Determination of Magnetic Dipole Moment of Permanent Disc Magnet with Two Different Methods

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Abstract

In this work, we present two different experiments to estimate the magnetic dipole moment of cylindrical neodymium magnets, which can be used by physics teachers and students at college or university level. One experiment is using computer data acquisition system and sensors to record measurements in real time. And the other one is employing a precision digital balance and meter-stick to measure the force between two the magnets. The magnetic dipole moment is determined from the slope of the magnetic force as a function of the inverse fourth power of the distance. The estimated average magnetic dipole moment by force sensor and digital balance is 1.5±0.05 Am^2 and 1.29±0.04 Am^2, respectively. The experiments details are described and the results discussed.

1. Introduction

The magnetic force interaction between two magnets is beyond monopole-monopole interaction; it is considered magnetic dipole-dipole interaction. This relevant topic is discussed in many text books of electricity and magnetism [1].

In this paper, we present two different experiments to investigate the relationship between the magnetic force as a function of the distance of separation between two identical disc neodymium magnets, and to estimate the magnetic dipole moment of permanent magnet.

The first proposed experiment uses a computer data acquisition system and commercially available sensors of Pasco [2],wherethe data of themagnetic force as a function of distance of distance between the two magnets are recorded and plotted graphically in real time measurements. The second one is classical experiment that employs a precision balance with a digital readout measurement of the force between two magnets as a function of distance.

The magnetic dipole moment of permanent magnet, for both methods, was determined from the slope of the linear best fit of the graph representing magnetic force versus inverse fourth power of the distance of separation between the two magnets. The experimental details of both experiments are described and the results are discussed.

The simplicity and relatively small laps of time to perform data measurements of the activities presented in this work can probably contribute to creating appropriate learning environments and promote the successful students learning of magnetic concepts.

2. Force between two disc magnets

The magnetic force between two identical disc magnets with the vectors of magnetization lying on their common axis was approximated by the expression[3].

$$F(x) = \frac{\pi\mu_o M^2 R^4}{4}\left[\frac{1}{x^2} + \frac{1}{(x+2h)^2} - \frac{2}{(x+h)^2}\right] \quad (1)$$

Where $\mu_o = 4\pi \times 10^{-7}\ TmA^{-1}$ is the magnetic permeability, M the magnetization of the magnets, $x$the distance of separation between the two magnets, $h$the thickness (height) of the magnets and $R$the radius of the magnet.

The effective magnetic dipole moment is expressed as,

$$m = MV \quad (2)$$

Where, V represents the volume of the magnet. For a cylindrical magnet $V = 2\pi R^2 h$. In the case where $x \gg h$, the expression (1) reduces to the following.

$$F(x) = \frac{3\mu_o m_1 m_2}{2\pi}\cdot\frac{1}{x^4} \quad (3)$$

This relation demonstrates that the magnetic force is proportional to the inverse fourth power of the distance between the two magnets. The magnetic dipole moment, m, can be determined from the slope of the plot of magnetic force versus the inverse fourth power of the distance.

3. Experiments and procedures

3.1. Computer-aided experiment

This experiment is based on the use of computer data acquisition system and commercially available force and rotary motion sensors. The force sensor is to measure the magnetic force of the magnet and the motion sensor indicates the distance of separation between the disc magnets.

Data measurements obtained by both sensors were recorded and displayed graphically in real time. The magnetic dipole moment of a permanent neodymium magnet was determined after the magnetic force as a function of distance between two identical disc neodymium magnets was performed by means of the experimental setup, as shown in Fig.1.

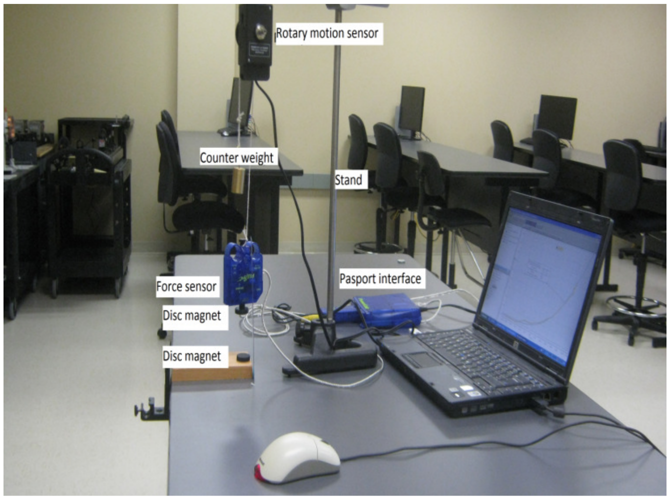

Fig.1. Experimental setup, showing computer data acquisition system, force and motion sensors, and dic magnets.

In this work, we used modified experimental procedure analysis and apparatus that are employed by [4].Two identical disc neodymium magnets, magnetized through the thickness, of volume (12.7 x 12.7 x 6.35 mm^3) were used to investigate the magnetic force as a function of distance between them and to estimate the magnetic dipole moment of the magnet. One of the magnets is glued on a table and the other magnet is fixed to the *Pasport force sensor* of Pasco. The two discs are placed in repelling position. In order to measure the exact distance of separation between the two magnets, a *rotary motionsensor* of Pascois placed on a stand, this unit has three pulleys on it of different diameters, 10 mm, 29 mm and 48 mm (*three-step pulley*). A cord is wrapped around a medium pulley of this sensor, one end of the string is connected to a counterweight mass of 200 g and the other end is attached to the *force sensor*. The two sensors were connected to the computer via the *Pasport interface* of Pasco; where data measurements are recorded, displayed and analyzed in real time using *Datastudio software* of Pasco.

A file is created in *Datastudio*to display graphically the vertical magnetic force as a function of distance of separation between magnets. The sampling rate of *force* and *rotary motion sensors* are set at 10 Hz, the sensitivity of both of them is adjusted at *Low,* and for a linear calibration of medium pulley of the rotary motion sensor a *360 divisions/rotation* was chosen for this purpose.

The force sensor with the magnet is pushed smoothly from a height of approximately 12 cm down to about 3 cm away from the fixed magnet on the table. At this distance, i.e., 3 cm the pushing repulsive force is more powerful. A graph representing a plot of data measurements of magnetic force as a function of distance of separation is shown in Fig.2. As can be seen from this graph, the magnetic force varies inversely as the fourth power of the distance with a power exponent n = - 4.02.

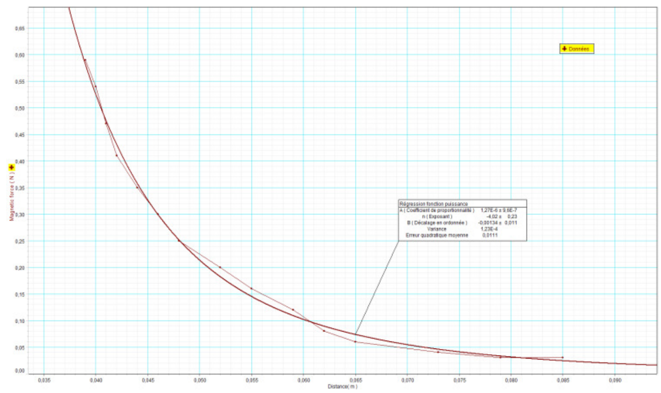

Fig.2. Magnetic force vs. distance – Computer-aided measurements.

3.2. Balance and meter-stick experiment

A digital balance of 0.01 g of precision and meter-stick are used in this experiment to explore the magnetic force as a function of distance between two identical disc neodymium magnets, as shown in Fig.3,the same magnets are employed as in the computer-aided experiment. One of the magnets is taped onto the balance pan, and the other magnet is fixed to the end of the meter stick, near the zero mark, with transparent glue. These magnets are placed in repulsive position, and the meter-stick was held by a clamp mounted onto a stand. A reference point is created in the meter and data measurements are performed in the interval of 3 to 18 cm of separation between discs with a 1 cm increment.

The graph of data measurements of magnetic force versus distance of separation between the two discs magnets is shown of Fig.4. The best fit data of this plot is a curve of inverse fourth power, with a power exponent n = - 3.882 and coefficient of determination R^2 = 0.9994.

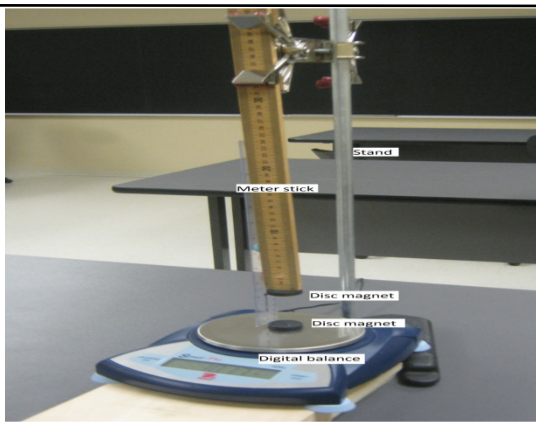

Fig.3. Experimental setup – Digital balance and meter stick

4. Results analysis and discussion

In order to correct systematic errors, special attention is paid when calculating magnetic forces F whichare obtained from the measurement of the masses displayed on the digital balance and so in the distance X between the two disc magnets as well. The estimated systematic error in the magnetic force ΔF and the distance of separation ΔX are 1.96 x 10^{-4} N and 2 x10^{-3} m, respectively. These systematic errors are taken in to account such that ΔF and ΔX are subtracted from F and X, respectively; when using equation (3).

In the absence of systematic errors, the mean value approaches the true (accepted) values as the number of measurements (trials) increases. To get the average value and the standard deviation of the magnetic dipole moment of the neodymium magnet, five trials are performed for each experiment, thecomputer-aided one and the digital balance-meter stick.

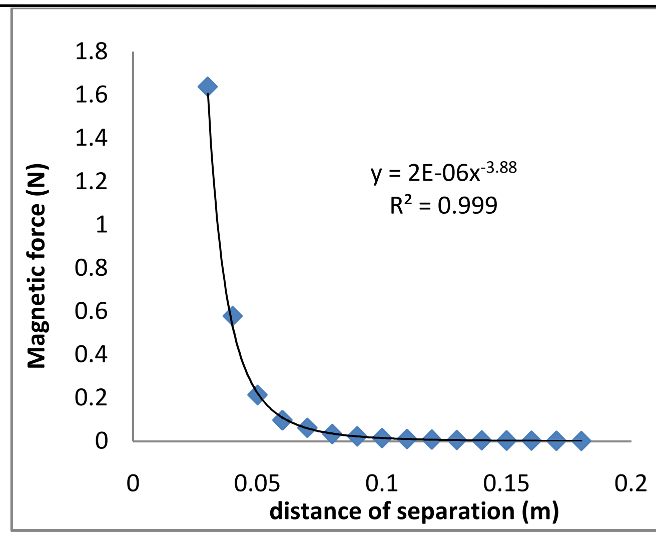

Fig.4. Magnetic force as a function of distance – Digital balance and meter-stick measurements.

As the magnetic dipole of the magnet is determined from the slope of the plot of magnetic force as a function of inverse fourth power of the distance between the disc magnets, the magnetic force versus distance between the two magnets are obtained for five trials for both experiments. Examples of graphs from the five trials of data measurements of magnetic force as a function of distance between the disc magnets for the computer data acquisition system experiment and the digital balance-meterstick are shown in Figure 2 and 4, respectively. As can be seen from these plots, the force varies inversely as the fourth power of the distance between the two magnets. The average value of the power exponent n, in equation (3), for the computer-aided experiment and the balance-meter stick one is - 4.05±0.16 and – 3.90±0.12, respectively. These results are in good agreement with the expected value closeto– *4*.

The data measurements of a force as a function of the distance are plotted as *log F* versus *log x* for the computer data acquisition system and balance-meter stick as shown in Fig.5 and 6, respectively. A linear best fit data is obtained for both experiments, the value of the slope, m, represents the power exponent, n, in equation (3).

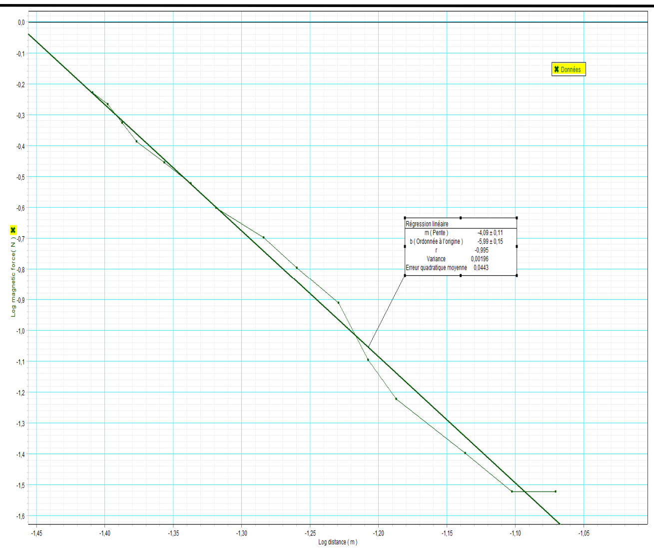

Fig.5. Log force vs. log distance – Computer-aided measurements

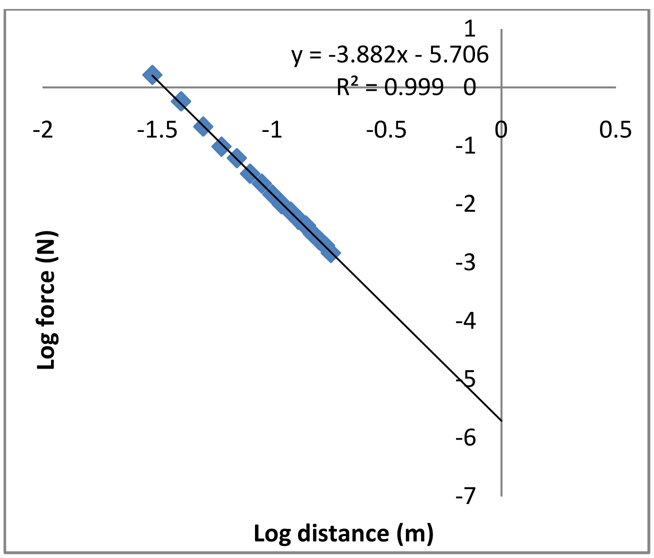

Fig.6. Log force versus log distance - Balance –meter stick measurements

The magnetic dipole moment of disc neodymium magnet for each experiment is determine from the slope of the plot of the force as function of the inverse fourth power of distance.

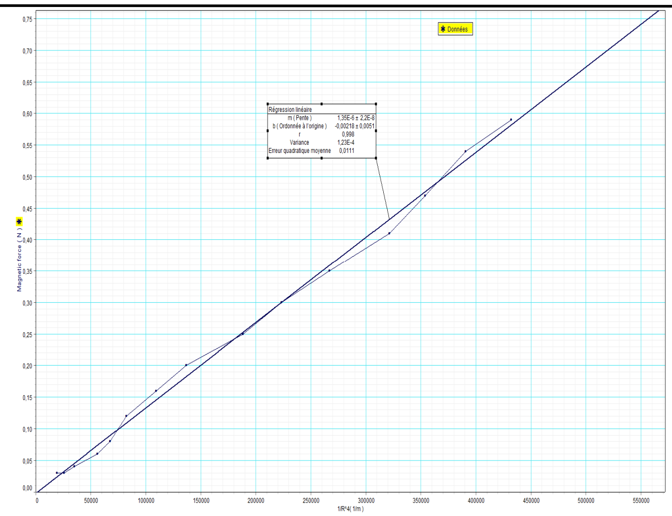

Fig.7. Force vs. inverse fourth power of distance ($1/x^4$)–computer –aided measurements.

The slope $k = \frac{3\mu_o m^2}{2\pi}$, where$m^2$is the magnetic dipole of the two identical disc neodymium magnetsand $\mu_o$the magnetic permeability. As can be seen from the plot of *force* versus $1/x^4$ the value of the slope for the computer aided experiment and the balance–meter stick one is 1.35 x 10^{-6} and 1.0 x 10^{-6}, respectively, as illustrated in Fig.7 and 8.

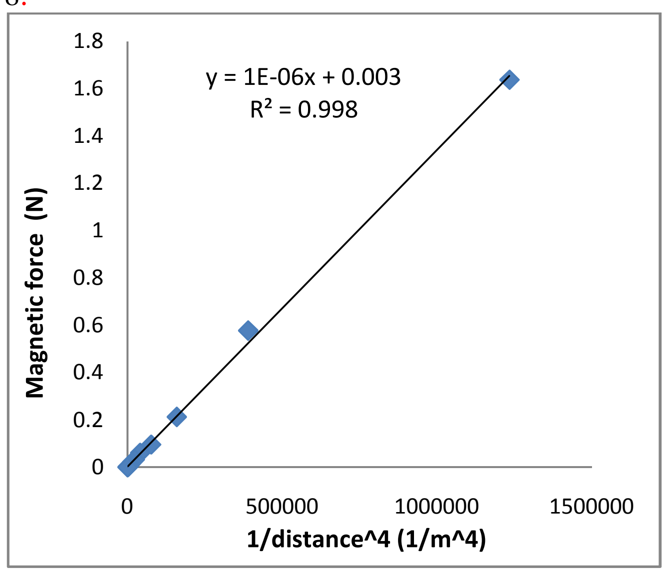

Fig.8. Magnetic force vs. inverse fourth power of distance – Balance –meter stick measurements

The average value of the magnetic dipole moment for five trials for the data acquisition system and the balance-meter stick experiments, is 1.50 ± 0.05Am^2 and 1.29±0.04 Am^2, respectively. The difference between these values is approximately 21%.

5. Conclusion

The experimental procedures and the results obtained by both experiments in this work provide a supplementary source of knowledge to college or university level students and teachers when exploring the relationship of the magnetic force in permanent cylindrical magnets. The other references are reported in literature [5].

The comparison between the two experimental methodsto estimate the magnetic dipole moment of disc magnet has confirmed that both of them are consistent due to their reproducibility in the same conditions. Both experiments can be performed in relatively small laps of time, and can be carried out as laboratory experiments by physics students or as demonstrationsin classrooms or laboratory by teachers. They investigate the relationship of the magnetic force-distance, the magnetic field-distanceand provide an estimate of the magnetic moment dipole of magnets.

The obtained results are found to be reliable andconsistent because the mean value of the magnetic moment dipole for five trials for computer aided experiment and balance meter-stick was 1.5 Am^2 and 1.29 Am^2 with standard deviation, σ ±0.05Am^2 and ±0.04 Am^2, respectively.

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