A Century of Planck Constant Measurement

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Abstract

The Planck constant 'h' is a fundamental physical constant which plays an important role in understanding the behaviour of matter at the subatomic level. It is a cornerstone of the theory of quantum mechanics, which links wave-like and particle –like properties. Although the first numerical estimate of h was given by Planck himself, the American Physicist R. A. Millikan measured its value first time using the photoelectric effect in 1914. That is why the year 2014 is considered as centenary of Planck constant measurement. The measurement of the Planck constant, h, is now entering a new phase as its importance has been linked to a proposed redefinition of a kilogram unit of mass. In this article, we briefly review on measurements of Planck constant.

Keywords: Planck constant, Watt balance, Photoelectric effect, Josephson effect, Quantum Hall effect

Introduction

The Planck constant 'h' is a fundamental physical constant that plays a pivotal role in the exotic province of quantum mechanics. The constant started as a theoretical hypothesis just over 100 years ago in 1900 by Max Planck. While thinking about the conditions of equilibrium between matter and radiation, Max provided an explanation of the observed properties of black body radiation. He assumed that atoms emit and absorb discrete quanta of radiation with energy $E=hf$, where f is the frequency of the radiation and 'h' is a fundamental constant of nature. Even theoretical constants have real value, so numerical measurements of 'h' within the International System of Units (SI) soon started. The SI units were originally and solely based upon artifact standards. Precise measurement of Planck constant 'h' had created many challenges in past. In the early history of Planck constant measurements, precise voltage measurement, frequency measurement, resistance measurement etc. had added barrier to measuring 'h' in SI units [1].

The first numerical estimate of h was given by Planck himself as $h = 6.55 \times 10^{-34}\ Js$. In 1914, the American Physicist R. A. Millikan (Noble prize 1923) measured its value first time using the photoelectric effect. The result was 6.626×10^{-34} Js [2]. Since last one century the numerical value of *h* has been determined many times using more and more sophisticated techniques. In 1969 the task group on fundamental constants called CODATA (Committee on Data for Science and Technology) was established. The 1987 report of CODATA gives the value of 'h' = 6.6260755 (40)x$10^{-34}\ Js$ (which one should read as h = (6.6260755±0.0000040)x $10^{-34}\ Js$).

According to CODATA 2010 recommendation the accuracy of the Planck constant has now improved to disagreement with stated uncertainties in 7th digit (6.62606957(43)x $10^{-34}\ Js$ (which one should read as (6.62606957 ± 0.00000043) x$10^{-34}\ Js$). On-going experiments measuring the value of 'h' using mass as a variable by reversing the calculation will soon overcome the last artifact barrier. The proposed revision of SI units would embed Planck constant into the definition of the Kilogram [1,3], as a fixed constant of nature. Since mass will be defined by 'h' and the speed of light 'c', obtaining a highly accurate value of the Planck constant is driving the interest in this topic. Watt balance systems are now the best method used to measure 'h'. New Watt balance 'h' and the worldwide effort to measure the Avogadro's constant [5,7] have recently become more newsworthy because of the kilogram redefinition.

A brief review of some of the early attempts to measure 'h' is indispensable because measurement of the Planck constant, h, is now entering a new phase. Transition from accepted values to newer results seems to be more interesting.

Historical Background

Planck constant was started as a hypothesis just over 100 years ago to explain the spectral distribution of 'Black body radiation' mathematically. The story begins on 19 October, 1900 when a German Physicist, Max Karl Ernst Ludwig Planck (1858-1947, Nobel 1918) presented a formula that he had guessed through his postulate that energy is quantized, to explain: why black body radiation has a finite energy spectrum. How light must have a discreteness that was not experimentally observed was a real puzzle for the scientists of those days. Although the discovery was presented to the members of the Berlin Physical Society, the lecturer himself was not able to derive the new result formally at that time. Two months later on 14 December, 1900 during a similar lecture in Berlin Planck proposed a solution to this problem. He announced revolutionary theoretical concept invented or perhaps more properly discovered by him. Planck postulated that the total energy of the cavity (the blackbody) can be attributed to the atoms of the cavity only in a very special way. He argued that each of the atoms, oscillating with frequency, can only gain a multiple of some energy E=hf, which Planck called the quantum of energy; E=hf. The letter 'h' was accepted to denote a new fundamental constant of nature. The latter date i.e. 14 December, 1900 has been considered by the physics community of the world as the birth of Planck constant – the beginning of a new era of physics, quantum physics.

Earliest Measurement (Indirect Measurement)

i) Photoelectric effect

Besides blackbody radiation experiment there was another experiment with unexplained results; the photoelectric effect, where electrons are ejected from a metal surface under bombardment by electro-magnetic radiation. These experiments were the earliest, although indirect determination of ‘h’. The unusual aspect of the effect was that the electrons had the same energy regardless of the intensity of light, but the electron energy did change with colour or frequency of light. Five year after Planck’s paper, in 1905, Einstein subsequently guessed that the photoelectric effect was due to the energy being discrete or quantized [3]. He utilized Planck’s mathematical relation to describe the discrete photon energy E, absorbed by or emitted from an atom as proportional to Planck constant ‘h’, to the frequency f as E = hf.

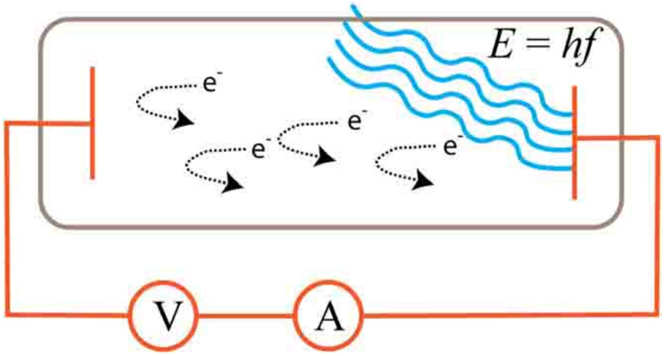

Fig.1: Photoelectric Effect.

In photoelectric effect inside a vacuum chamber, light of different frequencies ejects electrons from a metal plate. The energy of electrons is found by increasing the voltage ‘V’, until there is no more measured current. The energy of impinging electromagnetic energy must exceed a work function, W, a characteristics of the metal. Graphing the current stopping voltage against the impinging light’s frequency produces a line where the slope is the ratio h/e, ‘e’ being the elementary charge. Einstein’s photoelectric equation in terms of voltage,V and work function,W describes the relation, eV = hf – W.

To determine value of ‘h’ a value of ‘e’ was needed to be measured independently. It is interesting to see here that voltage units have been historically involved in and are still intrinsic in measuring ‘h’.

The value of ‘h’ is measured in units of energy times a unit of time i.e. in the SI system of units we have [Js], which is also known to be a unit for the quantity of action [6] and angular momentum. The first numerical [2] estimate ‘h’ was given by Planck himself as $h = 6.55 \times 10^{-34}$ Js.

The earliest reported photoelectric experimental results of Hughes, Richardson and Compton [1,8] in 1912, were explorations into the waves versus particle nature of the effect. Early 1913, Millikan pursued this experimental line of inquiry. In 1914, he compiled his and other’s researcher’s data using various physics relationships to infer the values for several constants including, e, h, Avogadro’s constant N_A and Boltzmann constant k_β. The 1913 paper was the initial effort to assemble physical constant data, which constituted the Committee on Data for Science and Technology (CODATA) work of today. Millikan’s 1916 paper first time published the value for $h = 6.57 \times 10^{-34}\ Js$ with stated uncertainty of 0.5%. This ‘h’ value was resulted by combining h/e measurement with his own evaluation of ‘e’ from the famous Oil drop experiment [3].

ii) X-ray Diffraction Experiment

Later on several methods for measuring of h/e were improved and consequently became more significant in determining another physical constant, Avogadro’s number, N_A.

In the sequence new method utilized the strong X-rays emitted from Coolidge tubes when high voltages applied to cathode tube. Duane and Hunt [1,3] used the relation $eV = hf = \frac{hc}{\lambda}$. The wavelength λ was measured using famous Bragg's relation $2a\sin\theta = n\lambda$ (where 'a' is grating spacing and θ is the diffraction angle). However, along with 'e', a value of 'c', the speed of light was borrowed from other experiments. From these early experiments, the relation for $\frac{h}{e}$ is

$$\frac{h}{e} = \lambda_{\min}\frac{V}{c} \qquad (1)$$

The method had several limitations. Particularly the voltage standard and measuring the crystal lattice spacing, a, of the diffraction material in au (atomic units) and its conversion to SI units incorporated significant uncertainties. With the data accumulated upto 1963, the inferred value [3] from the various indirect methods was $h = 6.62559\,(16)\times\overline{10}^{34}$ Js.

iii) Josephson Effect (SI units)

The discovery of Josephson effect in 1962 opened up new hope to adopt a new technique, not dependent upon the X-ray frequency and atomic lattice measurement for determination of $\frac{h}{e}$ ratio. By that time national volt unit was somehow standardized [8] with Weston standard Voltage cell and voltage measurement had become more précised. When a Josephson junction (junction of superconductors through a thin resistive layer) is irradiated by electromagnetic waves (microwave frequency or above) at a frequency 'f' constant voltage step, V appear across the junction as the current increases along I-V curve following the integral multiple of $\left(\frac{h}{2e}\right)$. The relation becomes

$$V = nf\left(\frac{h}{2e}\right), \qquad (2)$$

where $\left(\frac{h}{2e}\right)$ ratio is the quantization of magnetic flux in superconductor. Adopting Josephson device as a voltage standard $\frac{h}{2e}$ ratio was evaluated in SI. The changeover from artifact voltage laboratory standard to electronic standards marked the beginning of quantum electrical metrology.

In fact the so called voltage standard controversy was a desirable prelude for international consistency in changing from kilogram mass unit artifact definition to an electronic definition based on fundamental constants. It is worth mentioning here three other important experiments that were historically linked to the determination of Planck constant. These are the (i) volt balance or electrometer (ii) the current balance/ampere balance, and (iii) Quantum Hall Effect experiment (QHE).

(i) SI Volt Balance: The improved version of electrometer by Sir William Thompson and Lord Kelvin in 1868 used the electrical energy in a charged capacitor for electrostatic measurement.

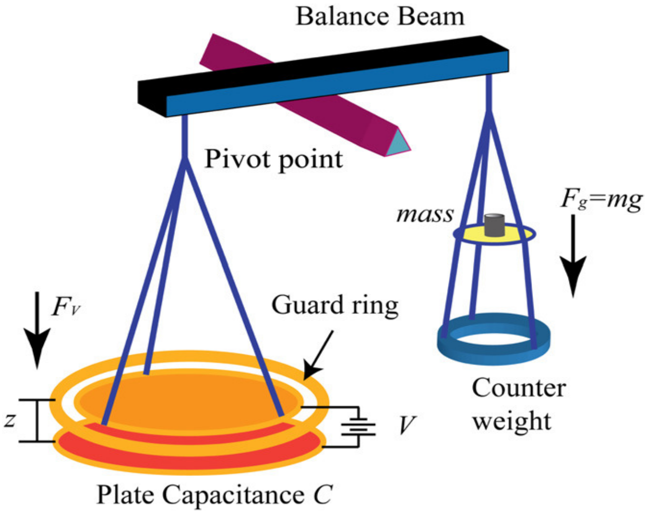

Fig.2: Absolute voltage balance diagram [1].

A gravitational force on a test mass mg is balanced by the electrical force of the differential capacitance ‘C’ of two surfaces separated by a distance ‘z’ charged to a voltage V.

$$mg = \frac{V^2}{2}\left(\frac{dC}{dz}\right)$$

$$V = \left[2mg\left(\frac{dz}{dC}\right)\right]^{\frac{1}{2}} . \qquad (3)$$

The major drawbacks to this experiment were due to use of heavy electrodes to supply kilovolts of electrical potential. To avoid the limitations of the small force relative to the heavy electrodes Clothire[1] in 1965 suggested an alternative, liquid electrometer.

(ii) SI Ampere Balance

Ampere is one of the base SI units. The definition of SI unit of current as ‘Ampere’ became official in 1948. The ampere is that constant current, which when maintained in two straight parallel conductors of infinite length of negligible circular cross-section and placed 1m apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} Newton per meter length. The roll of ampere balance is very crucial in the history of measurement of Planck constant. It initiated the interest and motivation for independent and precise measurement of Planck’s constants.

Improved Ampere balance by Rosa and Guthe [1,3] in 1912 used a pair of induction coils in a fixed position arranged vertically over one another. At the middle of these two induction coils there is a smaller movable coil. This improved geometry had the advantage of allowing the Centre coil to move to a position where the vertical force is maximum, and also varies at a minimum rate over small displacement (dz).

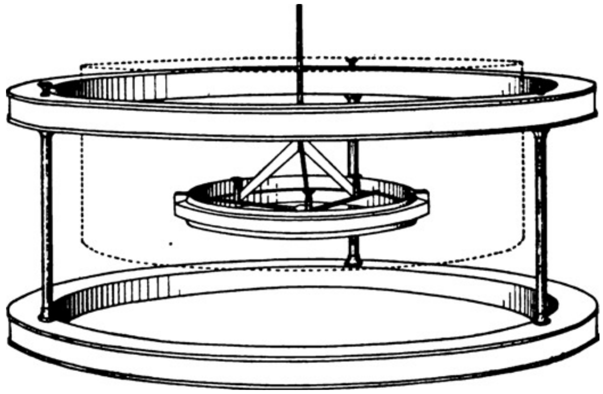

Fig.3: Absolute ampere coil configuration.

The moving coil was mechanically connected to a balance beam mass comparator. A current I_1 is passed through the fixed outer coils and current I_2, into the central movable coils. The central coil interacts with the magnetic field generated by the outer coils. This interaction creates an electromagnetic force to balance the force mg, on a reference mass ‘m’ due to gravity ‘g’. Of course, value of g was borrowed from a separate experiment.

The basic equation is

$$mg = I_1 I_2 \frac{\Delta M_{12}}{\Delta Z} , \qquad (4)$$

$\Delta M_{12} \rightarrow$ Mutual inductance between the coils at different position (ΔZ).

This established the link between the ampere and the SI units of mass, length and time. The ampere balance measured the ratio of force to current. As per suggestion of Kibble in 1976 the two quotient measurements were taken as a comparison of two different kinds of powers. Using equation-4 we can equate mechanical force multiplied with velocity (mechanical power watt) to electrical voltage multiplied with current (electrical power watt). This is the significant contribution of ampere balance which created a platform for construction of watt balance.

(iii) Quantum Hall effect Device, Resistance Standard

Interestingly while voltage and ampere controversies were coming to an end, Klaus Von Klitzing in 1980 discovered experimental evidence for quantized resistance region in Hall Effect. The equation relating the resistance steps, R in QHE device to the ratio of h/e^2 is

$$R = \frac{1}{(n)}\left(\frac{h}{e^2}\right), \qquad (5)$$

where n is the quantum number for the step. For the discovery of QHE, Klaus von Klitzing received the Nobel Prize for physics in 1985.

Electrical Unit Adjustment in 1990

With the new and improved absolute volt measurement, ampere measurement, resistance measurement, the volt and resistance units were adjusted internationally in 1990 (Previous adjustment in 1969). The Josephson constant $K_J = \frac{2e}{h}$ was given a defined value of $K_{J-90} = 483597.9 \text{x} 10^9 \, \text{Hz} \, \text{V}^{-1}$.

The symbol V_{J-90} was adopted to represent the volt as obtained via Josephson Effect, consistent with K_{J-90}. Using the new QHE system the resistance standard was adjusted and symbol Ω_{90} was adopted. Consequently a fixed value for Von Klitzing constant $R_K = \frac{h}{e^2}$ was assigned as $R_{K-90} = 25812.807\Omega$ where Ω_{90} is the conventional unit.

Modern Direct Measurement of ‘h’

Combining the Josephson Effect of equation (2) and Quantum Hall Effect (QHE) equation (5) Taylor in 1985 explicitly published a simple formula for h in terms of frequency.

$$\frac{V^2}{R} = \frac{\left[f\left(\frac{h}{2e}\right)\right]^2}{\left(h/e^2\right)} = f^2\left(\frac{h}{4}\right) \qquad (6)$$

This equation simply relates electrical energy to Planck constant and frequency. Looking at the simplicity and direct relation to ‘h’, it was assumed that direct measurement of ‘h’ was now possible. Hence, interest in several experiments like superconducting magnetic levitation and the Joule balance was intensified for direct measurement of ‘h’. However, the most popular, advance experiment for direct measurement of ‘h’ has been conducted using watt balance.

The Watt Balance

The watt balance scheme is a simple adaptation from the ampere balance. The concept of balance signifies here to null balancing of mechanical and electromagnetic forces, and consequently equating the computed mechanical and electromagnetic power. The four criteria which are basically required for the watt balance is summarized in the recent paper [1]

a. A Stable magnetic field needs to be generated where the spatial gradient of flux density $(\vec{\nabla}\phi)$ has a preferential geometry.
b. Considering the opted stable magnetic field an induction coil operating at reference values for voltage, current, resistance and mass is designed.
c. The coil needs to be connected to a support mechanism that has balancing pivot point, so the coil moves with small driving force.
d. The whole mechanism is to be aligned in such a way that the coil

moves nearly vertical over a long distance.

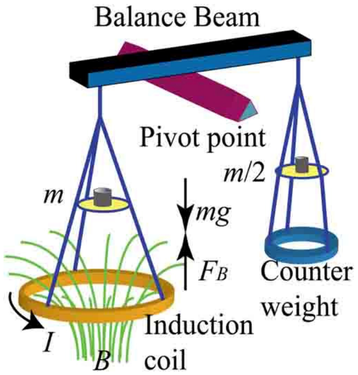

Fig.4: The watt balance scheme

To start with, without test masses the main coil with no current passing through it is first balanced with a counter weight. Then in the force mode (with current in main coil) a tare mass on the counter mass side, of half the test mass is initially balanced by the downward component of the electromagnetic force. This is because the electromagnetic applied force is half the total gravitational force on the test mass. The balance beam facilitates the coil to move mostly vertical. After this the current is reversed to balance the main test mass. Current reversal not only eliminates voltage and balance force off sets, but also maintains constant Ohmic heating of the coil during weighing mode.

Basic Watt Equation

The differential Lorentz force equation describes a gravitational force on mass ‘m’ as equal to a force on a coil with a current I, within a magnetic flux density dependent on the z-gradient

$$F_Z = mg = -I\frac{\partial\phi}{\partial z} \qquad (7)$$

In the differential form of Faraday’s law a coil produces a voltage V (in fact emf $\in$), when moving at a velocity $\frac{\partial z}{\partial t} = v_z$.

$$V = -\frac{\partial\phi}{\partial t} = -\frac{\partial\phi}{\partial z}\frac{\partial z}{\partial t} = -v_z\frac{\partial\phi}{\partial z} \qquad (8)$$

Assuming magnetic flux gradient to be identical in equation (7) and (8) we can have the two equations

$$mgv_z = VI = \frac{V^2}{R} \qquad (9)$$

where I is measured as the voltage drops (V), across a reference resistance R.

Expanding equation (6) (ignoring quantum numbers)

$$\frac{V^2}{R} = \frac{(f\,{}^{h}\!/_{2e})^2}{({}^{h}\!/_{e^2})} = f^2\,\frac{h}{4}[W] \qquad (10)$$

Using $\left(\frac{2e}{h}\right)^2 = K_{J-90}^2$ and $\frac{h}{e^2} = R_{K-90}$, we can rewrite equation (10) as

$$\frac{V^2}{R} = \frac{f^2}{K_{J-90}^2 R_{K-90}}[W_{90}] \qquad (11)$$

Comparing (10) and (11)

$$[W_{90}] = \frac{h}{4}\left(R_{K-90}\,K_{J-90}^2\right)[W] \qquad (12)$$

Thus putting conventional electrical measurements into the SI units of mechanical power measurements requires a conversion from [W$_{90}$] to [W], given by equation (12).

From equation (9), we have

$$mgv_z = \frac{V^2}{R} = \left(\frac{V^2}{R}\right)_{90}[W]_{90}$$

$$\left[\frac{mgv_z}{\left(V^2/R\right)_{90}}\right] = [W_{90}] \qquad (13)$$

$$\frac{mgv_z}{\left(V^2/R\right)_{90}} = \frac{h}{4}\left[R_{K-90}K^2_{J-90}\right]$$

using equation (12)

$$h = \frac{mgv_z}{\left(V^2/R\right)_{90}} \frac{4}{\left(R_{K-90}K^2_{J-90}\right)} \qquad (14)$$

It is important to note from equation (14) that, measurement of ‘h’ does not depend upon any adopted values for $K_{j\text{-}90}$ and $R_{k\text{-}90}$. Recently [8], a watt balance at the National Institute of Standards and Technology (NIST) has determined the value of Planck constant as $h = 6.62606979(30) \times 10^{-34}$ Js.

Conclusion

The Planck constant plays an important role in understanding the behaviour of matter at the subatomic level. It is a cornerstone of the theory of quantum mechanics, which describes the strange behaviour of particles at this level. The accuracy of Planck constant has recently become a newsworthy issue because of proposed revision of SI units. Since indirect method (1914) to direct method (2010) the Planck constant has been measured with ever better resolution (Fig.5).

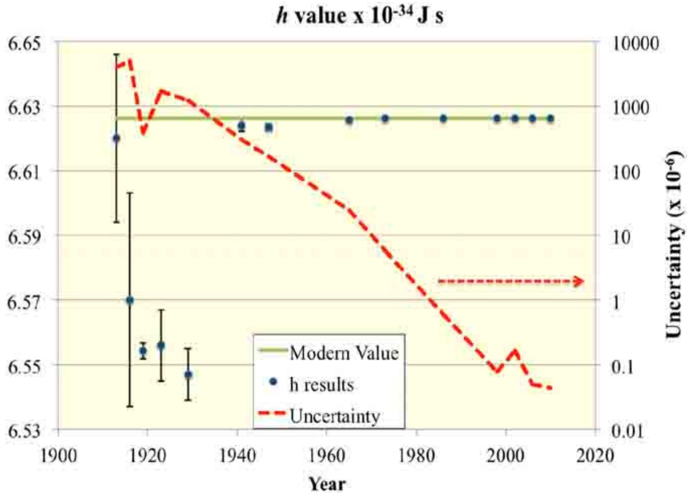

Fig.5: A graph summarizing the complete history of Planck constant determinations [1]. Dotted line shows the decrease of uncertainty over the years.

Several of the base units within the SI system are in the process of being redefined with an aim of linking the units to fundamental constants of nature [4,6]. This is the proposal of Bureau International des Poids et Mesures (BIPM-2010). The proposed revision of SI units would embed Planck constant into the definition of kilogram as a fixed constant of nature. In this present article, brief review on measurements of Planck constant has illuminated its impressive history. It is also amazing to realize that the best measurement of Planck constant ‘h’, which is purely a quantum concept, is attainable by a watt balance which combines an ‘Old Classical Physics’ involving force balance, electromagnetic induction/ interaction etc. to a new quantum based time, frequency and electrical measurements. Many review articles are available in the literature but original papers are hard to find. However, the conclusion is that as measurement uncertainties pertaining to Planck constant has declined substantially (few parts in 10^8) from watt balance experiments and Avogadro determination, its importance has been linked to a proposed redefinition of a kilogram unit of mass.

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