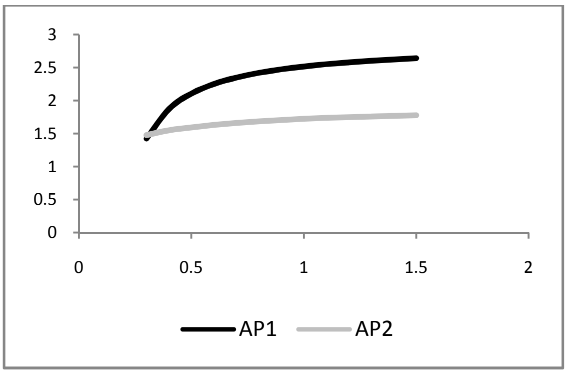
$$\frac{1}{\sqrt{r}}+\sqrt{r}e^{-r}$$



As an exercise, I tried to scale down the variable r in the equation using division of r by a constant (10). And I obtained:

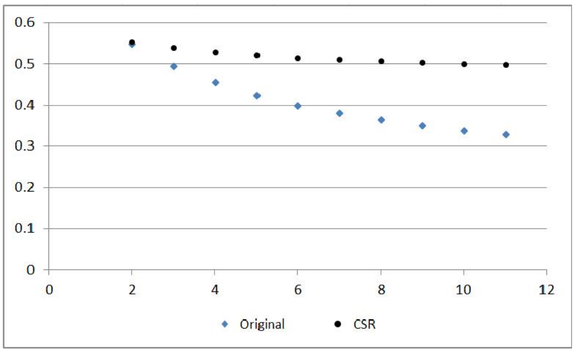


However, the most interesting result was obtained when each point of the data-set was evaluated after r was divided by arithmetic progressions for successive values of r (A.P.$_1$ with a = 1 and r = 1; and A.P.$_2$ with a = 0.1 and r = 0.1):



This, along with my previous conjecture that the discrepancy could possibly be because of varying divisors for the distance variable in the function for v(r), took me to test if I can use the idea of Cosmological Transformation and check the changes brought about.

Now, I have considered a hypothetical case of a galaxy. I have considered 10 data-points at distances 4 to 13 kpc. Also, I have considered the AP case with a = 1.2 and r = 0.2. This means, in terms of Cosmological Transformation, γ is taking values from 1.2 to 0.2. In other words, we have t ~ 7.5 Billion Years to ~13 Billion Years. The radius of the galaxy is taken to be r = 13.92 kpc.



The data-sets have been normalized to meet at the left end-point

However, I would like to point out a few points here, for the given hypothetical case:

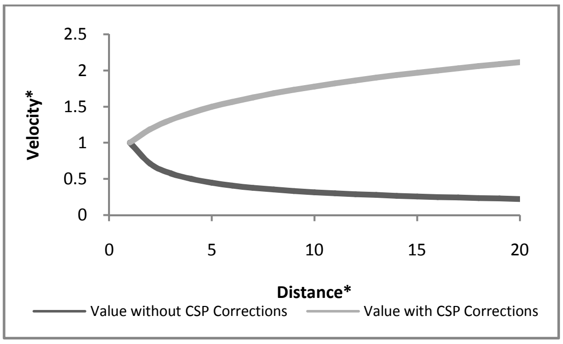
- Firstly, the progression I have taken begins from a large value of t. t~7.6 Billion Years is about 55% of the age of the universe. Thus, we are talking of distant galaxies that are at a distance of about 2.33 mpc or more.
- I have limited the data-points to 13 kpc to limit it within the radius of the galaxy.
- Here one has taken Milky-Way-like configuration with a negligible core radius and a uniform mass distribution.

Use of Newtonian Dynamics

One can use the formula

$$\omega = \frac{v}{R} = \frac{1}{r}\sqrt{\frac{GM}{r}}$$

And try and see if a system with CSP Corrections would give expected results. For Distance from Earth to Galactic Centre = 900 kpc, and uniform mass distribution (with unit mass and unit G, for convenience),



**Variables have been normalized (hypothetical case) with M=1 and G=1*

One final line of thought that can be studied and analyzed is that of trying to explain the galaxy rotation curves using a combination of the MOND theory and Cosmological Special Relativity. It is seen that some galaxies, such as NGC 2841, have a discrepancy in MOND fits and observed values. One can try to see if CSR corrections can be used to make the fitted curve closer to the experimentally obtained values.

Short-Term Targets

For near Galaxies, as one can see, the Cosmological factor γ is negligible and hence the correction is minimal. Hence, by the velocity function given by *Brownstein et al*[12], one cannot find significant corrections. I would like to work on the Mass-corrections if any, after extending the present work to Cosmological General Relativity for various galaxies, as worked on by *J. Hartnett*.

I would also like to extend the present study to more galaxies, with application to particular systems, with emphasis on mass distribution.

Conclusion

I have tried work on the fundamentals and applications of Cosmological Special Relativity, as modelled primarily by Dr. Moshe Carmeli, and develop the theory further. I have worked on proving the validity of the model, as compared to the Friedmann Model, in describing the accelerated expansion of the Universe. I have also studied red-shifts and shown that the apparent disparity between theoretically expected and observed data-points for the *Rotational Velocity vs. Distance* graph can be explained by Cosmological Special Relativity.

Though Cosmological Special Relativity is not a standard paradigm in Cosmology today, it is an interesting theory put forth by Moshe Carmeli, which has been studied in the present paper.

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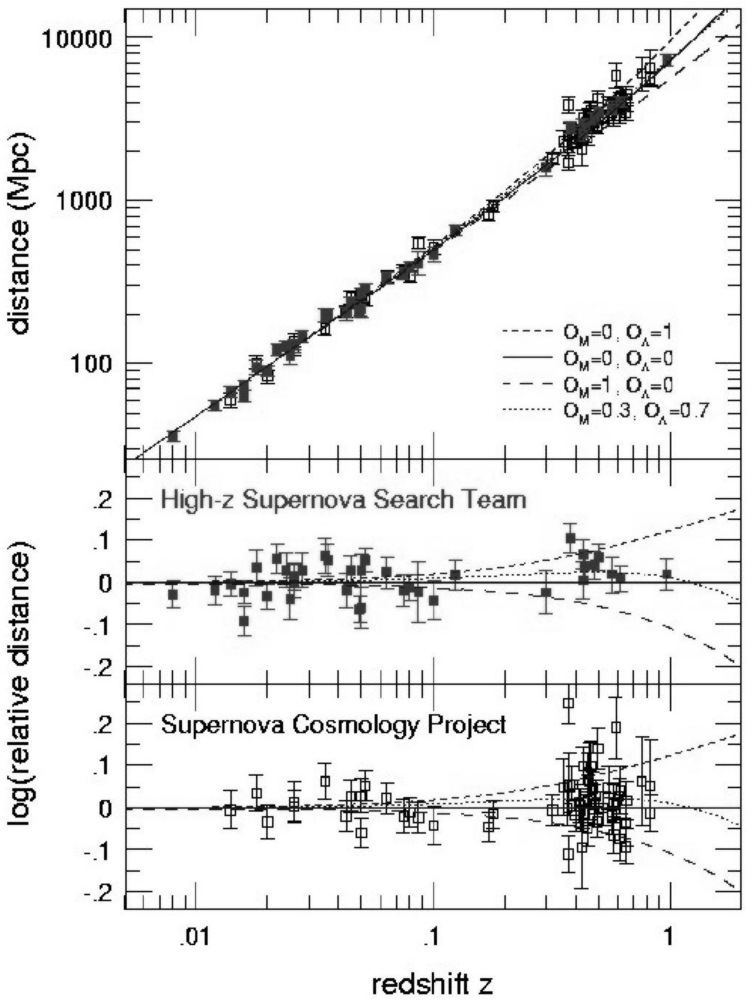

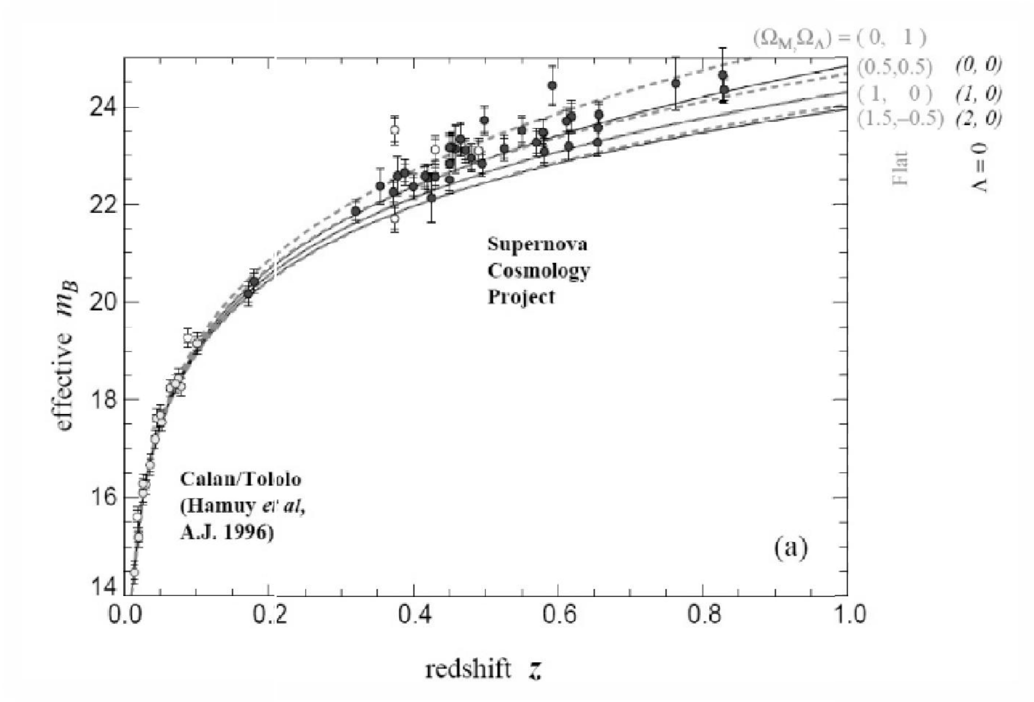

Figure 6

The Hubble diagram for 42 high redshift type Ia supernovae from SCP and 18 low redshift supernovae from the Calan/Tololo Supernova Survey. The dashed curves show a range of "flat" models where$\Omega M + \Omega\Lambda = 1$.

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