Creation of electric potential minima in 3D: Trapping charged particles

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Abstract

An electrostatic potential minima does not exist in three dimension. However, an alternating electric field can produce a dynamical potential minima in three dimensional space and charged particles can be trapped within such potential well. A system of trapped ion/s is almost free from unknown external perturbations and hence such a system finds enormous applications in different fields. This article explains how such a potential minima can be developed with electric field only and how the charged particles can be trapped within it. Some important applications of trapped particles have been outlined here with a demonstrative experiment for realization of the technique.

1 Introduction

'Let us consider a particle at rest'-this is often the introductory sentence in our text books or while teaching in classroom. But can we have a particle at rest in practice? A famous remark from Erwin Schroedinger may be quoted in this regard, 'We never experiment with just one electron or atom or (small) molecule. In thought-experiments we sometimes assume that we do; this invariably entails ridiculous consequences'. However, it has now become a reality to have a particle almost at rest. A single particle like an atom, or even an electron can be confined in space within a region of few micrometers. For confinement of charged particles, two different techniques were developed by two pioneers Wolfgang Paul (the device, named after him, known as Paul trap [1]) and Hans Georg Dehmelt (the device, named after Frans Michel Penning, known as the Penning trap [2]). In Paul trap, the charged particles can be trapped by using a static electric field together with a time varying electric field while in Penning trap, a static magnetic field is required in association with a static electric field. Both of these devices are regularly used as important tool in different fields, both of fundamental physics interests and commercial applications. Here we will restrict ourselves in discussions related to the Paul trap. The readers are, however, referred to an article [3] which covers discussions on both the Paul trap and the Penning trap.

The article has been arranged in the following way. In section 2, the fundamental technique of creation of potential minima using only the electric field (as associated with the Paul trap) has been described. The equation of motion of a single charged particle within such a dynamic potential well is reviewed in section 3. In section 4, a demonstrative experiment has been presented. The applications of trapped ion system in different fields have been outlined at the end of this article (section 5).

2 How to create a potential minima in 3D?

A particle in one dimension can be confined by a restoring force proportional to its displacement from the equilibrium position (the force as associated with a simple harmonic motion). In other words, it requires a quadrupole potential (proportional to the square of the displacement). Naturally, for three dimensional trapping of a particle, the potential should be quadrupolar in all three dimensions and is described as follows:

$$\Phi(x,y,z) = Ax^2 + By^2 + Cz^2, \tag{1}$$

where A, B, C are constants. For a charged particle, this potential can be chosen as the electric potential. Thus the force on a particle of charge e under the influence of this potential is given by

$$\begin{aligned} \vec{F}(x,y,z) &= -e\vec{\nabla}\Phi(x,y,z) \\ &= -2e(Ax\hat{x} + By\hat{y} + Cz\hat{z}). \end{aligned} \tag{2}$$

As is necessary for trapping, the force $\vec{F}$ should be restoring in nature and thus it follows that the constants A, B and C are all positive (for a positively charged particle). However, any electrostatic potential in free space should satisfy the Laplace's equation ($\nabla^2\Phi(x,y,z) = 0$), following which, at least one constant must be negative for this electrostatic potential. Thus it can be concluded that no electrostatic potential minima exists in three dimension [1].

So how to create the electric potential minima in three dimension? The answer to the this challenge was addressed by Wolfgang Paul who demonstrated that a time varying electric potential can produce a 'dynamic minima' in three dimension. The idea is to vary A, B and C with respect to time, such that the potential having its minima in one direction at an instant, rotates to the other direction at a later instant. If the rotation of the potential minima is faster as compared to the motion of the charged particle, the particle will experience a time-averaged potential minima in all directions. The particle will be confined within the potential well if the average potential depth is larger than its kinetic energy. This can be compared to a ball placed on a rotating saddle (see, for reference, a nice demonstration in youtube, the mechanical analogue of Paul trap [4]).

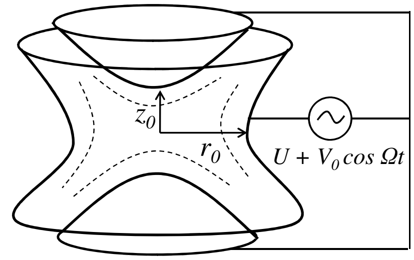

Figure 1: The hyperbolic geometry of the electrodes with necessary electrical connections for developing the quadrupole potential as defined in eqn. 5. The dotted lines show hyperbolic equipotential surfaces (eqn. 3).

In order to produce the quadrupole electric potential, suitable geometry of the electrodes is required. If there exists a rotational symmetry about the z axis, $A = B$, and hence $C = -2A$. Consider, for example, the electrode geometry depicted in fig. 1. As can be seen from fig. 1, two coaxial bowl-shaped electrodes at the ends, together with the ring electrode at the middle are hyperboloids of revolution about the z axis. If the radial and axial dimensions of the trap are respectively r_0 and z_0, the equations for the hyperbolic electrode surfaces are given by [5]

[1] This is, in literature, known as Earnshaw's theorem

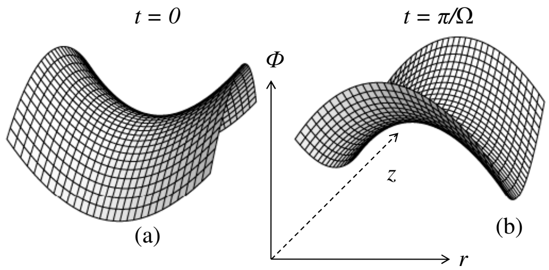

Figure 2: Potential surface in the $r-z$ plane at different instant. (a) Harmonic oscillator Potential along r(in $x-y$ plane) and inverted oscillator potential along z at $t = 0$. (b) Harmonic oscillator Potential along z and inverted oscillator potential along r at $t = \pi/\Omega$. The minima of the potential surface rotates in the $r-z$ plane with angular frequency Ω, the frequency of the applied alternating potential.

$$\begin{aligned} r^2 - 2z^2 &= r_0^2, \\ r^2 - 2z^2 &= -2z_0^2, \end{aligned} \tag{3}$$

where $r^2 = x^2 + y^2$. When voltage is applied to the middle electrode or on the end electrodes, it produces equipotential surfaces defined by eqn. 3. The potential inside the trap can therefore be written as

$$\Phi(r, z) = A(r^2 - 2z^2). \tag{4}$$

Now, the coefficient A should be chosen in such a way that the potential has its minima along r (i.e. in the $x-y$ plane) at an instant, and in the z direction at a later instant. To elucidate the statement, let us consider the following form of the potential:

$$\Phi(r, z, t) = \frac{U + V_0 \cos \Omega t}{2r_0^2}(r^2 - 2z^2). \tag{5}$$

It is seen from eqn. 5 that, at time $t = 0$ the potential resembles that of simple harmonic oscillator along r and inverted harmonic oscillator along z [fig. 2(a)]. However, the vice-versa hold at $t = \pi/\Omega$ [fig. 2(b)].

3 Motion of a Trapped Ion

The equation of motion of a single particle of charge e and mass m under the influence of the potential (defined by eqn. 5) follows from Newton's law of motion and can be described by the following equations:

$$\begin{aligned} \frac{d^2 r}{dt^2} &= -\frac{e}{mr_0^2}(U + V_0 \cos \Omega t) r, \\ \frac{d^2 z}{dt^2} &= \frac{2e}{mr_0^2}(U + V_0 \cos \Omega t) z. \end{aligned} \tag{6}$$

Using a common symbol u for both r and z, and introducing the dimensionless parameters a_u, q_u and ζ the eqn. 6 can be rewritten as

$$\frac{d^2 u}{d\zeta^2} + (a_u - 2q_u \cos 2\zeta) u = 0, \tag{7}$$

where

$$\begin{aligned} a_z &= -2a_r = -\frac{8eU}{mr_0^2\Omega^2}, \\ q_z &= -2q_r = \frac{4eV_0}{mr_0^2\Omega^2}, \\ \zeta &= \frac{\Omega t}{2}. \end{aligned} \tag{8}$$

The equation of motion (eqn. 7) is a standard differential equation in mathematics, known as Mathieu differential equation. The solutions of this equation result in either stable or unstable motion depending on the values of the parameters a_u and q_u, defined in eqn. 8. There exists a region in a_u vs. q_u diagram for which the ion-motion is stable along a particular direction, for example along r (fig. 3). A similar stability region exists for the motion along z direction. An intersection between these two stability regions (the shaded region in fig. 3) is where the stable motion in three dimension is sustained.

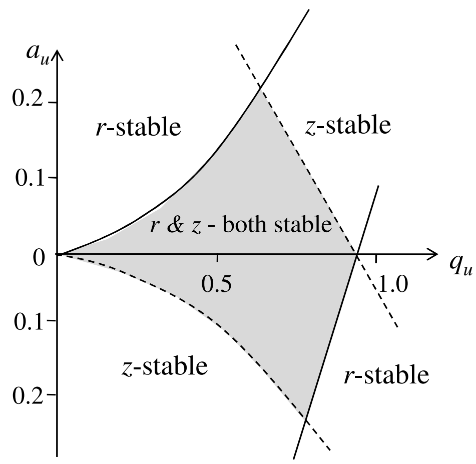

Figure 3: Stability region in a_u vs. q_u diagram for an ion trap. The regions bounded by the dotted line and bold line correspond to stable motion along r and z respectively. The motion is stable along both r and z directions in the shaded region and the trap operating parameters a_u and q_u are chosen in this region.

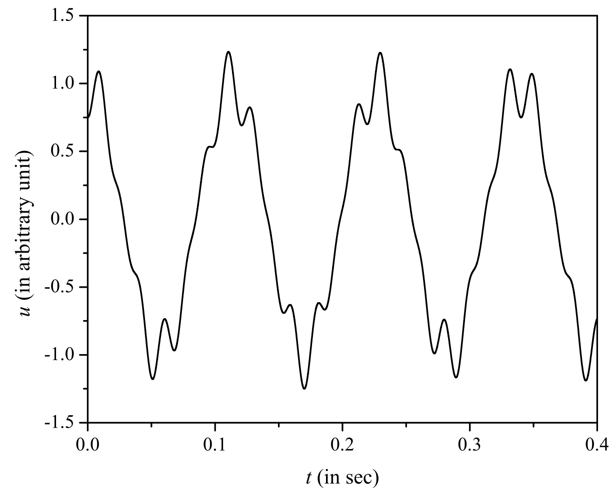

Figure 4: The motion of a trapped ion in one direction. A low frequency motion, called the secular motion is modulated by a high frequency motion, the micromotion. The simulation of the trajectory is done for $q_r = 0.5$, $\Omega = 2\pi \times 50$ rad/s and $\omega_{0r} = 2\pi \times 8.5$ rad/s.

In the 'adiabatic approximation' (i.e. for small a_u and q_u values), the solution of Mathieu differential equation can be represented in the following form [5]:

$$u = c\left(1 - \frac{q_u}{2}\cos\Omega t\right)\cos\omega_{0u}t, \tag{9}$$

where c is a constant and

$$\omega_{0u} = \frac{\beta_u \Omega}{2}. \tag{10}$$

The parameter β_u, for small a_u and q_u, can be defined as

$$\beta_u \approx \sqrt{a_u + \frac{q_u^2}{2}}. \tag{11}$$

Eqn. 9 shows that the ion oscillates with a frequency ω_{0u} and its motion is modulated with the frequency Ω of the applied alternating potential (fig.4). For $\beta_u < 1$, $\omega_{0u} < \Omega$. The slow frequency motion (at ω_{0u}) is called the macromotion or secular motion while the higher frequency motion (at Ω) is termed as the micromotion.

4 An Experiment

In this section, a demonstrative experiment has been described. Dust particles, here chalk dust, have been trapped in a ring trap at the line frequency, at 50 Hz.

The trap setup is shown schematically in fig. 5. The surfaces of two end electrodes are hyperboloids of revolution about the z axis and the ring electrode at the middle has hyperbolic cross section, a similar geometry that is described in fig. 1. In the experiment, the electrodes are made of brass. The ring is taken of diameter ~ 10 mm ($r_0 = 5$ mm) and the end cap electrodes are separated by a distance ($2z_0$) of 7 mm (note that, $r_0^2 = 2z_0^2$, a dimensional constraint

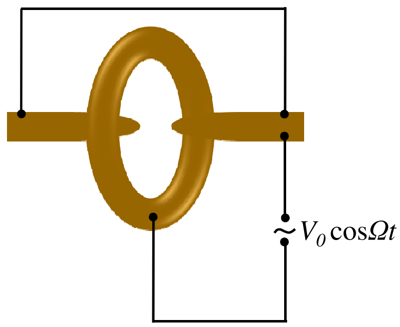

Figure 5: Schematic of a ring trap setup. The end electrodes are connected together and the alternating potential is applied between the ring and the end electrodes.

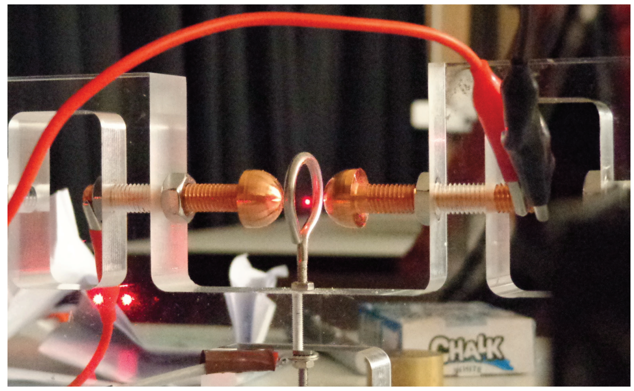

Figure 6: A photograph of the setup with trapped chalk dust. A laser beam is incident on the trapped dust at the center and is scattered by the dust particles for clear visualization of ion trapping.

of this electrode geometry as necessary for efficient trapping). The end electrodes are electrically connected together and an alternating voltage is applied between the ring electrode and the end electrodes. The line voltage (230 V, 50 Hz) is passed through a variac and fed to a step-up transformer for necessary voltage amplification. A typical voltage used for trapping is 1500 V. It is to be noted that no dc potential is applied here i.e. $U = 0$ and hence $a_u = 0$ (dc potential just modifies the effective potential depth). The chalk dust are taken in a syringe and injected inside the trap. The dust get ionized due to injection and are trapped inside. A photograph of the experimental setup with trapped dust particles at the center is presented in fig. 6.

The dust particles form thread-like clusters and oscillate inside the trap. If the trapping voltage is stabilized and the system is adequately isolated from the surroundings, the particles can be stored for days within the trap. It is possible to estimate the charge-to-mass ratio of the trapped dust clusters. For stable and efficient trapping, the q parameter should be around 0.5. With the applied ac voltage $V_0 = 1800$ V, at frequency $\Omega = 2\pi \times 50$ rad/s, the charge-to-mass ratio (e/m) is estimated from eqn. 8 as 3×10^{-4} C/kg.

5 Applications

Ion traps provide best realization of 'particle at rest' and hence it is used as an important tool in many applications. The thermal motion of trapped ion can be reduced by laser cooling technique and it can be localized within its de Broglie wavelength which is few μm [6]. Thus a single trapped and laser cooled ion represents a perturbation-free quantum system. A series of experiments are being performed for testing and demonstrating wide aspects of fundamental physics. A single or few ions are used for precision measurement of various atomic properties such as lifetime of atomic states [7], transition frequency or ac Stark shift [8], quadrupole moment of atomic states [9], atomic parity violation [10] *etc.* A single trapped ion is used for developing atomic frequency standard [11]. Single or few trapped ions are used for quantum teleportation [12], quantum information processing [13] and designing quantum computer [14]. Large ion traps are used for Coulomb crystal study [15], mass spectrometric applications [16] and many more [17].

6 Acknowledgments

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