Squaring and Cubing of Binary Numbers

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Abstract

The book on 'Vedic Mathematics' written by Jagadguru Sankaracarya Sri Bharati Krisna Tirthaji Maharaja contains 16 Sutras or formulas and 13 Sub-Sutras or corollaries to carry out various arithmetical operations. The sutra 'Urdhva Tiryagbhyam' is used for multiplication of two decimal numbers containing equal number of digits. The squaring of a binary number can be done using this sutra. The sub-sutra 'Anurupyena' is used to find the cube of a decimal number. The cube of a binary number can be evaluated using this sub-sutra. The multiplication of binary numbers is used in the field of digital signal processing for the design of digital multipliers.

1.Introduction

The book on 'Vedic Mathematics' written by Jagadguru Sankaracarya Sri Bharati Krisna Tirthaji Maharaja was first published in 1965. It contains 16 Sutras or formulas and 13 Sub-Sutras or corollaries to carry out various arithmetical operations. The meaning of the sutra 'Urdhva Tiryagbhyam' is vertically and cross-wise. This sutra is used for multiplication of two decimal numbers containing equal number of digits. The squaring of binary numbers can be done using this sutra. The sub-sutra 'Anurupyena' is used for cubing a decimal number. This sub-sutra can be used for finding the cube of a binary number. A large amount of research work has so far been done for understanding these sutras.

The multiplication of binary numbers is used in the field digital signal processing for the design of digital multipliers. The speed of a digital signal processor is largely determined by the speed of its multipliers. The squaring operation also forms the backbone in cryptography.

The rest the paper is organized as follows. Multiplication of two decimal numbers using 'Urdhva Tiryagbhyam' sutra is given in Section 2. In Section 3, the squaring of a binary number using this sutra is given. The multiplier based on 'Urdhva Tiryagbhyam' sutra for squaring a 2-bit binary number is presented in Section 4.The cubing of a decimal number using the sub-sutra

'Anurupyena' is given in Section 5. In Section 6, the cubing of a binary number using this sub-sutra is discussed.

2. Multiplication of Two Decimal Numbers Using 'Urdhva Tiryagbhyam' Sutra

The meaning of the sutra 'Urdhva Tiryagbhyam' is vertically and cross-wise. This sutra is used for the multiplication of two decimal numbers containing equal number of digits. The multiplication of two decimal numbers basing on this sutra is illustrated in the following examples.

Example 1: Multiplication of two 2-digit decimal numbers 84 and 52.

Let the product of 84 and 52 is $P_3P_2P_1P_0$.

$$\begin{array}{rl} & 84 \text{ - Multiplicand} \\ \times & 52 \text{ - Multiplier} \\ \hline & P_3P_2P_1P_0 \\ \hline \end{array}$$

The multiplication of these numbers is carried out as per the following steps.

Step1: Multiply the right-hand-most digit 4 of the multiplicand vertically by the right-hand-most digit 2 of the multiplier.

$$\begin{array}{c} 4 \\ \downarrow \\ 2 \\ \hline 4 \times 2 = 8 \\ \hline \end{array}$$

Step 2: Multiply 8 and 2, and 4 and 5 cross-wise. Add the products.

$$\begin{array}{cc} 8 & 4 \\ 5 & 2 \\ \hline \multicolumn{2}{c}{8\times 2 + 4\times 5 = 36} \\ \hline \end{array}$$

Step 3: Multiply the left-hand-most digit 8 of the multiplicand with the left-hand-most digit 5 of the multiplier.

$$\begin{array}{c} 8 \\ \downarrow \\ 5 \\ \hline 8 \times 5 = 40 \\ \hline \end{array}$$

Step 1: 8; 8 carry 0; $P_0 = 8$

Step 2: 36 + 0 = 36; 6 carry 3; $P_1 = 6$

Step 3: 40 + 3 = 43; 3 carry 4; $P_2 = 3$

Carry of *step 3* = $P_3 = 4$

Thus, the product is 4368.

Example 2: Multiplication of two 3-digit decimal numbers 395 and 746.

$$\begin{array}{r} 395 \\ \times 746 \\ \hline P_5P_4P_3P_2P_1P_0 \\ \hline \end{array}$$

The steps for multiplication are given below.

Step 1

5

↓

6

5 × 6 = 30

Step 2

9 5

4 6

9×6 + 5×4=74

Step 3

3 9 5

7 4 6

3 × 6 + 9 × 4 + 5 × 7 = 89

Step 4

3 9

7 4

3×4+9×7 = 75

Step 5

3

↓

7

3 × 7 = 21

Step 1: 30; 0 carry 3; $P_0 = 0$

Step 2: 74 + 3 = 77; 7 carry 7; $P_1 = 7$

Step 3: 89 + 7 = 96; 6 carry 9; $P_2 = 6$

Step 4: 75 + 9 = 84; 4 carry 8; $P_3 = 4$

Step 5: 21 + 8 = 29; 9 carry 2; $P_4 = 9$

Carry of *step 5* = $P_5 = 2$

Thus, the product is 294670.

Example 3: Multiplication of two 4-digit decimal numbers 9731 and 4682.

$$\begin{array}{r} 9731 \\ \times\, 4682 \\ \hline P_7P_6P_5P_4P_3P_2P_1P_0 \\ \hline \end{array}$$

9 5

This multiplication is done as per the following steps.

Step 1

1

↓

2

1 × 2 = 2

Step 2

3 1

8 2

3×2 + 1×8=14

Step 3

7 3 1

6 8 2

7 × 2 + 3 × 8 + 1 × 6 = 44

Step 4

9 7 3 1

4 6 8 2

$9 \times 2 + 7 \times 8 + 3 \times 6 + 1 \times 4 = 96$

Step 5

9 7 3

4 6 8

$9 \times 8 + 7 \times 6 + 3 \times 4 = 126$

Step 6

9 7

4 6

$9 \times 6 + 7 \times 4 = 82$

Step 7

9

↓

4

$9 \times 4 = 36$

Step 1: 2; 2 carry 0; $P_0 = 2$

Step 2: 14 + 0 = 14; 4 carry 1; $P_1 = 4$

Step 3: 44 + 1 = 45; 5 carry 4; $P_2 = 5$

Step 4: 96 + 4 = 100; 0 carry 10; $P_3 = 0$

Step 5: 126 + 10 =136; 6 carry 13; $P_4 = 6$

Step 6: 82 + 13 = 95; 5 carry 9; $P_5 = 5$

Step 7: 36 + 9 = 45; 5 carry 4; $P_6 = 5$

Carry of *step 7* = $P_7 = 4$

The product is 45560542.

Similarly for the multiplication of two decimal numbers containing any number of digits, the above multiplication techniques can be applied.

3. Squaring of a Binary Number Using 'Urdhva Tiryagbhyam' Sutra

The method of multiplication using 'Urdhva Tiryagbhyam' sutra given in Section-2 can be implemented for squaring of a binary number and multiplication of two *n*- bit binary numbers. As an example, the squaring of 4-bit binary number 1111 is given below. The decimal equivalent of this number is 15.

Let 1111 × 1111 = $P_7P_6P_5P_4P_3P_2P_1P_0$

Step1

1 1 1 1

↓

1 1 1 1

$1 \times 1 = 1$

Step2

1 1 1 1

1 1 1 1

$1\times1+1\times1=10$

Step 3

1 1 1 1

1 1 1 1

$1\times1+1\times1+1\times1=11$

Step 4

1 1 1 1

1 1 1 1

$1\times1+1\times1+1\times1+1\times1=100$

Step 5

1 1 1 1

1 1 1 1

$1\times1+1\times1+1\times1=11$

Step 6

1 1 1 1

1 1 1 1

$1\times1+1\times1=10$

Step 7

1 1 1 1

↓

1 1 1 1

$1 \times 1 = 1$

Step 1: 1; 1 carry 0; $P_0 = 1$

Step 2: 10 + 0 = 10; 0 carry 1; $P_1 = 0$

Step 3: 11 + 1 = 100; 0 carry 10; $P_2 = 0$

Step 4: 100 + 10 = 110; 0 carry 11; $P_3 = 0$

Step 5: 11 + 11 = 110; 0 carry 11; $P_4 = 0$

Step 6: 10 + 11 = 101; 1 carry 10; $P_5 = 1$

Step 7: 1 + 10 = 11; 1 carry 1; $P_6 = 1$

Carry of *step 7* = $P_7 = 1$

Thus, the square of 1111 is 11100001, whose decimal equivalent is $225 = 15^2$.

The general method of squaring of a 4-bit binary number $a_3a_2a_1a_0$ is illustrated in *Table* 1. The square operation $a_3a_2a_1a_0 \times a_3a_2a_1a_0$ will be of 8-bits long. Let the product term in 8-bits is $P_7P_6P_5P_4P_3P_2P_1P_0$. C_1 to C_5 represent the carry from the previous product term.

Table 1.Method of squaring a 4-bit binary number.

Product term	**Sum + Carry from previous term**
P_0	a_0
P_1	$a_1\times a_0+a_1\times a_0$
P_2	$a_2\times a_0+a_2\times a_0+a_1+C_1$
P_3	$a_3\times a_0+a_3\times a_0+a_2\times a_1+a_2\times a_1+C_2$
P_4	$a_3\times a_1+a_3\times a_1+a_2+C_3$
P_5	$a_3\times a_2+a_3\times a_2+C_4$
P_6	a_3+C_5
P_7	Carry bit of P_6

The comparison between the number of multiplications (M) and additions (A) required in conventional and 'Urdhva Tiryagbhyam' method is shown in *Table* 2.

Table 2. Comparison between conventional method of multiplication and 'Urdhva Tiryagbhyam' method of multiplication.

Multiplication	**Conventional Method**		**Urdhva Tiryagbhyam Method**	
	M	**A**	**M**	**A**
2×2-bit multiplication	4	2	4	1

3×3-bit multiplication	9	7	9	4
4×4-bit multiplication	16	15	16	9

4. Multiplier Based on ‘Urdhva Tiryagbhyam’ Sutra for Squaring 2-Bit Binary Number

A multiplier is one of the key hardware blocks in most of the digital signal processing systems. The hardware implementation of 2×2 bit multiplier using the concept of ‘Urdhva Tiryagbhyam’ sutra is shown in Fig.1. Let the product of two 2-bit binary numbers a_1a_o and b_1b_o is $P_3P_2P_1P_o$. It consists of four AND gates and two half adders. A half adder is a logic circuit that adds two binary digits and produces a 2-bit data, i.e., sum and carry. The 1st half adder is used to add a_1b_o and a_ob_1, the outputs of the 2nd and 3rd AND gates respectively. The 2nd half adder is used to add the carry c_1 generated from 1st half adder and the output a_1b_1 of the 4th AND gate. The output of 1st AND gate is P_o. The sum of 1st half adder is P_1 and the sum of 2nd half adder is P_2 . The carry of 2nd half adder gives P_3. The same architecture can be used for squaring a 2-bit binary number.

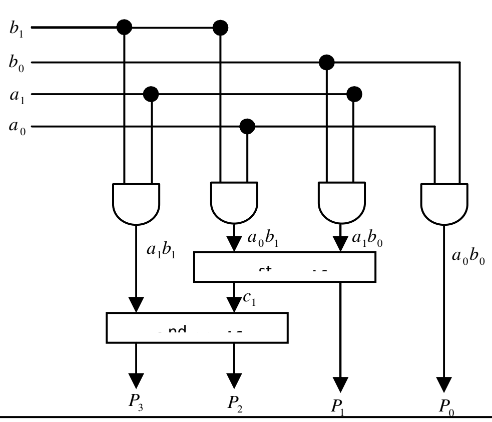

Figure 1. Block diagram of 2×2-bit multiplier based on ‘Urdhva Tiryagbhyam’ sutra.

5. Cubing of a Decimal Number Using “Anurupyena” Sub-Sutra

The cubing operation using ‘Anurupyena’ sub-sutra is much better than the conventional cubing. According to this sub-sutra,

$$(ab)^3 = a^3 + 3ba^2 + 3ab^2 + b^3 \tag{1}$$

where a and b are the digits of a decimal number. Here (+) does not indicate ordinary addition. Two examples are given below for finding the cube of a decimal number using this sub-sutra.

Example 1: Consider a two digit decimal number 32. The cube of this number can be found out as per the following steps.

Step 1: Let $a = 3$ and $b = 2$

Step 2: Applying the ‘Anurupyena’ sub-sutra (1),

$$(32)^3 = 3^3 + 3\times2\times3^2 + 3\times3\times2^2 + 2^3$$

Step 3: Add the partial products in *Step 2* from right by shifting them one digit, as b contains one digit.

$$\begin{array}{rll} b^3 & = 2^3 & = \quad 8 \\ +3ab^2 & = 3\times3\times2^2 & = \; 36 \\ +3ba^2 & = 3\times2\times3^2 & = 54 \\ +a^3 & = 3^3 & = 27 \\ \hline (ab)^3 & = 32^3 & = 32768 \\ \hline \end{array}$$

Example 2: The cube of the decimal number 423 can be calculated as given below.

Step 1: Let $a = 4$ and $b = 23$

Step 2: According to the sub-sutra (1),

$$423^3 = 4^3+3\times23\times4^2+3\times4\times23^2+23^3$$

Step 3: Add the partial products in *Step 2* from right by shifting them by two digits, as *b* contains 2 digits.

$$\begin{array}{rl}
23^3 & = \quad 12167 \\
+3\times4\times23^2 & = \quad 6348 \\
+3\times23\times4^2 & =1104 \\
+4^3 & =64 \\
\hline
(ab)^3 = 423^3 & =75686967 \\
\hline
\end{array}$$

6. Cubing of a Binary Number Using 'Anurupyena' Sub-Sutra

The method of cubing a decimal number given in Section-5 can be applied for cubing a binary number. Two examples are given below for finding the cube of a binary number using the 'Anurupyena' sub-sutra.

Example 1: Let us consider the 3-bit binary number 101, whose decimal equivalent is 5. The cubing of this binary number is done by the following steps.

Step 1: Let $a = 10$ and $b = 1$

Step 2: According to the 'Anurupyena' sub-sutra (1),

$$(ab)^3 = (101)^3 = (10)^3+11\times1\times(10)^2+11\times10\times1^2+1^3$$

Instead of 3 in the formula, its binary equivalent 11 shall be taken while adding the partial products.

Step 3: Add the partial products in *Step 2* from right by shifting them by one bit, as *b* contains one bit.

$$\begin{array}{rll}
b^3 & = 1^3 & = \quad 1 \\
+3ab^2 & = 11\times10\times1^2 & = \quad 110 \\
+3ba^2 & = 11\times1\times(10)^2 & = \; 1100 \\
+a^3 & = (10)^3 & =1000 \\
\hline
(ab)^3 & = (101)^3 & =1111101 \\
\hline
\end{array}$$

The decimal equivalent of 1111101 is $125 = 5^3$.

Example 2: Let us consider a 4-bit binary number 1001, whose decimal equivalent is 9. The cubing of 1001 is done as per the following steps.

Step 1: Let $a = 10$ and $b = 01$.

Step 2: Using the 'Anurupyena' sub-sutra (1),

$$(ab)^3 = (1001)^3 = (10)^3+11\times01\times(10)^2+11\times10\times(01)^2+(01)^3$$

Step 3: Add the partial products in *Step 2* from right by shifting them by 2 bits, as *b* contains 2 bits.

$$\begin{array}{rll}
b^3 & = 01\times01\times01 & = \qquad 0001 \\
+3ab^2 & = 11\times10\times01\times01 & = \; 00110 \\
+3ba^2 & = 11\times01\times10\times10 & = \; 01100 \\
+a^3 & = 10\times10\times10 & = 1000 \\
\hline
(ab)^3 & = (1001)^3 & = 1011011001 \\
\hline
\end{array}$$

The decimal equivalent of 1011011001 is $729 = 9^3$

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