Simulation of Earth orbit around Sun by Computational Method

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Abstract

Solar system is filled with many celestial bodies like planets, meteors, asteroids etc. Most of these bodies are rotating around the Sun. The study of orbital motions of the celestial bodies is important for astronomical point of view. In this paper the orbital motion of Earth is being simulated by using Keplar's Model of orbital Motion and Newton's Laws of motions. The simulation is achieved by using numerical method (Euler Cromer Method) and MATLAB software. Results show that varying time step in years and initial velocities of Earth motion, the Earth is getting unstable from its orbital motion.

1. Introduction

From the beginning of human civilizations, humans showed his great interest in exploring physical phenomenon and processes of the Earth and universe. Now, It is well known that the solar system consist Sun at its centre and having celestial bodies like planets, asteroids, meteors, comets etc. A planets move around Sun. Earth is the third planet after mercury and Venus from the Sun.

Studying orbital motions of celestial bodies have great importance in astrophysics. Earlier record shows that Nicolaus Copernicus was the first to develop an equation for a heliocentric system of Sun**.** Galileo Galilei, Johannes Kepler and Isaac Newton, (in the 17th Centuary) developed an understanding of physics that led to the gradual acceptance of the idea that Earth moves around the physical laws that governed Earth [1].

The Earth orbit is the motion of Earth around Sun with an average distance 149.59787 million kilometers away. A complete orbit of the Earth around the Sun occurs every nearly in a one year. The orbital speed of the Earth around the Sun averages about 30 km/s which is fast enough to cover the planet's diameter (about 12,700 km) in seven minutes, and the distance to the Moon of 384,000 km in four hours [2].

Computer is now providing solutions in each field of work and study. The modeled physical processes and phenomenon can now be easily solved by numerical methods and using computers

[3]. Until 1950s the only methods used for obtaining numerical result lies on physical methods such as separation of variables, contour integration and conforming mapping [4]. Solving physical problems by Numerical methods have more advantages than analytical methods. Euler Cromer method, Newton Remption, Ruge Cutta Methods, Monte Carlo Methods and lot are major examples of numerical methods.

In this paper the simulation of the Earth orbit around Sun has been presented. Euler Cromer method [5] is used as a numerical technique and MATLAB is used for computation. The simulation of the orbital motion at different velocities and time steps is also analyzed. The obtained results are presented in the result section of the article.

2. Modeling and Computation

Orbit modeling is the process of creating mathematical models to simulate motion of a massive body as it moves in orbit around another massive body due to gravity. Earth orbit is modeled by using Keplar's laws and Newton's laws of Motion. These are discussed as follows:

A. Kepler's laws are given by the following three statements:

- The planets move in elliptical orbits around the sun. The sun resides at one focus.
- The line joining the sun with any planet sweeps out equal areas in equal times.
- Given an orbit with a period T and a semi major axis a: the ratio T^2/a^3 is a constant.

B. Newton's Second Law
Let us consider the motion of the Earth around the Sun. Let r be the distance and M_s and M_e be the masses of the Sun and the Earth respectively. After neglecting the effect of the Other planets and the motion of the Sun (i.e. we assume that $M_s >> M_e$). Our goal is to calculate the position of the Earth as a function of time

From the Newton's Second law of motion

$$M_e \frac{d^2\vec{r}}{dt^2} = -\frac{GM_eM_s}{r^3}\vec{r}$$

$$= -\frac{GM_eM_s}{r^3}(x\vec{i} + y\vec{i}) \quad \text{.................................1}$$

We get the two equations

$$\frac{d^2x}{dt^2} = -\frac{GM_s}{r^3}x \quad \text{............................2}$$

$$\frac{d^2y}{dt^2} = -\frac{GM_s}{r^3}y \quad \text{............................3}$$

We replace these two second-order differential equations by the four first-order differential equations

$$\frac{dx}{dt} = v_{x.} \quad \text{..4}$$

$$\frac{dv_x}{dt} = -\frac{GM_s}{r^3}x \quad \text{................................5}$$

$$\frac{dy}{dt} = v_y \quad \text{..6}$$

$$\frac{dv_y}{dt} = -\frac{GM_s}{r^3}y \quad \text{................................7}$$

We recall

$$r = \sqrt{x^2 + y^2} \quad \text{..................................8}$$

C. Euler Cramer modeling of the Newton's equation of motions is as follows:

The time discretization is

$$t \equiv t(i) = i\Delta t, i = 0,........, N \quad \text{................9}$$

The total time interval is $T = N\Delta t$. We define;
$x(t) = x(i)$, $v_x(t) = v_x(i)$, $y(t) = y(i)$, $v_y(t) = v_y(i)$.

Equations (4), (5), (6),(7) and (8) become for
i = 0, ...,N

$$v_x(i+1) = v_x(i) - \frac{GM_s}{(r(i))^3}\{x(i).\Delta t\} \quad \text{...........10}$$

$$x(i + 1) = x(i) + v_x(i)\ \Delta t \quad \text{..................11}$$

$$v_y(i+1) = v_y(i) - \frac{GM_s}{(r(i))^3}\{y(i).\Delta t\} \text{...........12}$$

$$y(i+1) = y(i) + v_{y}(i)\,\Delta t. \text{ ....13}$$

$$r(i) = \sqrt{x(i)^2 + y(i)^2} \text{.................................14}$$

D. The MALAB code of Euler Cromer equations for simulation of orbital motion of Earth around Sun is given below for time step 0.002 in years and initial velocity 0 in AU/year. The initial position of planet is taken at 1 AU from the Sun.

```
>> npoints=500;

>> dt=0.002;

>> x=1;

>> y=0;

>> v_x=0;

>> v_y=2*pi;

>>plot(0,0,'oy','MarkerSize',30,'MarkerFaceColor', 'yellow');

>> axis([-1 1 -1 1]);

>> xlabel('x(AU)');

>> ylabel('y(AU)');

>> hold on;

>> for step=1:npoints-1;

radius=sqrt(x^2+y^2);

v_x_new=v_x-(4*pi^2*x*dt)/(radius^3);

v_y_new=v_y-(4*pi^2*y*dt)/(radius^3);

x_new=x+v_x_new*dt;

y_new=y+v_y_new*dt;

plot(x_new,y_new,'-k');

drawnow;

v_x=v_x_new;

v_y=v_y_new;

>> x=x_new;

>> y=y_new;

>> end;
```

3. Results and Discussions

The obtained result from the MATLAB simulation using following sets of values (dt=0.002, V_Y=2*Pi), (dt=0.05, V_Y=2*pi), (dt=0.05, V_Y=4), (dt=0.002, V_Y=4) are shown in figures 1, 2, 3and 4 respectively.

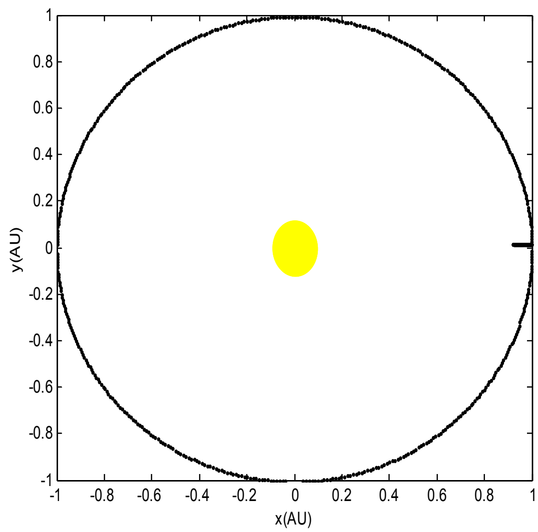

Fig. 1: Simulation of Earth Orbit around Sun (dt=0.002, V_Y=2*pi)

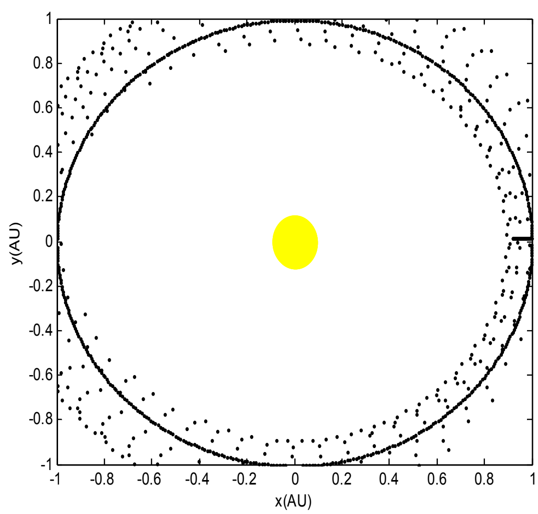

Fig. 2: Simulation of Earth Orbit around Sun (dt=0.05, V_Y=2*pi)

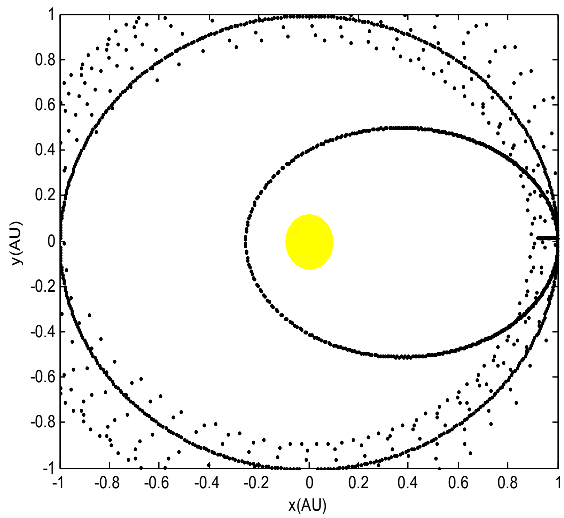

Fig. 3: Simulation of Earth Orbit around Sun (dt=.05 V_Y=4)

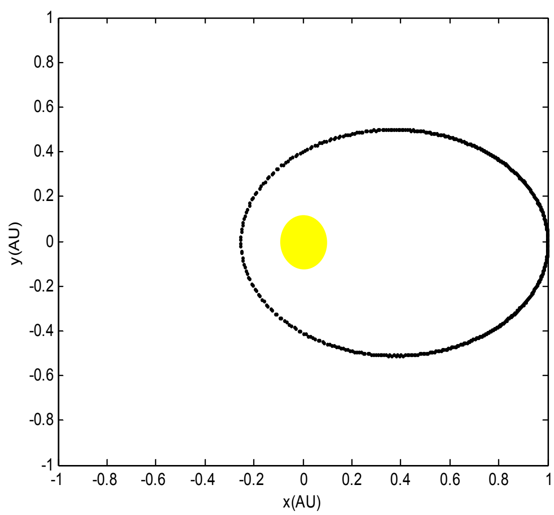

Fig. 4: Simulation of Earth Orbit around Sun (dt=.002 and v_y=4)

The results are very significant as the value of time step increases from the 0.002 years. The orbital motion is unstable in case of 0.05 years. In figure 2 the time step is increased to 0.05 years which shows a clear unstable orbit of Earth. It is also visible in figure 3. While in figure 4 the stable orbit obtained with an increased velocity. This is in accordance with the rule of thumb that the time step should be less than 1% of the characteristic time scale of the problem.

4. Conclusions

From the above simulation it is clear that varying time step the earth orbit get unstable. This is according to the rule of thumb that the time step should be less than 1% of the characteristic time scale of the problem.
Secondly computational methods have great advantage in simulating physical problems by using computer software and numerical methods.

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