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Verification of laws of reflection and refraction from quantum model of light

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Abstract

Light exhibits dual characteristics – wave and particle. Although the laws of reflection and refraction were verified in corpuscular theory, wave theory and electromagnetic theory of light no explanation of it was given in old quantum model of light. Here a simplified version of verification has been dealt with using conservation of linear momentum and continuity of tangential component of electric and magnetic fields across the interface between two media.

1. History

Light is something that allows us to see the objects around us [1-4]. About 300 BC Euclid of Alexandria in his book *Optica* noted that light travels in straight lines and described the law of reflection. Aryabhatta, an Indian philosopher, reiterated that it was light arriving from an external source that illuminated the world around us. Ibn al-Haytham (965 – 1039) in his *Book of Optics* gave a lucid description of the optical system of the eye which led to the belief that light consists of rays which originate in the object seen and not in the eye, a view contrary to that of the ancient Greek and Indian philosophers. Willebrord Snell Van Royen (1580 – 1626) discovered the law of refraction in 1621 that is now popularly known as the Snell's Law.

2. The laws

In brief the laws of reflection and refraction are as follows:

(i) The first law of reflection and refraction state that the incident ray, the reflected ray, the refracted ray and the normal to the point of incidence at the interface between two different optical media with the refractive indices (n_1) *and* (n_2) *respectively, lie in one plane for any wavelength of incident light.*

(ii) The second law of reflection states that the angle of incidence (θ) *is equal to the angle of reflection* (θ') *for any wavelength of incident light.*

(iii) Finally the second law of refraction or Snell's law states that for a particular wavelength of incident light the sine of angle of incidence (θ) *bears a constant ratio to the sine of angle of refraction* (θ_m). Thus there are different constants for different colours and mathematically,

$$\sin\theta / \sin\theta_m = n_2 / n_1 \qquad -(1)$$

Clearly this is the relative refractive index of second medium with respect to the first and for free space as first medium it is absolute refractive index n_m. Of course the refractive index of free space is unity.

3. The corpuscular theory

However, true nature of light was not clear. It is invisible, but it influences growth of plants by transfer of energy. But, how this energy transfer takes place between source and object was not clear. The interesting point is that René Descartes (1596 – 1650) first gave corpuscular model of light in 1637 and Sir Isaac Newton (1643 – 1727) presented this model of light in 1675 in a Royal Society paper. Later on Newton popularized this concept in the book 'opticks' and thus this theory is very often, attributed to him. According to this theory, light is a stream of minute, invisible material particles called corpuscles which shot out from a luminous source with a very high speed in a straight line much like the shots fired from a gun. Corpuscles have no mass and can spread in all directions. Different colours are due to different sizes of corpuscles. Reflection occurs due to repulsion between the reflecting surface and the corpuscles. Refraction is due to attraction between the corpuscles and the refracting surface. For refraction trajectory of the corpuscle is determined by the conservation of the horizontal component of momentum along the interface. If the angles of incidence and refraction are θ and θ_m with the momentum of the corpuscle in the two media be p and p_m respectively, then $p\sin\theta = p_m \sin\theta_m \Rightarrow \sin\theta/\sin\theta_m = p_m / p = c_m / c$. This gives the incorrect explanation of Snell's law and predicts higher speed if the ray moves towards normal and inconsistence with the experimental results that speed of light in free space or vacuum is maximum [1-4].

4. The wave theory

In 1678 by geometrical construction, Christian Huygens (1629 – 1695) proposed the wave theory of light in a communication to the Academie des Science in Paris and in particular demonstrated how waves might interfere to form a wave front in a straight line. This theory showed that speed of light in denser medium is less than that measured in free space which agrees well with the experimental value of speed of light or $\sin\theta/\sin\theta_m = c/c_m$. It also explained satisfactorily the laws of reflection and refraction. However the main drawback of this theory was that light was assumed as a mechanical wave and it was not clear how light came from the sun through the space. Huygens assumed that light is transmitted through all pervading ether medium that is made up of small elastic particles, which was quite unsatisfactory. So the wave nature was not really accepted until the interference experiments by Thomas Young (1773 – 1829) and Augustin-Jean Fresnel (1788 – 1827). The final death blow to the corpuscular theory and the universal acceptance of the wave theory was achieved by Foucault in 1850, who determined the speed of light in different media and showed conclusively that its speed is definitely greater in a rarer medium - a fact supported by the wave theory and contrary to the corpuscular theory [1-4].

5. The electromagnetic theory

In the year 1873 James Clark Maxwell (1831 – 1879) developed a completely different nature of light while working with the variation of electric and magnetic field intensities in space due to oscillatory electric currents. The nature of transfer of energy was thought of as mechanical one at that time. But Maxwell developed wave theory further from his theoretical research and put forward the hypothesis that light was not a mechanical wave but electromagnetic wave and thus transfers only electromagnetic energy. He theoretically deduced that electric and magnetic field are in phase and are at right angles to each other and also to the direction of propagation. The electric vector plays the role of light vector. He called it electromagnetic field wave and deduced the speed of this wave, which is numerically equal to the speed of light. In 1845 Faraday first obtained the definite indication of electromagnetic nature of light when plane of polarization of plane polarized light was rotated in presence of strong magnetic field parallel to beam of light. Hertz remarkably confirmed experimentally Maxwell's predictions in the year 1888. He produced and detected electromagnetic wave and at the same time he demonstrated that these waves could be reflected, refracted, focused, polarized, etc. similar to light wave. Also, the necessity of all pervading medium called ether is not required here for the propagation of this wave and Maxwell's electromagnetic wave can move through free space or vacuum with speed governed by the relation $c = 1/\sqrt{\varepsilon_0 \mu_0}$. Here ε_0 is the permittivity and μ_0 the permeability of free space and it correctly describes the speed of light. So Huygens' and Fresnel's ideas did not meet any opposition in electromagnetic theory of light but only the mechanical wave was replaced by electromagnetic wave and interpretation was done in terms of electrodynamics in place of mechanics. Thus absolute refractive index of a medium will be [1-6] $n_m = c/c_m = \sqrt{\varepsilon_m \mu_m / \varepsilon_0 \mu_0} = \sqrt{\varepsilon_r \mu_r}$.

6. The quantum theory

In 1887, Heinrich Rudolf Hertz (1857 – 1894) discovered the photoelectric effect which did not agree with the established wave theory of light. In 1905 Albert Einstein (1879 – 1955) interpreted the photoelectric effect by putting forward the famous photon theory according to which light consisted of quanta of energy $E = h\nu$. Here ν is the frequency of incident light and h is the Planck's constant. Einstein also showed that the photons in addition to having an energy $h\nu$ should have momentum in free space given by $p = h\nu/c$. This was verified experimentally in 1923 by Arthur Holly Compton (1892 – 1962). Thus dual nature of light was established. One is the particle nature supported by the photoelectric effect and the other is the wave nature supported by interference, diffraction and polarization experiments [7-8].

7. The necessity

Although the refractive index of a medium was not explained properly in corpuscular theory using particle nature of light still the laws of reflection and refraction are simply verified in corpuscular theory. They are also verified in wave theory and also in electromagnetic theory of light. But in quantum theory of light this is not explained properly [9]. Here a simplified and elementary explanation of refractive index of a medium has been given from analogy with other classical concepts and ideas from quantum theory of light.

8. Analogy with other physical phenomena

(i) de Broglie hypothesis and Compton effect

Nature manifests itself in the form of matter and energy and so by symmetry matter possess the same dual nature of particle and wave just like energy. This was de Broglie's hypothesis. Compton effect used the momentum and energy conservation for photon and electron collision [7-8].

(ii) Classical mechanics and optics

The fundamental laws governing mechanics and optics are also very similar. The principle of least action in mechanics (a moving particle always chooses a path of minimum action) is similar to Fermat's principle in optics (light ray always chooses the least path) [1-4,10].

(iii) Conservation of linear momentum in classical mechanics

It can be stated as the time integral of force or impulse is equal to the integral of momentum. Mathematically we can write it as $\vec{p} = \int d\vec{p} = \int \vec{F} dt$. *Linear momentum conservation theorem* can be verified from this theorem. If there is no applied force then $\vec{F} = 0$ and from it we have $\vec{p} = \int d\vec{p} =$ conserved.

(iv) Continuity of electric and magnetic field across the interface separating two different media

A charge free and current free region of space is considered with $\oint \vec{E}\cdot d\vec{l}=0$. Thus, the tangential component of the electric field is continuous across the boundary $\vec{E}_1\cdot\hat{n}=\vec{E}_2\cdot\hat{n}\Rightarrow E_{1t}=E_{2t}$, where $\hat{n}$ is a unit vector along the tangent to the surface. Thus, the tangential component of the electric field is continuous across the boundary. Next, magnetic field $(\vec{H})$ of electromagnetic wave is taken and Ampere's Circuital Law is applied, $H_{1t}=H_{2t}$. Thus, in the absence of any surface current, the tangential components of magnetic intensity are continuous across the interface. Hence, the tangential components of these fields are continuous across the interface or the boundary of the two medium [7-8].

(v) Pressure exerted by radiation.

An electromagnetic radiation exerts a small but finite pressure on the incident surface. It was first observed by Kepler, when the tail of comets continuously veers round so as to be always opposite to the sun. Maxwell's electromagnetic theory gives a theoretical proof while quantum theory of light relates it to the momentum.

(vi) Momentum of electromagnetic wave

The momentum of electromagnetic wave in free space can be easily identified using Poynting's theorem because of known values of electric field $\vec{E}$, electric displacement vector $\vec{D}=\varepsilon_0\vec{E}$, magnetic field $\vec{H}$ and magnetic induction vector $\vec{B}=\mu_0\vec{H}$. But within a medium, we have to make a choice between $(\vec{E},\vec{D})$ and $(\vec{H},\vec{B})$ because there are two entirely reasonable and rival forms for the momentum of light in a medium [11-13]. These are the Minkowski (1908) momentum $\vec{p}_{Minkowski}=\vec{D}\times\vec{B}$ and the Abraham (1909, 1910) momentum $\vec{p}_{Abraham}=\dfrac{\vec{E}\times\vec{H}}{c^2}$. It is 100 years since Minkowski $(p_{Minkowski}=\hbar\omega n_m/c)$ and Abraham $(p_{Abraham}=\hbar\omega/cn_m)$ first gave rival expressions for the momentum of light in a material medium. At the single-photon level, these correspond, respectively, either to multiplying or dividing the free-space value $\hbar\vec{k}$ by the refractive index n_m. The conclusion is that both the Abraham and Minkowski forms of the momentum are correct, with the former being the kinetic momentum and the latter the canonical momentum.

9. Verification of laws of reflection

A photon of frequency ν and wavelength λ falls on an interface MN separating two media as shown in figure 1. We know that light exhibits dual nature – wave like (i.e., as an electromagnetic wave) and particle like (i.e., as photon). Initially it has momentum $h\nu/c$ along the direction AO according to De Broglie hypothesis. Its tangential component will be $(h\nu/c)\sin\theta$ before reflection and $(h\nu'/c)\sin\theta'$ after reflection, where ν' is the frequency of photon after reflection and c is the speed of photon in free space. Since frequency remains unaltered on reflection, so $\nu=\nu'$. Applying linear momentum conservation principle along the tangent to the surface or along MN, we get

$$\frac{h\nu}{c}\sin\theta=\frac{h\nu'}{c}\sin\theta'$$

Thus $\theta=\theta'$ - (2)

We arrive at the conclusion that angle of incidence is equal to the angle of reflection. Now, the path followed by incident photon and reflected photon and the normal at the point of incidence lie on the plane of the paper. Thus, the laws of reflections are verified using quantum model of light.

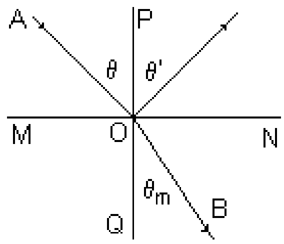

Figure 1 Verification of laws of reflection and refraction

10. Verification of laws of refraction

A photon of frequency ν and wavelength λ falls on an interface MN separating two media as shown in figure 1 of refractive indices, n and . We know that light exhibits dual nature – wave like (i.e., as an electromagnetic wave) and particle like (i.e., as photon). Initially the photon has momentum $h\nu/c$ according to De Broglie hypothesis. The photon will bend towards the normal as $n < n_m$ and away as $n > n_m$. We apply linear momentum conservation principle along the tangent to the interface MN as

$$\frac{h\nu}{c}\sin\theta = \frac{h\nu'}{c_m}\sin\theta_m$$

Thus $$\frac{\sin\theta}{\sin\theta_m} = \frac{c\nu'}{c_m\nu} = \frac{c}{c_m} = \frac{n}{n_m} \qquad \text{- (3)}$$

It is because the colour of light does not change on refraction so $\nu = \nu'$. Here $n = 1$ for free space and $n_m = c/c_m$ is the refractive index of the second medium. Thus Snell's law is verified using quantum model of light. Now, the path followed by incident photon and refracted photon and the normal at the point of incidence lie on the plane of the paper. Thus, the laws of refraction are also proved.

11. Conclusions

Thus as like the corpuscular theory, the wave theory and the electromagnetic theory of light the idea of photon momentum in quantum theory of light can be used to verify the laws of reflection and refraction although there is a debate for photon momentum for over hundred years. The photon momentum increases in a medium in comparison to that in free space as like refractive index. While going through the undergraduate syllabus I found it missing and a link is demonstrated here although elementary yet uses the gray area between the classical and quantum ideas. Emphasis is given so that undergraduate students can think of new ideas and nurture with problems.

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