# Damped Motion of Electron in Electric Conductor Due Presence of Magnetic and Alternative Electric Fields at Right Angles and Closed form Solutions

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## Abstract

Flow of current in an electric conductor is attributed to movement of electrons in the opposite directions. In this paper a second- order differential equation of an electron is formed taking into account [1] a damping force owing to collisions between electrons and two more applied forces, magnetic field and alternative electric field in perpendicular directions and has been completely solved in closed form subject to the prescribed initial conditions. The governing differential equation of motion of the electrons is separated into two tractable simultaneous equations by use of complex number iota ($I = \sqrt{-1}$). It is proved that the effect of damping partially dies away after sometime vis-à-vis theoretically after a long time, say after infinite time. The acceleration, velocity and distance described by the electron at any instant of time are determined. Thereafter their maximum and minimum values are found out.

# 1. Introduction

Mahindra Sing Sodha published a paper in Aug 07 issue of Bull IAPT wherein he has considered damped motion of an electron with formation of its relevant differential equation of motion in three cases but without going in for solution to the last case. "For presentation in 27th IAPT annual convention, Cochin University, Nov 02-04-2012." To, Dr. P. Radhakrishrian Email: radhak@cusat.ac.in".

1. When the damped force acting on the electron is proportional to its velocity
2. When an electric field of constant magnitude is coupled with the damping force.
3. When the electron is acted on by three forces altogether, magneticand electric fields of contact magnitudes at right angles and the damping force.

Sodh[1], however, examined its motion reaching the steady state and the heating effects together with current density are also discussed by him S. N. Maitra[2] solved the relevant differential equation of motion of an electron in presence of damping force and two mutually perpendicular magnetic and electric fields of constant magnitudes with a subtle technique of a complex number iota ($I = \sqrt{-1}$) subject to given initial conditions and subsequently explained the reasons for the electron attaining the steady state wherein the acceleration disappears. F acceleration acquired by the electron at any instant of time and the corresponding distance described by it. Nonetheless, in the present paper the entire problem vis-à-vis the third case is modified in a more cumbersome manner introducing an alternative electric field instead of electric filed with constant magnitude and is ultimately solved in closed form.

## 1. Differential equation of motion

Let

$\bar{r} = Ix + jy$ and $\bar{v} = Iv_x + jv_y$ be the position and velocity vectors of an electron of mass m with charge e at any instant of time t with respect to a fixed frame OXY with origin at O and i, j as unit vectors along axes OX and OY at right angles, respectively. The electron moves under the following forces: Alternative electric Field

$\overline{Electric\ Field} = \bar{E} \cos w_0 t = (IE_x + j\, E_y)$ …………………………(1)

Magnetic field perpendicular to the electric field of magnitude B,

$\bar{B} = \hat{k}\, B$ ………………………(2)

Where, $\hat{k}$ = unit vector perpendicular to i, j and damping force.

$\bar{F} = mk\bar{v}$ …………………………(3)

With constant damping factor k, $\widehat{k}$= unit vector perpendicular to I, j. Hence in view of (1) to (3), the vector equation governing the motion of the electron is

$$m\frac{d\bar{v}}{dt} + mk\bar{v} = -e\,[\bar{E}\,\cos w_0\,t + \bar{v}x\bar{B}]\,\ldots.(4)$$

$$\bar{v} = \frac{d\bar{r}}{dt}\,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Which, can be rewritten in scalar form

$$I\frac{dV_x}{dt} + j\frac{dV_y}{dt} + k(iV_x + jV_y) = -\frac{e}{m}[IE_x + jE_y)\cos w_0\,t\ + (IV_x + jV_y)x\widehat{k}\,B\ \ldots\ldots\ldots.(6)$$

Equating the coefficients of I and j from both side (6),

$$\dot{V}_x + kV_x = -a_x\cos w_0\,t - wV_y, \qquad \dot{V}_y + kV_y = -a_y\cos w_0\,t - wV_x\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (7)$$

Or

$$\dot{V}_x + kV_x + wV_y = -a_x\cos w_0\,t\ \ldots\ldots\ldots..(8)$$

$$\dot{V}_y + kV_y - wV_x = -a_y\cos w_0\,t\ldots\ldots\ldots\ldots(9)$$

Where some constant parameters involved are

$$a_x = \frac{eE_x}{m}, a_y = \frac{eE_y}{m}, w = \frac{eB}{m}\ldots\ldots\ldots\ldots\ldots(10)$$

And the dot sign derivatives with respect to time t.

## 2. Complete solution to motion of the Electron.

Multiplying equation (9) by (I= $\sqrt{-1}$) and thereafter adding to and subtracting from (8) we get two linear equations i.e. first order differential equations

$$\left(V_x + IV_y\right)^0 + (k - Iw)(V_x + IV_y) = -(a_x + Ia_y)\,(\cos w_0\,t)\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(11)$$

$$\left(V_x - IV_y\right)^0 + (k + Iw)(V_x - IV_y) = -(a_x - Ia_y)\,(\cos w_0\,t)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...(12)$$

It can be noted that (12) can be obtained by replacing I by –I in (11)

Let us introduce the initial conditions that at t=0, x=0, y=0, $V_x = 0$ and $V_y = 0$ …………...(13)

this, means that the electron starts from the origin at rest and is accelerated owing to the applied electric and magnetic fields;

In order to solve (11) we find its integrating factor

$$L = e^{(k-Iw)t}$$

So that

$$e^{(k-Iw)t}\frac{d}{dt}\left(V_x+IV_y\right)\quad+(k-Iw)(V_x+IV_y)e^{(k-Iw)t}=-(a_x+Ia_y)e^{(k-Iw)t}\cos w_0\,t$$

Or

$$\frac{d}{dt}\left\{\left(V_x+IV_y\right)\quad e^{(k-Iw)t}\right\}=\frac{1}{2}\left(a_x+Ia_y\right)(e^{Iw_0t}+e^{-Iw_0t})\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(14)$$

Therefore;

$$e^{\pm Iw_0t}=\cos w_0\,t\pm I\sin w_0\,t$$

$$\cos w_0\,t=\frac{1}{2}(e^{Iw_0t}+e^{-Iw_0t})$$

$$\sin w_0\,t=\frac{1}{2I}(e^{Iw_0t}-e^{-Iw_0t})\quad\ldots\ldots\ldots(15)$$

Integrating (1)subject to the conditions (13), once gets

$$\{V_x+IV_y\}e^{\{k-Iw\}t}=-\frac{1}{2}\left(a_x+Ia_y\right)\int_0^t e^{\{k+I(w_0-w)\}t}+e^{\{k-I(w_0+w)\}t}\,dt$$

$$V_x+IV_y=-\frac{1}{2}\left(a_x+Ia_y\right)$$

$$\left[\frac{e^{Iw_0t}\{k-I(w_0-w)\}}{k^2+(w_0-w)^2}+\frac{e^{-Iw_0t}\{k+I(w_0+w)\}}{k^2+(w_0+w)^2}-\{\frac{k-I(w_0-w)}{k^2+(w_0-w)^2}+\frac{k+I(w_0+w)}{k^2+(w_0+w)^2}\}e^{-\{k-Iw\}t}\right]$$

By use of (15)

$$V_x+IV_y=-\frac{1}{2}\left(a_x+Ia_y\right)$$

$$\Big[(\mathrm{k}\cos w_0\,t)\left\{\frac{1}{k^2+(w_0-w)^2}+\frac{1}{k^2+(w_0+w)^2}\right\}+(\sin w_0\,t)\left\{\frac{-(w_0-w)}{k^2+(w_0-w)^2}+\frac{(w_0+w)}{k^2+(w_0+w)^2}\right\}+I\left\{\frac{-(w_0-w)\,(\cos w_0t)}{k^2+(w_0-w)^2}+\frac{(w_0+w)\,(\cos w_0t)}{k^2+(w_0+w)^2}+\frac{k\ \sin w_0t}{k^2+(w_0-w)^2}-\frac{k\ \sin w_0t}{k^2+(w_0+w)^2}\right\}-e^{-kt}\left\{\frac{k\cos w_0t}{k^2+(w_0-w)^2}+\frac{k\cos w_0t}{k^2+(w_0+w)^2}-\frac{(w_0-w)\ \sin w_0t}{k^2+(w_0-w)^2}-\frac{(w_0+w)\ \sin w_0t}{k^2+(w_0+w)^2}\right\}-I\{\frac{(w_0+w)\cos\ \ wt}{k^2+(w_0-w)^2}-\frac{(w_0-w)\cos\ \ wt}{k^2+(w_0+w)^2}+\frac{k\sin wt}{k^2+(w_0-w)^2}+\frac{k\sin wt}{k^2+(w_0+w)^2}\}\Big]\ldots\ldots\ldots\ldots...(16)$$

Wherefrom equating the real and imaginary parts we get components of the electron along X and Y axes respectively.

$$\frac{dx}{dt} = V_x = \frac{a_y}{2}\left[\frac{(w_0+w)\cos w_0 t - k\ \sin w_0 t}{k^2+(w_0+w)^2} + \right.$$

$$\left.\frac{(w_0-w)\cos w_0 t + k\ \sin w_0 t}{k^2+(w_0-w)^2}\right]$$

$$-\frac{a_x}{2}\left\{\frac{(w_0+w)\ \sin w_0 t - k\cos w_0 t}{k^2+(w_0+w)^2} - \right.$$

$$\left.\frac{(w_0-w)\ \sin w_0 t - k\cos w_0 t}{k^2+(w_0-w)^2}\right\} -$$

$$e^{-kt}\left[\frac{a_x}{2}\left\{\frac{k\cos wt-(w_0+w)\sin wt}{k^2+(w_0+w)^2} + \right.\right.$$

$$\left.\frac{k\cos wt-(-w_0+w)\sin wt}{k^2+(w_0-w)^2}\right\} - \frac{a_y}{2}\left\{\frac{k\sin wt+(w_0+w)\cos wt}{k^2+(w_0+w)^2} + \right.$$

$$\left.\left.\frac{k\sin wt+(-w_0+w)\cos wt}{k^2+(w_0-w)^2}\right\}\right\} \text{………(17)}$$

$$\frac{dy}{dt} = V_y = -\frac{a_y}{2}\left[\frac{(w_0+w)\ \sin w_0 t + k\cos w_0 t}{k^2+(w_0+w)^2} + \right.$$

$$\left.\frac{k\cos w_0 t + (w_0-w)\ \sin w_0 t +}{k^2+(w_0-w)^2}\right]$$

$$-\frac{a_x}{2}\left\{\frac{(w_0+w)\cos w_0 t - k\ \sin w_0 t}{k^2+(w_0+w)^2} - \right.$$

$$\left.\frac{(w_0-w)\cos w_0 t - k\ \sin w_0 t}{k^2+(w_0-w)^2}\right\} -$$

$$e^{-kt}\left[\frac{a_y}{2}\left\{\frac{k\cos wt-(w_0+w)\sin wt}{k^2+(w_0+w)^2} + \right.\right.$$

$$\left.\frac{k\cos wt-(-w_0+w)\sin wt}{k^2+(w_0-w)^2}\right\} + \frac{a_x}{2}\left\{\frac{k\sin wt+(w_0+w)\cos wt}{k^2+(w_0+w)^2} + \right.$$

$$\left.\left.\frac{k\sin wt+(-w_0+w)\cos wt}{k^2+(w_0-w)^2}\right\}\right\} \text{………..(18)}$$

Integrating (17) and (18) and applying the initial conditions, we obtain the distance travelled by the electron along the axis:

$$x = \frac{a_y}{2}\left[\frac{\left(1+\frac{w}{w_0}\right)\ \sin w_0 t - \left(\frac{k}{w_0}\right)(1-\cos w_0 t)}{k^2+(w_0+w)^2} + \right.$$

$$\left.\frac{\left(\frac{w}{w_0}-1\right)\ \sin w_0 t + \left(\frac{k}{w_0}\right)(1-\cos w_0 t)}{k^2+(w-w_0)^2}\right]$$

$$-\frac{a_x}{2}\left\{\frac{\left(1+\frac{w}{w_0}\right)(1-\cos w_0 t) + \left(\frac{k}{w_0}\right)\ \sin w_0 t}{k^2+(w_0+w)^2} + \right.$$

$$\left.\frac{\left(\frac{w}{w_0}-1\right)(1-\cos w_0 t) - \left(\frac{k}{w_0}\right)\ \sin w_0 t}{k^2+(w-w_0)^2}\right\} +$$

$$e^{-kt}\left[\frac{a_x}{2}\left\{\frac{k\{k[(1-\cos wt)+w\sin wt]-(w_0+w)[w\,(1-\cos wt)-k\sin wt]\}}{\{k^2+(w_0+w)^2\}(k^2+w^2)}\right.\right.$$

$$\left.\frac{k\{k[(1-\cos wt)+w\sin wt]-(w-w_0)[w\,(1-\cos wt)-k\sin wt]\}}{\{k^2+(w-w_0)^2\}(k^2+w^2)}\right\} -$$

$$\frac{a_y}{2}\left\{\frac{k\{w[(1-\cos wt)-k\sin wt]+(w_0+w)[k\,(1-\cos wt)+w\sin wt]\}}{\{k^2+(w_0+w)^2\}(k^2+w^2)} + \right.$$

$$\left.\left.\frac{k\{w[(1-\cos wt)-k\sin wt]+(w-w_0)[k\,(1-\cos wt)+w\sin wt]\}}{\{k^2+(w-w_0)^2\}(k^2+w^2)}\right\}\right\}$$

………(19)

$$y = -\frac{a_y}{2}\left[\frac{\left(1+\frac{w}{w_0}\right)(1-\cos w_0 t) + \left(\frac{k}{w_0}\right)\ \sin w_0 t}{k^2+(w_0+w)^2} + \right.$$

$$\left.\frac{\left(\frac{k}{w_0}\right)\ \sin w_0 t - \left(\frac{w}{w_0}-1\right)(1-\cos w_0 t)}{k^2+(w-w_0)^2}\right]$$

$$-\frac{a_x}{2}\left\{\frac{\left(1+\frac{w}{w_0}\right)\ \sin w_0 t - \left(\frac{k}{w_0}\right)(1-\cos w_0 t)}{k^2+(w_0+w)^2} + \right.$$

$$\left.\frac{\left(\frac{w}{w_0}-1\right)\ \sin w_0 t + \left(\frac{k}{w_0}\right)(1-\cos w_0 t)}{k^2+(w-w_0)^2}\right\} +$$

$$e^{-kt}\left[\frac{a_y}{2}\left\{\frac{k\{k[(1-\cos wt)+w\sin wt]-(w_0+w)[w\,(1-\cos wt)-k\sin wt]}{\{k^2+(w_0+w)^2\}(k^2+w^2)}\right.\right.$$

$$\left.\frac{k\{k[(1-\cos wt)+w\sin wt]-(w-w_0)[w\,(1-\cos wt)-k\sin wt]\}}{\{k^2+(w-w_0)^2\}(k^2+w^2)}\right\} +$$

$$\frac{a_x}{2}\left\{\frac{k\{w[(1-\cos wt)-k\sin wt]+(w_0+w)[k\,(1-\cos wt)+w\sin wt]\}}{\{k^2+(w_0+w)^2\}(k^2+w^2)} + \right.$$

$$\left.\left.\frac{k\{w[(1-\cos wt)-k\sin wt]+(w-w_0)[k\,(1-\cos wt)+w\sin wt]\}}{\{k^2+(w-w_0)^2\}(k^2+w^2)}\right\}\right\}$$

………(20)

Nevertheless the expressions for the velocity components can be put as

$$V_x = \frac{a_y}{2}\left\{\frac{\cos(w_0 t+p)}{\sqrt{k^2+(w+w_0)^2}} + \frac{\cos(w_0 t-q)}{\sqrt{k^2+(w-w_0)^2}}\right\} -$$

$$\frac{a_x}{2}\left\{\frac{\sin(w_0 t+p)}{\sqrt{k^2+(w+w_0)^2}} - \frac{\sin(w_0 t-q)}{\sqrt{k^2+(w-w_0)^2}}\right\} -$$

$$e^{-kt}\{\frac{a_y}{2}\left\{\frac{\cos(wt-p)}{\sqrt{k^2+(w+w_0)^2}}+\frac{\cos(wt-q)}{\sqrt{k^2+(w-w_0)^2}}\right\}+$$

$$\frac{a_x}{2}\left\{\frac{\sin(wt-p)}{\sqrt{k^2+(w+w_0)^2}}-\frac{\sin(wt-q)}{\sqrt{k^2+(w-w_0)^2}}\right\}\} \quad \ldots\ldots(21)$$

$$V_y=-\frac{a_x}{2}\left\{\frac{\cos(w_0t+p)}{\sqrt{k^2+(w+w_0)^2}}+\frac{\cos(w_0t-q)}{\sqrt{k^2+(w-w_0)^2}}\right\}-$$

$$\frac{a_y}{2}\left\{\frac{\sin(w_0t+p)}{\sqrt{k^2+(w+w_0)^2}}-\frac{\sin(w_0t-q)}{\sqrt{k^2+(w-w_0)^2}}\right\}-$$

$$e^{-kt}\{\frac{a_x}{2}\left\{\frac{\cos(wt-p)}{\sqrt{k^2+(w+w_0)^2}}+\frac{\cos(wt-q)}{\sqrt{k^2+(w-w_0)^2}}\right\}-$$

$$\frac{a_y}{2}\left\{\frac{\sin(wt-p)}{\sqrt{k^2+(w+w_0)^2}}+\frac{\sin(wt-q)}{\sqrt{k^2+(w-w_0)^2}}\right\}\}\ldots\ldots(22)$$

Where *tan p* = $k/(w+w_0)$ and *tan q* = $k/(w+w_0)$

Or otherwise with *tan α* = $\frac{a_x}{a_y}$ ..... (23)

$$V_x=\frac{\sqrt{a_x{}^2+a_y{}^2}}{2}\left\{\frac{\cos(w_0t+p+\propto)}{\sqrt{k^2+(w+w_0)^2}}+\frac{\cos(w_0t-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\right\}-$$

$$e^{-kt}\left\{\frac{\cos(wt-p-\propto)}{\sqrt{k^2+(w+w_0)^2}}+\frac{\cos(wt-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\right\}\ldots\ldots\ldots(24)$$

$$V_y=-\frac{\sqrt{a_x{}^2+a_y{}^2}}{2}\left\{\frac{\sin(w_0t+p+\propto)}{\sqrt{k^2+(w+w_0)^2}}-\frac{\sin(w_0t-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\right\}-$$

$$e^{-kt}\left\{\frac{\sin(wt-p-\propto)}{\sqrt{k^2+(w+w_0)^2}}+\frac{\sin(wt-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\right\}\ldots\ldots\ldots(25)$$

Integrating (24) and (25) and employing the initial conditions (13), we can also find the position (x,y) of the electron at any time t.

But differentiating with respect to time t rather than {(17),(18)} or {(21),(22)} we can find the acceleration components in more simplified form:

$$f_x=\dot{V_x}=\frac{\sqrt{a_x{}^2+a_y{}^2}}{2}[w_0\left\{\frac{\sin(w_0t+p+\propto)}{\sqrt{k^2+(w+w_0)^2}}+\right.$$

$$\left.\frac{\sin(w_0t-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\right\}+e^{-kt}[w\left\{\frac{\sin(wt-p-\propto)}{\sqrt{k^2+(w+w_0)^2}}+\right.$$

$$\left.\frac{\sin(wt-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\right\}+k\{\frac{\cos(wt-p-\propto)}{\sqrt{k^2+(w+w_0)^2}}+$$

$$\frac{\cos(wt-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\}]\ldots\ldots\ldots(26)$$

$$f_y=\dot{V_y}=-\frac{\sqrt{a_x{}^2+a_y{}^2}}{2}[w_0\left\{\frac{\cos(w_0t+p+\propto)}{\sqrt{k^2+(w+w_0)^2}}-\right.$$

$$\left.\frac{\cos(w_0t-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\right\}-e^{-kt}[w\left\{\frac{\cos(wt-p-\propto)}{\sqrt{k^2+(w+w_0)^2}}+\right.$$

$$\left.\frac{\cos(wt-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\right\}-k\{\frac{\sin(wt-p-\propto)}{\sqrt{k^2+(w+w_0)^2}}+$$

$$\frac{\sin(wt-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\}]\ldots\ldots\ldots(26)$$

Squaring and adding (24) and (25) we have an expression for velocity

$$v^2=\frac{a_x{}^2+a_y{}^2}{4}\left[\left\{\frac{1}{k^2+(w_0+w)^2}+\frac{1}{k^2+(w_0-w)^2}\right\}(1+\right.$$

$$\left.e^{-2kt})+\frac{2\{\cos(2w_0t+p-q)+e^{-2kt}\cos(p+q)}{\sqrt{k^2+(w+w_0)^2}\sqrt{k^2+(w-w_0)^2}}\right]-$$

$$\frac{(a_x{}^2+a_y{}^2)}{2}e^{-kt}\,[\left\{\frac{\cos(w_0t+p+\propto)}{\sqrt{k^2+(w+w_0)^2}}-\right.$$

$$\left.\frac{\cos(w_0t-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\right\}\left\{\frac{\cos(wt-p-\propto)}{\sqrt{k^2+(w+w_0)^2}}+\frac{\cos(wt-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\right\}+$$

$$\{\frac{\sin(w_0t+p+\propto)}{\sqrt{k^2+(w+w_0)^2}}-\frac{\sin(w_0t-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\}\{\frac{\sin(wt-p-\propto)}{\sqrt{k^2+(w+w_0)^2}}+$$

$$\frac{\sin(wt-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\}\}]\ \ldots\ldots\ldots\ldots(28)$$

Discussion and Conclusion:

After a lapse of time t, $e^{-kt}\longrightarrow$

0 as t $\longrightarrow$ $\infty$ and so from (28), one gets, $v_1 =$

$$\frac{\sqrt{{a_x}^2+{a_y}^2}}{2}\left\{\frac{1}{\sqrt{k^2+(w-w_0)^2}}-\frac{1}{\sqrt{k^2+(w+w_0)^2}}\right\} \le v \le$$

$$\frac{\sqrt{{a_x}^2+{a_y}^2}}{2}\left\{\frac{1}{\sqrt{k^2+(w+w_0)^2}}+\frac{1}{\sqrt{k^2+(w-w_0)^2}}\right\} = v_2$$

…..(29)

Which, rectifies that as the time passes, the velocity of the electron attains the minimum and the maximum values $v_1$ *and* $v_2$ respectively as shown in (29).

Similarly, squaring and adding (26) and (27) we get its acceleration f:

$$f^2 = \frac{({a_x}^2+{a_y}^2){w_0}^2}{4}\left[\left\{\frac{1}{k^2+(w_0+w)^2} + \frac{1}{k^2+(w_0-w)^2}\right\} - \frac{2\{\cos(2w_0\,t+p-q)+e^{-2kt}\cos\,(p+q)\}}{\sqrt{k^2+(w+w_0)^2}\sqrt{k^2+(w-w_0)^2}}\right] + e^{-kt}\,f_1 + e^{-2kt}\,f_2$$

Where $\vec{f_1}$ *and* $\vec{f_2}$ are not constants but circular functions of time t and are finite as t $\rightarrow$ $\infty$. So as the time passes, $e^{-kt}, e^{-2kt} \rightarrow 0$, which confirms that in light of (29)

$v_1\, w_0 \le f \le v_2\, w_0$………………..(31)

Equating equation (19) and (20) representing the position (x, y) of the electron at time t can be written as

After a significant time i.e. as t $\longrightarrow$ $\infty$, $e^{-2kt} \longrightarrow 0$ so that (32) and (33) yields

$$x = \frac{\sqrt{{a_x}^2+{a_y}^2}}{2\,w_0}\left\{\frac{\sin(w_0t+p+\propto)}{\sqrt{k^2+(w+w_0)^2}}+\frac{\sin(w_0t-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\right\} - \frac{a_x}{2\,w_0}\left\{\frac{(w+w_0)}{\sqrt{k^2+(w+w_0)^2}}-\frac{(w-w_0)}{\sqrt{k^2+(w-w_0)^2}}\right\} - \frac{e^{-kt}}{2}\{f_1a_x + f_2a_y\} - \frac{a_yk}{2\,w_0}\left\{\frac{1}{\sqrt{k^2+(w+w_0)^2}}-\frac{1}{\sqrt{k^2+(w-w_0)^2}}\right\} \ldots(32)$$

$$x = \frac{\sqrt{{a_x}^2+{a_y}^2}}{2\,w_0}\left\{\frac{\cos(w_0t+p+\propto)}{\sqrt{k^2+(w+w_0)^2}}-\frac{\cos(w_0t-q-\propto)}{\sqrt{k^2+(w-w_0)^2}}\right\} - \frac{a_y}{2\,w_0}\left\{\frac{(w+w_0)}{\sqrt{k^2+(w+w_0)^2}}-\frac{(w-w_0)}{\sqrt{k^2+(w-w_0)^2}}\right\} - \frac{e^{-kt}}{2}\{a_x\,k_1(\cos wt, \sin wt) + a_y\,k_2(\cos wt, \sin wt)\} + \frac{a_xk}{2\,w_0}\left\{\frac{1}{\sqrt{k^2+(w+w_0)^2}}-\frac{1}{\sqrt{k^2+(w-w_0)^2}}\right\} \ldots(33)$$

$$R^2 = (x-x_1)^2 + (y-y_1)^2$$

$$= \frac{{a_x}^2+{a_y}^2}{4\,{w_0}^2}\left[\left\{\frac{1}{k^2+(w_0+w)^2} + \frac{1}{k^2+(w_0-w)^2}\right\} + \frac{2\cos(2w_0\,t+p-q)}{\sqrt{k^2+(w+w_0)^2}\sqrt{k^2+(w-w_0)^2}}\right] \ldots(34)$$

As t increases further i.e on the whole as

$t \longrightarrow \infty$ because of $-1 \leq \cos(w_0 t + p - q) \leq 1$ get,

$$\frac{\sqrt{a_x{}^2+a_y{}^2}}{2\,w_0}\left\{\frac{1}{\sqrt{k^2+(w-w_0)^2}}-\frac{1}{\sqrt{k^2+(w+w_0)^2}}\right\} \leq R \leq \frac{\sqrt{a_x{}^2+a_y{}^2}}{2\,w_0}\left\{\frac{1}{\sqrt{k^2+(w+w_0)^2}}+\frac{1}{\sqrt{k^2+(w-w_0)^2}}\right\}\text{.....(35)}$$

Where, R is the distance of electron from a fixed point P$(x_1, y_1)$, given by

$$x_1 = -\frac{a_x}{2\,w_0}\left\{\frac{(w+w_0)}{\sqrt{k^2+(w+w_0)^2}}-\frac{(w-w_0)}{\sqrt{k^2+(w-w_0)^2}}\right\} - \frac{a_y k}{2\,w_0}\left\{\frac{1}{\sqrt{k^2+(w+w_0)^2}}-\frac{1}{\sqrt{k^2+(w-w_0)^2}}\right\}.$$

$$y_1 = -\frac{a_y}{2\,w_0}\left\{\frac{(w+w_0)}{\sqrt{k^2+(w+w_0)^2}}-\frac{(w-w_0)}{\sqrt{k^2+(w-w_0)^2}}\right\} + \frac{a_x k}{2\,w_0}\left\{\frac{1}{\sqrt{k^2+(w+w_0)^2}}-\frac{1}{\sqrt{k^2+(w-w_0)^2}}\right\}\text{......(36)}$$

The above inequality suggests that as the time passes, the effect of damping partially dies away and the electron ultimately remains in motion with its distance R from the above fixed point fluctuating between two values $r_1$ $and$ $r_2$ given by

$$r_1 = \frac{\sqrt{a_x{}^2+a_y{}^2}}{2\,w_0}\left\{\frac{1}{\sqrt{k^2+(w-w_0)^2}}-\frac{1}{\sqrt{k^2+(w+w_0)^2}}\right\}.$$

$$r_1 = \frac{\sqrt{a_x{}^2+a_y{}^2}}{2\,w_0}\left\{\frac{1}{\sqrt{k^2+(w-w_0)^2}}-\frac{1}{\sqrt{k^2+(w+w_0)^2}}\right\}\text{.............(37)}$$

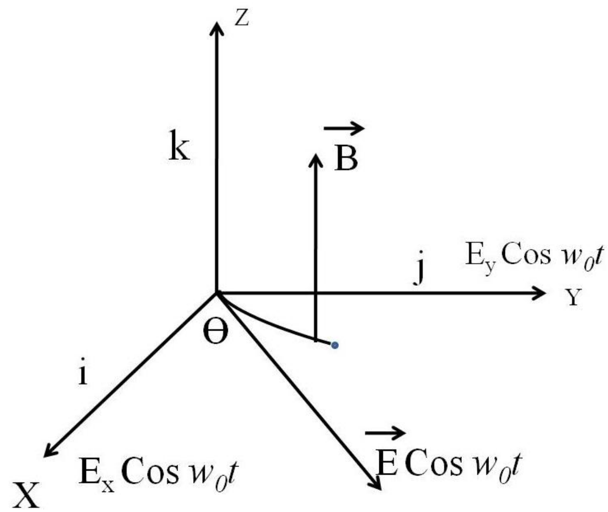


Figure 1, Path of an electron under mutually perpendicular alternative electric and constant magnetic fields.

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