The Time evolution of a Square Wave Packet and a Triangular Wave Packet

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Abstract

In this article, we discuss the time evolution of a square wave packet and a triangular wave packet. The approach followed in this study is to express a square wave packet and a triangular wave packet as a sum of several Gaussian wave packets. Specifically, the time evolution of a square wave packet has been derived here with three and five Gaussian wave packets; then the time evolution of a triangular wave packet has been derived with three Gaussian wave packets. Their evolution with time has been plotted using MatLab™ over appropriately chosen time intervals. The results are compared with those of a Gaussian wave packet.

Keywords:- Wave packet, Schrödinger equation, Gaussian wave packet, Triangular Wave Packet, Time Evolution, MatLab™.

1. Introduction

Wave packets are superposition of plane waves used in representing a particle. According to de Broglie's matter-waves hypothesis, material particles such as photons and electrons exhibit wave nature and show wave phenomena such as interference and diffraction. For a localized particle, the superposition of many plane waves results in a function called the wave function ψ. The wave packets are decomposed by Fourier Transformation and their time evolution is found which is of physical interest. In this article, the time evolutions of non-Gaussian wave packets such as the ones mentioned in the abstract are found. The time evolutions of square wave packet and the triangular wave packet are of interest as they are often encountered in wave analyses. Using Green's function approach, Mita (2007) shows that the probability amplitude of any non-Gaussian wave packet approximately becomes a Gaussian as it disperses [1]. Here we obtain the same result using a simpler approach of approximating a square wave packet and a triangular wave packet as a sum of several Gaussian wave packets. Mita (2007) points out the following advantages of using a Gaussian wave packet:

a) A Gaussian function is easy to analyze in closed form

b) The Fourier Transform of a Gaussian is also a Gaussian and

c) The Gaussian wave packet gives rise to a minimum uncertainty product at time t=0. [1]

In order to study the time evolution, we use the following form of the Gaussian wave packet as given by Greiner, W (2004) [2]. At time t=0,

$$\psi(x,0) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2} + ik_0x\right] \qquad (1)$$

where the wave function depends on the position and time coordinates, σ is the standard deviation from the mean μ, the term $\frac{1}{\sigma\sqrt{2\pi}}$ is the amplitude of the wave packet and k_0 is the wave number.

For the sake of comparison, we write the expressions or the time evolution and the probability distribution of the Gaussian wave packet. We have also plotted the time evolution of the Gaussian wave packet using MatLab™ for the sake of comparison. The evolution of the Gaussian wave packet at time t is given by

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}\sigma\left(1+\frac{i\hbar t}{m\sigma^2}\right)^{\frac{1}{2}}} exp\left(\frac{2\sigma^2 ik_0x - \frac{i\hbar t{k_0}^2\sigma^2}{m} - \frac{2\hbar tk_0\mu}{m} - (x-\mu)^2}{2\sigma^2\left(1+\frac{i\hbar t}{m\sigma^2}\right)}\right) \qquad (2)$$

The probability distribution is given by

$$|\psi(x,t)|^2 = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi\sigma^2} exp\left(-\frac{\left\{(x-\mu)-\frac{\hbar tk_0}{m}\right\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right) \qquad (3)$$

where $\hbar = \frac{h}{2\pi}$; h is the Planck's constant. This is also a Gaussian distribution with width $\sigma = \sqrt{1+\left(\frac{\hbar t}{m\sigma^2}\right)^2}$. We have assumed k_0 to be zero in order to simplify the calculation implying that the wave packet is at rest. The following values were used to plot the expression (3) in MatLab™:

Planck's constant $h = 6.6X10^{-34}$ Js

Mass of electron $m = 9.1X10^{-31}$kg

Mean value for the Gaussian wave μ = 0

Standard deviation of the wave σ = 0.5.

The following graph was obtained when the expression (3) was plotted with the above mentioned numerical values was plotted for different values of time t.

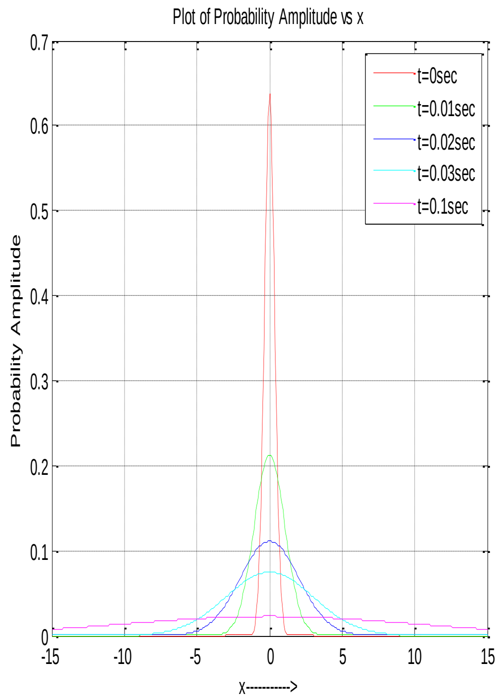

Figure 1: Time Evolution of a Gaussian wave function

2. Square Wave Packet

Consider a square wave packet with amplitude A and width Δx as shown below.

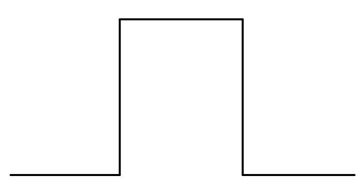

Figure 2: A square wave packet with amplitude A and width Δx

If we try to find its time evolution by the standard method, the integrations encountered are hard to solve. Hence, in order to simplify the calculations, the square wave packet is expressed as a sum of Gaussian wave packets of same width and amplitude as shown below. In order to find the time evolution of the approximated square wave packet, we find the time evolution of the system of Gaussian wave packets. Let us assume that the square wave packet is comprised of three Gaussian wave packets. Let their wave functions be ψ_1, ψ_2 and ψ_3; let their mean values be μ_1, μ_2 and μ_3 and let σ be the standard deviation. The forms of ψ at time t=0 and at a later time t for a single Gaussian wave packet is given by equations (1) and (2) respectively. Thus, the square wave packet is expressed as

$$\Psi(x,0) = \psi_1(x,0) + \psi_2(x,0) + \psi_3(x,0)$$

$$\Psi(x,0) = \frac{1}{\sigma\sqrt{2\pi}}\ \exp\left[-\frac{(x-\mu_1)^2}{2\sigma^2} + ik_0x\right] + \frac{1}{\sigma\sqrt{2\pi}}\ \exp\left[-\frac{(x-\mu_2)^2}{2\sigma^2} + ik_0x\right] + \frac{1}{\sigma\sqrt{2\pi}}\ \exp\left[-\frac{(x-\mu_3)^2}{2\sigma^2} + ik_0x\right]$$

(4)

and at a later time t>0, the square wave packet is expressed as

$$\Psi(x,t) = \psi_1(x,t) + \psi_2(x,t) + \psi_3(x,t)$$

$$\Psi(x,t) =$$

$$\frac{1}{\sqrt{2\pi}\sigma\left(1+\frac{i\hbar t}{m\sigma^2}\right)^{\frac{1}{2}}}\left\{exp\left(\frac{2\sigma^2 ik_0x - \frac{i\hbar t{k_0}^2\sigma^2}{m} - \frac{2\hbar tk_0\mu_1}{m} - (x-\mu_1)^2}{2\sigma^2\left(1+\frac{i\hbar t}{m\sigma^2}\right)}\right) + exp\left(\frac{2\sigma^2 ik_0x - \frac{i\hbar t{k_0}^2\sigma^2}{m} - \frac{2\hbar tk_0\mu_2}{m} - (x-\mu_2)^2}{2\sigma^2\left(1+\frac{i\hbar t}{m\sigma^2}\right)}\right) + \right.$$

exp2σ2ik0x−iħtk02σ2m−2ħtk0μ3m−x−μ322σ21+iħtmσ2 (5)

The probability distribution P(x,t) of the system of three Gaussian wave packets is given by

$$P(x,t) = |\Psi|^2 = |\psi_1 + \psi_2 + \psi_3|^2$$

$$|\Psi|^2 = {\psi_1}^2 + {\psi_2}^2 + {\psi_3}^2 + 2Re(\psi_1{\psi_2}^*) + 2Re(\psi_1{\psi_3}^*) + 2Re(\psi_2{\psi_3}^*) \qquad (6)$$

where the asterisk indicates complex conjugate. The probability distribution for a single Gaussian wave packet is given by equation (3). Therefore, we can write for ${\psi_1}^2$, ${\psi_2}^2$ and ${\psi_3}^2$in equation (6) as

$${\psi_1}^2 = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi\sigma^2} exp\left(-\frac{\left\{(x-\mu_1)-\frac{\hbar tk_0}{m}\right\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)$$

$${\psi_2}^2 = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi\sigma^2} exp\left(-\frac{\left\{(x-\mu_2)-\frac{\hbar tk_0}{m}\right\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)$$

$$\psi_3{}^2 = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi\sigma^2} exp\left(-\frac{\left\{(x-\mu_3)-\frac{\hbar t k_0}{m}\right\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)$$

For the term $2Re(\psi_1\psi_2{}^*)$ in equation (6), we write

$$2Re(\psi_1\psi_2{}^*) = 2\frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi\sigma^2} Re\ exp\left\{\left(\frac{2\sigma^2 ik_0x - \frac{i\hbar t k_0{}^2\sigma^2}{m} - \frac{2\hbar t k_0\mu_1}{m} - (x-\mu_1)^2}{2\sigma^2\left(1+\frac{i\hbar t}{m\sigma^2}\right)}\right) + \left(\frac{-2\sigma^2 ik_0x + \frac{i\hbar t k_0{}^2\sigma^2}{m} - \frac{2\hbar t k_0\mu_2}{m} - (x-\mu_2)^2}{2\sigma^2\left(1-\frac{i\hbar t}{m\sigma^2}\right)}\right)\right\}$$

Simplifying

$$2Re(\psi_1\psi_2{}^*) = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{\pi\sigma^2} exp\left\{\left(-\frac{\left((x-\mu_1)-\frac{\hbar k_0 t}{m}\right)^2 + \left((x-\mu_2)-\frac{\hbar k_0 t}{m}\right)^2}{2\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)\right\}$$

Similarly, we can get $2Re(\psi_1\psi_3{}^*)$ and $2Re(\psi_2\psi_3{}^*)$ as

$$2Re(\psi_1\psi_3{}^*) = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{\pi\sigma^2} exp\left\{\left(-\frac{\left((x-\mu_1)-\frac{\hbar k_0 t}{m}\right)^2 + \left((x-\mu_3)-\frac{\hbar k_0 t}{m}\right)^2}{2\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)\right\}$$

and,

$$2Re(\psi_2\psi_3{}^*) = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{\pi\sigma^2} exp\left\{\left(-\frac{\left((x-\mu_2)-\frac{\hbar k_0 t}{m}\right)^2 + \left((x-\mu_3)-\frac{\hbar k_0 t}{m}\right)^2}{2\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)\right\}$$

We assume k_0 to be zero and use the determined forms of the terms in the LHS of equation (6) and rewrite it in the final form as

$$|\Psi|^2=\frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{\pi\sigma^2}\left[\frac{1}{2}\left\{exp\left(-\frac{\{(x-\mu_1)\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)+exp\left(-\frac{\{(x-\mu_2)\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)+exp\left(-\frac{\{(x-\mu_3)\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)\right\}+\right.$$
$$\left.exp\left(-\frac{(x-\mu_1)^2+(x-\mu_2)^2}{2\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)+exp\left(-\frac{(x-\mu_1)^2+(x-\mu_3)^2}{2\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)+exp\left(-\frac{(x-\mu_2)^2+(x-\mu_3)^2}{2\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)\right] \quad (7)$$

For the estimating with three Gaussian wave packets, a square wave function with some arbitrary amplitude within x=0 to x=2 and zero elsewhere was estimated. The Gaussian wave packets had a full width at half maxima equal to $\sigma = 2/10$. The three Gaussian wave packets had mean values at $\mu_1 = 1/3$, $\mu_2 = 1$ and $\mu_3 = 10/6$. For the purpose of approximation, a Gaussian wave packet of the form given in equation (1) was used with $k_0 = 0$. The plot thus obtained is as below

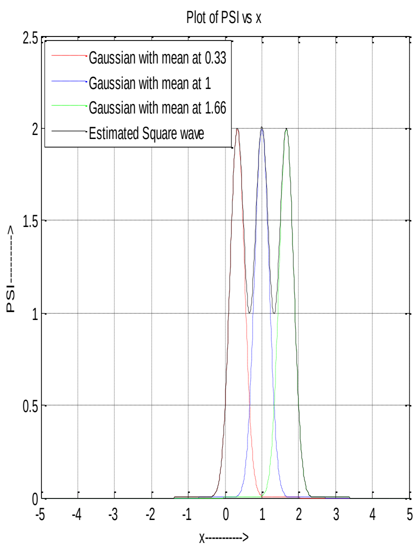

Figure 3: Approximation of a Square wave packet by three Gaussian wave packets

Now, with the same values of mean and standard deviation, the probability distribution of the approximated square wave packet given by equation (7) is plotted against x for different values of time't'. Here, again we use the same values of m and $\hbar$ as in section **1.** The plot obtained is as below.

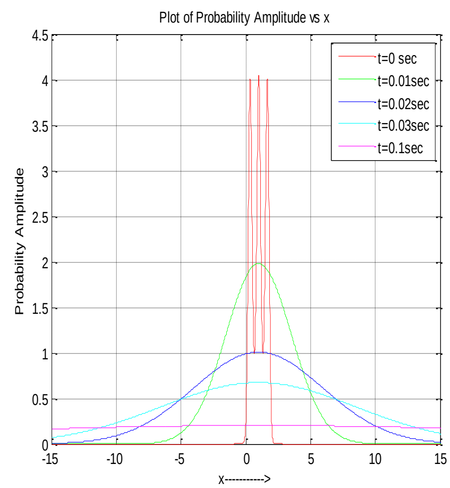

Figure 4: Time Evolution of a Square Wave packet approximated by three Gaussian wave packets

Thus, we see that as the square packet evolves with time, it spreads and approximately becomes a Gaussian.

The square wave packet was also approximated by five Gaussian wave packets with standard deviation $\sigma = 1/5$. The mean values for the Gaussian wave packets were taken to be $\mu_1 = 2/15$, $\mu_2 = 17/30$, $\mu_3 = 1$, $\mu_4 = 43/30$ and $\mu_5 = 28/15$. The width of the square wave packet was fixed to be a=2 and then the interval was divided into 5 parts a/15, 17a/60, a/2, 43a/60 and 14a/15. The resulting figure is shown below.

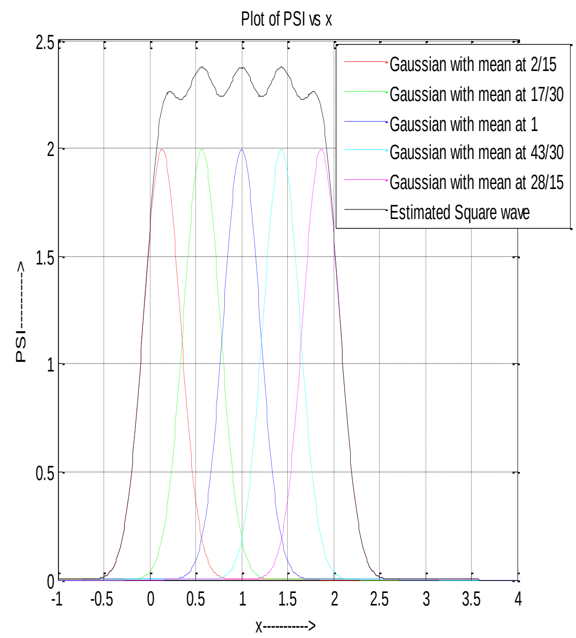

Figure 5: Estimation of Square wave packet by five Gaussian wave packets

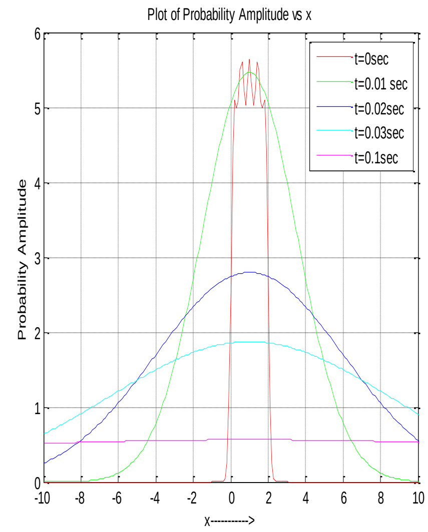

Figure 6: Time evolution of Square wave packet with five Gaussian wave packets

The probability distribution of the Gaussian approximation of the square wave packet above is then plotted and is shown below.

We note that the square wave packet estimated by five Gaussian wave packets gives a better approximation. See section **4** for a detailed discussion.

3. Triangular Wave Packet

In this section, we discuss the time evolution of a triangular wave packet expressed as a sum of Gaussian wave packets. Consider a triangular wave packet which is comprised of three Gaussian wave packets as shown below. Let the wave functions of the three Gaussian wave packets be ψ_1 , ψ_2 and ψ_3 and let their mean values be μ_1, μ_2 and μ_3. Let the standard deviations of the three wave functions be σ_1 and σ_2. Here the two Gaussians on either side of the central Gaussian wave have the same standard deviation. The time evolution of a single Gaussian wave packet is given by equation (2). At time t=0, the wave function of the system of triangular wave packet resembles the form of equation (4). At a later time t>0, the wave function of the system is expressed as

$$\Psi(x,t) = \psi_1(x,t) + \psi_2(x,t) + \psi_3(x,t)$$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}\sigma_1\left(1+\frac{i\hbar t}{m{\sigma_1}^2}\right)^{\frac{1}{2}}}\left\{exp\left(\frac{2{\sigma_1}^2 ik_0x - \frac{i\hbar t{k_0}^2{\sigma_1}^2}{m} - \frac{2\hbar tk_0\mu_1}{m} - (x-\mu_1)^2}{2{\sigma_1}^2\left(1+\frac{i\hbar t}{m{\sigma_1}^2}\right)}\right)\right.$$

$$\left. + exp\left(\frac{2{\sigma_1}^2 ik_0x - \frac{i\hbar t{k_0}^2{\sigma_1}^2}{m} - \frac{2\hbar tk_0\mu_1}{m} - (x-\mu_3)^2}{2{\sigma_1}^2\left(1+\frac{i\hbar t}{m{\sigma_1}^2}\right)}\right)\right\}$$

$$+ \frac{1}{\sqrt{2\pi}\sigma_2\left(1+\frac{i\hbar t}{m{\sigma_2}^2}\right)^{\frac{1}{2}}} exp\left(\frac{2{\sigma_2}^2 ik_0x - \frac{i\hbar t{k_0}^2{\sigma_2}^2}{m} - \frac{2\hbar tk_0\mu_2}{m} - (x-\mu_2)^2}{2{\sigma_2}^2\left(1+\frac{i\hbar t}{m{\sigma_2}^2}\right)}\right)$$

The probability distribution of the system is given by

$$P(x,t) = |\Psi|^2 = |\psi_1 + \psi_2 + \psi_3|^2$$

$$|\Psi|^2 = {\psi_1}^2 + {\psi_2}^2 + {\psi_3}^2 + 2Re(\psi_1{\psi_2}^*) + 2Re(\psi_1{\psi_3}^*) + 2Re(\psi_2{\psi_3}^*) \qquad (8)$$

The first three terms on the LHS of the above equation are

$${\psi_1}^2 = \frac{\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi{\sigma_1}^2} exp\left(-\frac{\left\{(x-\mu_1)-\frac{\hbar tk_0}{m}\right\}^2}{{\sigma_1}^2\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)}\right)$$

$${\psi_2}^2 = \frac{\left(1+\left(\frac{\hbar t}{m{\sigma_2}^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi{\sigma_2}^2} exp\left(-\frac{\left\{(x-\mu_2)-\frac{\hbar tk_0}{m}\right\}^2}{{\sigma_2}^2\left(1+\left(\frac{\hbar t}{m{\sigma_2}^2}\right)^2\right)}\right)$$

$${\psi_3}^2 = \frac{\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi{\sigma_1}^2} exp\left(-\frac{\left\{(x-\mu_3)-\frac{\hbar tk_0}{m}\right\}^2}{{\sigma_1}^2\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)}\right)$$

In the above equation for $\psi_3{}^2$, σ_1 appears as we have assumed that the Gaussians on either side of the central Gaussian gave the same standard deviation i.e. $\sigma_1 = \sigma_3$. Also, the amplitude of ψ_2 depends on the slope of the triangle. The remaining terms in equation (8) are written as

$$2Re(\psi_1\psi_2{}^*)$$

$$= \frac{\left(1+\frac{\hbar^2t^2}{m^2\sigma_1{}^2\sigma_2{}^2}\right)^{-\frac{1}{2}}}{\pi\sigma_1{}^2\sigma_2{}^2} exp\left[-\frac{(\sigma_1{}^2+\sigma_2{}^2)\left(x-\frac{\hbar t k_0}{m}\right)^2-2(\mu_2\sigma_1{}^2+\mu_1\sigma_2{}^2)\left(\frac{\hbar t k_0}{m}-x\right)+\mu_2\sigma_1{}^2+\mu_1\sigma_2{}^2}{2\sigma_1{}^2\sigma_2{}^2\left(1+\frac{\hbar^2t^2}{m^2\sigma_1{}^2\sigma_2{}^2}\right)}\right]$$

And similarly

$$2Re(\psi_1\psi_3{}^*)$$

$$= \frac{\left(1+\frac{\hbar^2t^2}{m^2\sigma_1{}^2\sigma_3{}^2}\right)^{-\frac{1}{2}}}{\pi\sigma_1{}^2\sigma_3{}^2} exp\left[-\frac{(\sigma_1{}^2+\sigma_3{}^2)\left(x-\frac{\hbar t k_0}{m}\right)^2-2(\mu_3\sigma_1{}^2+\mu_1\sigma_3{}^2)\left(\frac{\hbar t k_0}{m}-x\right)+\mu_3\sigma_1{}^2+\mu_1\sigma_3{}^2}{2\sigma_1{}^2\sigma_3{}^2\left(1+\frac{\hbar^2t^2}{m^2\sigma_1{}^2\sigma_3{}^2}\right)}\right]$$

$$2Re(\psi_2\psi_3{}^*)$$

$$= \frac{\left(1+\frac{\hbar^2t^2}{m^2\sigma_3{}^2\sigma_2{}^2}\right)^{-\frac{1}{2}}}{\pi\sigma_3{}^2\sigma_2{}^2} exp\left[-\frac{(\sigma_3{}^2+\sigma_2{}^2)\left(x-\frac{\hbar t k_0}{m}\right)^2-2(\mu_2\sigma_3{}^2+\mu_3\sigma_2{}^2)\left(\frac{\hbar t k_0}{m}-x\right)+\mu_2\sigma_3{}^2+\mu_3\sigma_2{}^2}{2\sigma_3{}^2\sigma_2{}^2\left(1+\frac{\hbar^2t^2}{m^2\sigma_3{}^2\sigma_2{}^2}\right)}\right]$$

Once again, we assume the value of k_0 to be zero and since we have also assumed $\sigma_1 = \sigma_3$, we rewrite equation (8) in the final form as

$$\Psi^2 = \frac{\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi{\sigma_1}^2} exp\left(-\frac{\{(x-\mu_1)\}^2}{{\sigma_1}^2\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)}\right) + \frac{\left(1+\left(\frac{\hbar t}{m{\sigma_2}^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi{\sigma_2}^2} exp\left(-\frac{\{(x-\mu_2)\}^2}{{\sigma_2}^2\left(1+\left(\frac{\hbar t}{m{\sigma_2}^2}\right)^2\right)}\right)$$

$$+ \frac{\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi{\sigma_1}^2} exp\left(-\frac{\{(x-\mu_3)\}^2}{{\sigma_1}^2\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)}\right)$$

$$+ \frac{\left(1+\frac{\hbar^2 t^2}{m^2{\sigma_1}^2{\sigma_2}^2}\right)^{-\frac{1}{2}}}{\pi{\sigma_1}^2{\sigma_2}^2} exp\left[-\frac{({\sigma_1}^2+{\sigma_2}^2)x^2 + 2(\mu_2{\sigma_1}^2+\mu_1{\sigma_2}^2)x + \mu_2{\sigma_1}^2+\mu_1{\sigma_2}^2}{2{\sigma_1}^2{\sigma_2}^2\left(1+\frac{\hbar^2 t^2}{m^2{\sigma_1}^2{\sigma_2}^2}\right)}\right]$$

$$+ \frac{\left(1+\frac{\hbar^2 t^2}{m^2{\sigma_1}^2{\sigma_1}^2}\right)^{-\frac{1}{2}}}{\pi{\sigma_1}^2{\sigma_1}^2} exp\left[-\frac{({\sigma_1}^2+{\sigma_1}^2)x^2 + 2(\mu_3{\sigma_1}^2+\mu_1{\sigma_1}^2)x + \mu_3{\sigma_1}^2+\mu_1{\sigma_1}^2}{2{\sigma_1}^2{\sigma_1}^2\left(1+\frac{\hbar^2 t^2}{m^2{\sigma_1}^2{\sigma_1}^2}\right)}\right]$$

$$+ \frac{\left(1+\frac{\hbar^2 t^2}{m^2{\sigma_1}^2{\sigma_2}^2}\right)^{-\frac{1}{2}}}{\pi{\sigma_1}^2{\sigma_2}^2} exp\left[-\frac{({\sigma_1}^2+{\sigma_2}^2)x^2 + 2(\mu_2{\sigma_1}^2+\mu_3{\sigma_2}^2)x + \mu_2{\sigma_1}^2+\mu_3{\sigma_2}^2}{2{\sigma_1}^2{\sigma_2}^2\left(1+\frac{\hbar^2 t^2}{m^2{\sigma_1}^2{\sigma_2}^2}\right)}\right]$$

In our numerical analysis, the standard deviation of the central Gaussian wave packet i.e. σ_2 was chosen to be 0.5 and σ_1 and σ_3 were chosen to be 1. The mean values were chosen to be -5/2.5, 0 and 5/2.5.

The probability distribution of the approximated triangular wave packet is plotted against x for different values of time t. The values of m and $\hbar$ were the same as the ones used in sections **1** and **3.**

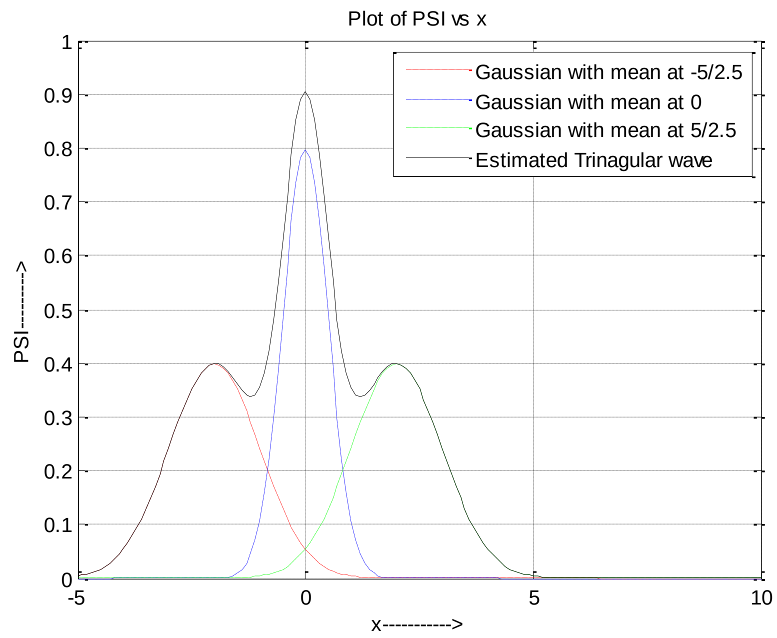

Figure 7: Estimation of Triangular wave packet by three Gaussian wave packets

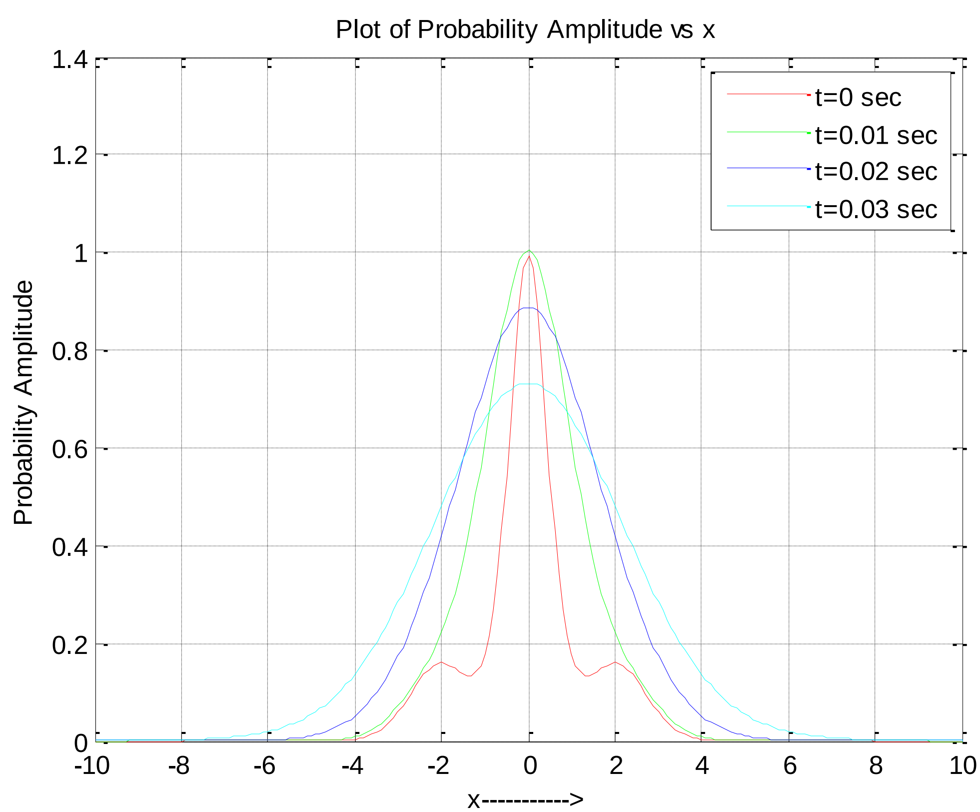

Figure 8: Time Evolution of Triangular wave approximated by 3 Gaussians

4. Results and Discussion

When the Square wave packet was approximated with three Gaussians, its amplitude was found to be 2 (fig. 4). Here the estimated square wave consisted of dips where the overlapping of Gaussian waves was not as significant. This is due to the intermediate terms in equation (7). Its probability amplitude was doubled, i.e. approx. 4 at time t=0, which is as expected. The probability amplitude at time t=0 consisted of some irregularities in the peak. With time, as the Gaussian waves evolved, so did the estimated square wave and thus the probability distribution of the square wave became smoother, broader and assumed an almost Gaussian shape which is clearly visible in fig. 5. In the case of Square wave approximated with five Gaussians, the dip in the final wave form reduced considerably due to significant overlapping thus estimating the square wave packet better than the one with the three Gaussian wave packets. Thus we see that the accuracy increases when the number of wave packets is increased. The probability distributions at t=0 and at later times are assumed to have the same behavior as mentioned earlier.

Similar results were obtained in the case of a Triangular wave packet being approximated by three Gaussian wave packets.

In the method adopted, care must be taken while choosing the standard deviation and mean values for the approximating Gaussian waves. The advantage of this method is that it is applicable for any wave packet in principle. The number of iterations can be improved by computer programming since there is an increase in the number of wave functions and also the number of times they are added

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