Pedagogical Framework of Elementary Mechanics

Comparable to Elementary Electromagnetism

Introductory Approach without Reliance on Equation of Motion

Yukio Kobayashi
Department of Information Systems Science, Faculty of Engineering,
Soka University, 1-236 Tangi-cho, Hachioji-shi, Tokyo 192-8577, Japan
koba@t.soka.ac.jp

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Abstract

A pedagogical framework of elementary mechanics is developed from temporal and spatial viewpoints. In contrast to conventional physics courses, the equation of motion is not a starting point for elucidating mechanical phenomena but an equation that summarizes three propositions on (1) the change of the linear momentum of a particle caused by impulse, (2) the change in the kinetic energy of a particle caused by the work done on the particle by applied force, and (3) the change of the angular momentum of a particle caused by torque. This construction is comparable to the formulation in the elementary course of electromagnetism in which Maxwell's equations are not a starting point but, instead, summarize the four laws of electromagnetic fields. The formulation that derives the above three propositions from the equation of motion cannot necessarily help a student understand the mechanisms of mechanical phenomena. For students, the acquisition of temporal and spatial viewpoints in regard to mechanical phenomena and the understanding of the essence of the intensity of motion are important in stimulating physical thought.

1. Introduction

The law of motion is a principle of causality in that the change of motion is induced by an applied force. The state of motion of a particle at any time t is specified by its position r and velocity *v*. According to the law of motion, the acceleration of a particle is determined under the influence of an applied force. If the acceleration is known, the future state of a particle can be predicted on the basis of kinematics. The causality of motion is summarized in a mathematical expression called the equation of motion. In undergraduate elementary physics courses, students learn some applications of the equation of motion. Certainly, they can write the equation of motion for the respective phenomena and solve it formally. However, from a pedagogical viewpoint, there are some problems in regard to the equation of motion.

First, some students regard the equation of motion as only a formula stating that mass times acceleration equals force. Thus, they do not necessarily understand the causality of motion. Some students cannot distinguish between an applied force and *m*a (the product of mass and acceleration). Other students regard *m*a as a physical quantity similar to linear momentum *m*v (the product of mass and velocity) although it is not. In addition, some students think that the aim of mechanics is solving the equations of motion as differential equations instead of elucidating natural phenomena. The processes of obtaining the velocity and position of a particle are only exercises in kinematics.

Second, acceleration is the second derivative of position with respect to time and, thus, is more difficult to realize intuitively than is velocity. In undergraduate advanced courses in science and engineering, generalized momentum, rather than acceleration, is related with generalized force.[1] Therefore, the framework of elementary mechanics can be constructed without reliance on acceleration.

Third, the conventional approach to elementary mechanics is in contrast with the elementary course of electromagnetism. Coulomb's law, Ampere's law, and Faraday's law are introduced through experimental results, and the laws of electromagnetic fields are then summarized in Maxwell's equations.[2] In contrast, in elementary mechanics, the equation of motion is a starting point for elucidating mechanical phenomena. The equation of motion called Newton's equation is comparable to Maxwell's equations. An approach to elucidate electromagnetic phenomena on the basis of Maxwell's equations is difficult for some students in undergraduate elementary courses on electromagnetism. For example, there are also several approaches in the theory of probability: probability based on axioms and probability as a limit of the relative frequency. The probability axioms are difficult for students to understand. In a graduate electromagnetic wave engineering course, however, electromagnetic phenomena are elucidated axiomatically on the basis of Maxwell's equations. In addition, in an undergraduate elementary course on mechanics, the formulation that derives experimental results from the equation of motion cannot necessarily help a student understand the mechanisms of mechanical phenomena. Indeed, Newton did not express the law of motion as the equation of motion. The so-called Newton's equation was enunciated by Euler[3]. Although the use of Newton's name is not entirely accurate here, it is used for contrast with Maxwell's in this paper.

Motivated by the considerations reported above, the present article will attempt to change the order in which the basic notions of elementary mechanics are introduced. The starting point is not the equation of motion but the three propositions on the causality of motion that are described later. The principle of these propositions is that the mechanisms of the same mechanical phenomenon are explained from both the temporal and spatial viewpoints. These viewpoints are the foundation of the exploration of physical phenomena. The motion of a particle in a uniform gravitational field is examined here. This is the most familiar mechanical phenomenon in daily life. Motion in a uniform gravitational field is a key to the essential problem of "how quantities for describing the intensity of motion are defined." Surveying the history of physics, Galileo and Descartes conducted an elaborate examination of the motion of a particle in a uniform gravitational field.[4] Some textbooks, however, treat this phenomenon as only an example. In the next section, the motion of a particle in a uniform gravitational field is discussed quantitatively from a kinetic viewpoint.

Defining the intensity of motion of a particle is an essential problem. If the intensity of motion is not defined quantitatively, it cannot be used for representing the law of motion. In this article, the physical meaning of the intensity of motion is clarified by examining the features of a particle thrown upward and those of a particle thrown downward. Based on these features from a temporal viewpoint, the intensity of motion of a particle can be described by the linear momentum, and, from a spatial viewpoint, it can be described by the kinetic energy. In addition, an angular momentum is convenient for describing the intensity of rotation of a particle from a temporal viewpoint. The intensity of the motion of a particle changes as time passes or as the particle moves. Linear momentum, kinetic energy, and angular momentum are all at the same level of physical quantities for representing the intensity of motion. In the conventional framework of elementary mechanics, these quantities are formally introduced by integrating the equation of motion with respect to time or displacement. In contrast to this approach, after introducing the three quantities on the basis of both kinematics and the concept of mass, the three propositions are constructed on (1) the change of the linear momentum of a particle caused by impulse, (2) the change of the kinetic energy of a particle caused by the work done on the displacement of the particle by applied force, and (3) the change of the angular momentum of a particle caused by torque. The common factor of the three propositions is the equation of motion; in other words, this equation summarizes these propositions in the same manner as Maxwell's equations summarize the laws of electromagnetic fields. The temporal viewpoint can be translated into the spatial viewpoint; that is, propositions (1) and (2) are two different representations of the same phenomenon. The temporal viewpoint of rotational motion is described in proposition (3), while the spatial viewpoint is described in proposition (2) in the same manner as translational motion.

For students, the acquisition of temporal and spatial viewpoints in regard to mechanical phenomena and the understanding of the essence of the intensity of motion are important for stimulating physical thought. The most important achievement is not to solve typical problems formally through the equation of motion but to understand the causal relationship that the intensity of motion is changed by the temporal or spatial action of force. Temporal and spatial points of view are also essential for learning about electromagnetic phenomena and wave phenomena. In linguistics, there are two viewpoints: the change of language throughout time and dialect with a regional difference.

2. Causality of motion

The propositions on the causality of motion of a particle can be constructed without reliance on the equation of motion by recognizing the two physical meanings of mass through the kinetic observations of a particle in a uniform gravitational field. Here, the shape and volume of a particle are disregarded, and, thus, a particle is treated as a mass point. After introducing impulse, work, and torque for describing the effects of force on a particle and linear momentum, kinetic energy, and angular momentum for describing the intensity of motion of a particle, the causality of motion is considered.

A. The temporal and spatial effects of force on a particle

In a uniform gravitational field, a particle thrown downward is accelerated vertically downward, while a particle thrown upward is decelerated vertically upward. The change of the velocity of the particle is caused by gravity, which attracts the particle toward the center of the earth. Impulse and work for describing the temporal and spatial consecutive effects of force are defined by observing the change in the velocity of a particle caused by gravity. The longer the time spent in falling or rising, the greater the change in velocity. Thus, impulse is defined as (Force acting on a particle) X (Time of force acting) for force other than gravity as well. Essentially, the concept of impulse is applicable to any mechanical phenomenon, although collision problems often deal with impulse. Looking at the same phenomenon differently, we have the greater the displacement of the particle, greater is the change of velocity. Thus, work done on a particle by applied force is defined as (Force acting on a particle) X (Component of the displacement of the particle in the direction of force) for forces other than gravity as well.

Torque is defined to describe the rotational motion of a particle as follows. Greater the force, greater is the work done on a particle by a force during rotation, even when the rotational angle or the distance between the particle and the center of rotation is the same. The greater the distance between the particle and the center of rotation, the greater the work, even if the applied force or the rotational angle is the same. The greater the rotational angle, the greater the work, even if the applied force or the distance between the particle and the center of rotation is the same. Thus, work done on a particle by applied force during rotation is defined as (Component of force in the direction of rotation) X (Displacement of a particle during rotation). The displacement of a particle during rotation is represented as (Distance between a particle and the center of rotation) X (Rotational angle), and, thus, work done on a particle by applied force during rotation is (Distance between a particle and the center of rotation) X (Component of force in the direction of rotation) X (Rotational angle). Thus, torque is defined as (Distance between a particle and the center of rotation) X (Component of force in the direction of rotation), which changes the velocity of a particle by rotating it. In the advanced courses of mechanics, the component of force in the direction of rotation and the rotational angle are regarded as generalized force and a generalized coordinate[1] respectively, which are important quantities in the advanced courses of electromagnetism as well.

These three quantities describing the effects of force, impulse, work done on a particle, and torque, are related to linear momentum, kinetic energy, and angular momentum, respectively, as discussed in the following.

B. Quantitative description of the intensity of the motion of a particle

To describe the intensity of the motion quantitatively, its physical meaning is essential. Let us again observe the motion of a particle in a uniform gravitational field. This phenomenon is familiar to students and, thus, suitable for discussion. From a temporal viewpoint, a particle thrown upward gradually slows down and finally stops over time. The intensity of the motion is greater if the time spent before a particle stops is longer. If the speed is at time t and the particle stops at time t', $t' - t$ is proportional to $|v|$ from a simple calculation based on kinematics. The same is also true for a particle thrown downward. It is convenient to consider velocity a vector quantity instead of considering speed a scalar quantity to distinguish between rising and falling. Thus, velocity is a temporal determining factor of the intensity of motion. From a spatial viewpoint, a particle thrown upward gradually slows down and finally stops as the particle rises. The intensity of motion is greater if the displacement before the particle stops is greater. If the speed is $|v|$ at height z and the particle stops at height z', $z' - z$ is proportional to $(1/2)|v|^2$ from a simple calculation based on kinematics. Thus, (1/2) X (velocity)^2 is a spatial determining factor of the intensity of motion. The two quantities, velocity and (1/2) X (velocity)^2, in reverse proportion to acceleration, are essential factors because acceleration is common to the representations of $t' - t$ and $z' – z$. Thus, velocity and (1/2) X (velocity)^2 also dominate the intensity of motion of a particle under the influence of forces other than gravity.

It is pedagogically important for students to infer the intensity of motion from the experimental results. Let us ask students the following questions: *Exercise*. If a heavier particle and a lighter one are both thrown upward from the same position at the same time, which moves with a greater intensity of motion? Although a heavier particle is attracted toward the center of the earth by a greater force of gravity, in a vacuum, both time and displacement are the same as those of a lighter particle until the particles stop. A heavier particle rises with a greater intensity of motion sufficient to overcome the greater force of gravity than does a lighter particle. Thus, there are two physical meanings of mass: one is the property that a particle is accelerated by gravity, and the other is the property in which a particle retains its velocity. The latter is called inertia, which is a property that does not depend on the applied force and is not restricted to the state in which no applied force acts on a particle. In any case, the greater the mass of a particle, greater is the intensity of motion. In some cases, gravitational mass and inertial mass are distinguished. For beginning students, however, these technical terms should be avoided from a pedagogical point of view because daily experience makes it clearly evident that a heavier particle is less accelerated and decelerated than a lighter one.

Let us turn to the rotational motion of a particle and consider rotation with a constant angular acceleration about any axis passing through a fixed reference point. Even if the angular velocity is the same, the velocity is greater at a greater distance from the center of rotation. If the time before a particle stops is the same, the greater the area swept, the greater the intensity of rotation. The rate at which an area is swept is represented by the product of the distance between the particle and the center of rotation and the velocity of the particle. Therefore, the rate at which an area is swept is a temporal determining factor of the intensity of rotation. From a spatial point of view, a particle gradually slows down and finally stops as it rotates. The intensity of rotation is greater if the displacement before the particle stops is greater. Using similar reasoning to that for the temporal viewpoint, (1/2) X (velocity)^2 is a spatial determining factor of the intensity of rotation.

According to the considerations discussed above, the three physical quantities, i.e., linear momentum (mass X velocity), kinetic energy [(1/2) X mass X (velocity)^2], and angular momentum [(distance between the particle and the center of rotation) X mass X velocity], describe the intensity of motion. Originally, velocity had only been a kinetic quantity defined as displacement per unit time. Mechanics enlarged the concept by giving velocity the physical meaning of linear momentum per unit mass.

Next, let us consider quantitatively the changes of linear momentum, kinetic energy, and angular momentum to relate the force acting on the particle with the change of the intensity of motion, i.e., the cause/effect relationship between them. From the definitions of acceleration and velocity, $d\mathbf{v} = \boldsymbol{a}dt$ and $d\mathbf{r} = \boldsymbol{v}dt$ are obtained, where **r** and **a** are the position relative to the origin and the acceleration of a particle, respectively. The changes of the three physical quantities are expressed as $d(m\mathbf{v}) = m\boldsymbol{a}dt$, $d[(1/2)mv^2] = md\boldsymbol{v}\cdot\mathbf{v} = m\boldsymbol{a}\cdot\boldsymbol{v}dt = m\mathbf{a}\cdot d\boldsymbol{r}$, $d(\boldsymbol{r} \times m\boldsymbol{v}) = \boldsymbol{r} \times md\boldsymbol{v} + d\boldsymbol{r} \times m\boldsymbol{v} = \boldsymbol{r} \times m\boldsymbol{a}dt + \boldsymbol{v}dt \times m\boldsymbol{v} = \boldsymbol{r} \times m\boldsymbol{a}dt$. The quantities $d(m\mathbf{v})$, $d\,[(1/2)mv^2]$, and $d(\mathbf{r} \times m\boldsymbol{v})$ have a common factor, $m\boldsymbol{a}$. By regarding the amount of force necessary for the accelerated motion with acceleration **a** as $m\boldsymbol{a}$, a method of measuring force is established. Here, it is not necessary to consider the origins of the applied forces, such as a gravitational field, a hand, and a rope. A force with the amount of $m\mathbf{a}$ is acting on a particle. In other words, $m\mathbf{a} = \mathbf{F}$ indicates that a particle with mass m is accelerated by the net applied force **F** with an amount of $m\boldsymbol{a}$. The equals sign relates the effect that a particle is accelerated with the reason that applied force is acting on the particle. From temporal point of view, quantitatively, the intensity of motion of a particle is changed by the applied force during the time when the force is applied to the particle. From spatial point of view, quantitatively, the intensity of motion of a particle is changed by the applied force in the displacement where the force is applied to the particle. Remembering the definitions of impulse, work done on a particle, and torque (or the moment of impulse) expressed as $\boldsymbol{F}\,dt$, $\boldsymbol{F}\cdot d\boldsymbol{r}$, and $\mathbf{r} \times \boldsymbol{F}\,dt$, respectively, it is reasonable to measure the net applied force by $m\boldsymbol{a}$. Thus, three propositions are obtained: (1) The change of the linear momentum of a particle is equal to the impulse during the time interval that the applied force is acting on the particle: $d(mv)= F\,dt$. (2) The change of the kinetic energy of a particle is equal to the work expended by the applied force acting on the particle: $d\,[(1/2)mv^2]= \boldsymbol{F}\cdot d\boldsymbol{r}$. (3) The change of the angular momentum of a particle is equal to the torque during the time interval that the applied force is acting on the particle: $d(\boldsymbol{r} \times m\boldsymbol{v}) = \boldsymbol{r} \times \boldsymbol{F}\,dt$.

These three propositions are comparable to the laws of electromagnetism, such as Coulomb's law, Ampere's law, and Faraday's law, on the basis of experimental results. The physical quantities of mass and gravitational field correspond to the electric charge and electromagnetic field, respectively. The equation of motion can be regarded as a goal of elementary mechanics in the manner of Maxwell's equations in elementary electromagnetism. For students, the physical meanings of the statements of the three propositions are easier to understand than the equation of motion, which expresses that mass times acceleration is equal to applied force. Students can learn these propositions through the interesting equation poems contrived by Prentis.[6] In the present approach to elementary mechanics, these three propositions, rather than the equation of motion, are the starting points. Students can solve typical exercises on mechanical phenomena from the temporal or spatial point of view on the basis of the three propositions. The keys to these exercises through the equation of motion cannot necessarily help students learn the causality of motion and the temporal and spatial points of view on mechanical phenomena.

*C. Remark*s on *th*e *equatio*n of *motio*n

The starting points of the present approach are the three propositions on the causality of motion. The equation of motion, however, is assumed in these propositions as reported above and, thus, is not disregarded in this approach. Remarks on the equation of motion are presented below.

1. Inertial mass is defined through proposition (1) by subjecting different masses to the same force and measuring their changes of velocity in the present approach in accordance with the conventional method. Here, significant remarks are necessary for the equal sign of the equation of motion. In the equation describing causality, it is convenient to set the effect (output) on the left and the cause (input) on the right in the manner form of function y = $f(x)$. For example, $0 = \Sigma \boldsymbol{F}_i$ implies zero acceleration in the equation of motion, while $\Sigma \boldsymbol{F}_i = 0$ implies that the amount of combined forces is zero. In contrast, force is measured by the amount of $m\boldsymbol{a}$. The action of force is observed only by the acceleration of a particle because force is invisible. The form of $\boldsymbol{F} = d\boldsymbol{p}/dt$ is similar to the definition of velocity, $\boldsymbol{v} = d\boldsymbol{r}/dt$, where $\boldsymbol{p}$ is the linear momentum. The law of motion is the only key to the transformation of the qualitative representation into the quantitative one. Thus, if the inertial mass is defined through proposition (1), $\boldsymbol{F} = m\boldsymbol{a}$ can be regarded as the quantitative definition of force. The equations $m\boldsymbol{a} = \boldsymbol{F}$ and $\boldsymbol{F} = m\mathbf{a}$ seem to be only the exchange of the respective sides because the same symbol $\boldsymbol{F}$ is used. The meaning of the symbol, however, depends on the equations. The meaning of the equals sign of $m\boldsymbol{a} = \boldsymbol{F}$ is different from that of $\boldsymbol{F} = -kx\boldsymbol{i}$ describing Hooke's law, where $\boldsymbol{i}$ is a unit vector in the x-direction and k is the spring constant. The symbol $\boldsymbol{F}$ denotes the representation of applied forces, such as gravity, restoring force, and resisting force in $m\boldsymbol{a} = \boldsymbol{F}$, while it is only a designation of the product of mass and acceleration in $\boldsymbol{F} = m\boldsymbol{a}$, as shown by the symbol $\boldsymbol{p}$ in the definition of linear momentum $\boldsymbol{p} = m\boldsymbol{v}$. Therefore, $m\boldsymbol{a} = \boldsymbol{F}$ and $\boldsymbol{F} = m\boldsymbol{a}$ are different equations. The problem is whether it is the law or the definition that the same equation represents.

2. The equation of motion describes the causality of motion, that is, the relationship between the change of velocity and the applied force. The motion of a particle in a uniform gravitational field also obeys the equation of motion. However, the expression of gravity mg is sometimes interpreted as mass times gravitational acceleration. This interpretation indicates that the right-hand side is only a rewritten form of the left-hand side. Thus, ma = mg is similar to "2x = 10 if x = 5". Some textbooks[5] fall into the circular reasoning that acceleration is g by solving the equation of motion after representing the applied force as mg with gravitational acceleration. The following concept is essential to the problem. The acceleration observed on earth is constant, and, thus, the force acting on a particle $\boldsymbol{F}$ is also constant. By solving the equation of motion $m\mathbf{g} = \boldsymbol{F}$, where gravitational acceleration is denoted by $\boldsymbol{g}$ and gravity $\boldsymbol{F}$ is unknown, $\boldsymbol{F}$ is represented by $m\boldsymbol{g}$. According to the meaning of the equation of motion, it is more appropriate to understand that the velocity of a particle is changed by a gravitational field $\boldsymbol{g}$, which denotes gravity acting

on a standard particle with unit mass. This point of view is also essential to learning about electromagnetic fields.

3. When the motion of a particle is observed in an accelerated reference framework, the equation of motion is more convenient than the proposition on the change of the linear momentum of a particle caused by impulse. The reason is that the problem is whether the force acting on a particle varies with the reference framework. Thus, the equation of motion $m(d\boldsymbol{v}/dt)= \boldsymbol{F}$ can be used rather than $d(m\boldsymbol{v})= \boldsymbol{F}\,dt$. In experiments in a noninertial frame, inertial force is included on the right-hand side of the equation of motion.

3. Discussion

The present approach treats equally propositions (1) and (2) shown in the previous section and is thus similar, as shown in the following, to Mach's interpretation[7]: Both natural and simple assumptions that the velocity of a particle is determined by the time it spends in falling and the distance traveled during falling are equivalent. The representations of both of these laws are given by empirical science. Therefore, $f\,t = mv$ and $f\,s = (1/2)mv^2$ are equivalently effective as a starting point, where $f\,t$ is an impulse and $f\,s$ is the work done on a particle with a distance of s. In the 17th Century,[4] the controversy had continued for more than half a century between the proposition that the same motion is proportional to the product of mass and velocity and the proposition that it is proportional to the product of mass and the square of velocity. To describe the intensity of the motion of a particle, Descartes considered the product of mass and velocity, while Leibniz considered the product of mass and the square of velocity. Today, it is believed that both propositions are the temporal and spatial points of view of the same motion. Let us consider the relationship between propositions (1) and (2). $d\,[(1/2)mv^2]= \boldsymbol{F}\cdot d\boldsymbol{r}$ can be derived by the inner product of $d(m\boldsymbol{v})= \boldsymbol{F}\,dt$ and $\boldsymbol{v}dt = d\boldsymbol{r}$. In the present approach, this derivation is regarded as the translation of the temporal point of view into the spatial point of view. Velocity $\boldsymbol{v}$ has a mathematical meaning of a mapping that time dt is input and displacement $d\boldsymbol{r}$ is output. Thus, $d\boldsymbol{r} = \boldsymbol{v}dt$ is the translation of time into displacement. The same mechanical phenomenon can be explained from each point of view. In the conventional approach, however, the proposition that the work done by the applied force is equal to the change of the kinetic energy of a particle is only a key to the solution by the reason that some exercises can easily be solved by applying this proposition rather than directly using the equation of motion.

In some textbooks[5], kinetic energy is formally introduced by eliminating time from the equations of displacement and velocity represented as the function of time. Thus, students cannot necessarily understand the physical meaning of the idea that kinetic energy describes the intensity of motion of a particle from a spatial point of view. The work done on a particle in a uniform gravitational field changes the kinetic energy of the particle. This in turn implies that a particle with kinetic energy can dissipate the intensity of motion by restoring the kinetic energy to the gravitational field. To understand this mechanism, it is advisable to emphasize that $\boldsymbol{g}$ denotes the gravitational field comparable to electromagnetic fields rather than only gravitational acceleration.

As reported in the previous section, solving exercises through the equation of motion does not necessarily help students learn the causality of motion and the temporal and spatial points of view on mechanical phenomena. Reif[8] also pointed out: "Instruction must ensure that students can adequately interpret any concept or principle before they use it to perform problem-solving tasks." In the present approach, the propositions on the causality of motion are constructed after introducing the concepts of force and the intensity of motion. This procedure is consistent with Reif's pedagogical suggestion. Reif emphasized that the ability to use knowledge depends on how well it is hierarchically organized. Mechanics deals with the motions of systems and the interactions between them and achieves its predictive power by laws of mechanics that specify the relationships between motion and interactions. Knowledge about the relationships between motion and interactions can be elaborated into the three basic laws of linear momentum, energy, and angular momentum. In addition, Reif indicated that this hierarchical structure of mechanics remains pertinent in more advanced courses, in which the laws of mechanics would also include Lagrangian and Hamiltonian formulations. The present approach is a specific formulation consistent with Reif's suggestions. The three laws playing principal roles are first constructed from daily experiences in the previous section.

There is also a new view for elementary electromagnetism. Imai[9] presented a new framework of electromagnetic theory based on the principles of the conservation of energy and momentum. There still remain some questionable points on fundamental physical quantities, although these days it is generally believed that the classical theory of electromagnetism is already well established. From a pedagogical point of view, similar circumstances are also true for elementary mechanics. The process of introducing linear momentum, kinetic energy, and angular momentum is essential. Momentum and energy are the physical quantities describing the intensity of motion from temporal and spatial points of view, respectively, although Imai has not mentioned this fact explicitly in the new framework of electromagnetism.

Finally, let us consider a related problem of thermodynamics. The first law of thermodynamics is essentially a statement of the principle of the conservation of energy for thermodynamical systems.[10] Some students may wonder why, in mechanics, the principle of the conservation of energy for dynamical systems can be derived from the equation of motion, while, in thermodynamics, the principle of the conservation of energy for thermodynamical systems must be assumed. The reason for this difference needs to be explained. In the conventional approach to elementary mechanics, there is no satisfactory explanation because the equation of motion is a starting point. In the framework of the present approach, however, proposition (2) shown in the previous section is a starting point and a hypothesis in the manner of the first law of thermodynamics. In practical terms, the equations of motion of the particles cannot be described in a thermodynamical system. It is only believed that the validity of these equations from the points of view of many experimental results can be explained by the principle of the conservation of energy. Based on this view, the equation of motion is not necessarily a practical starting point. Therefore, in a limited sense, the present approach without reliance on the equation of motion is comparable with thermodynamics.

4. Summary

In contrast to the traditional approach to elementary mechanics, the present approach can explain the physical meaning of the intensity of motion of a particle systematically and transparently. In the first place, the three physical quantities describing the intensity of motion are introduced naturally by observing the motion of a particle in a uniform gravitational field and a simple rotational motion. The propositions on these three quantities result in the equation of motion. It is assumed that the equation of motion is also valid for mechanical phenomena other than the above motions. In the advanced courses of mechanics, extension to the representation with the generalized coordinates is achieved. The construction of elementary mechanics and its expansion to analytical mechanics is summarized in a Chart.

It is advisable for beginners to understand the physical meaning of the intensity of motion of a particle from both the temporal and spatial points of view before learning keys to the typical exercises by formally applying the equation of motion[11]. In the advanced courses of mechanics, however, the equation of motion is essential, because students already understand the three propositions. The equation of motion is a differential equation that relates the change of velocity of a particle with the forces acting on the particle. By solving this differential equation formally, the positions and the velocities can be obtained at every moment. After understanding the physical meaning of the causality of motion on the basis of the three propositions, it is sufficient to calculate the positions and the velocities of particles numerically from the equation of motion in molecular dynamics and chaotic motion, for example. This approach is similar to that used in a graduate-level electromagnetic wave engineering course, in which electromagnetic phenomena are elucidated axiomatically on the basis of Maxwell's equations.

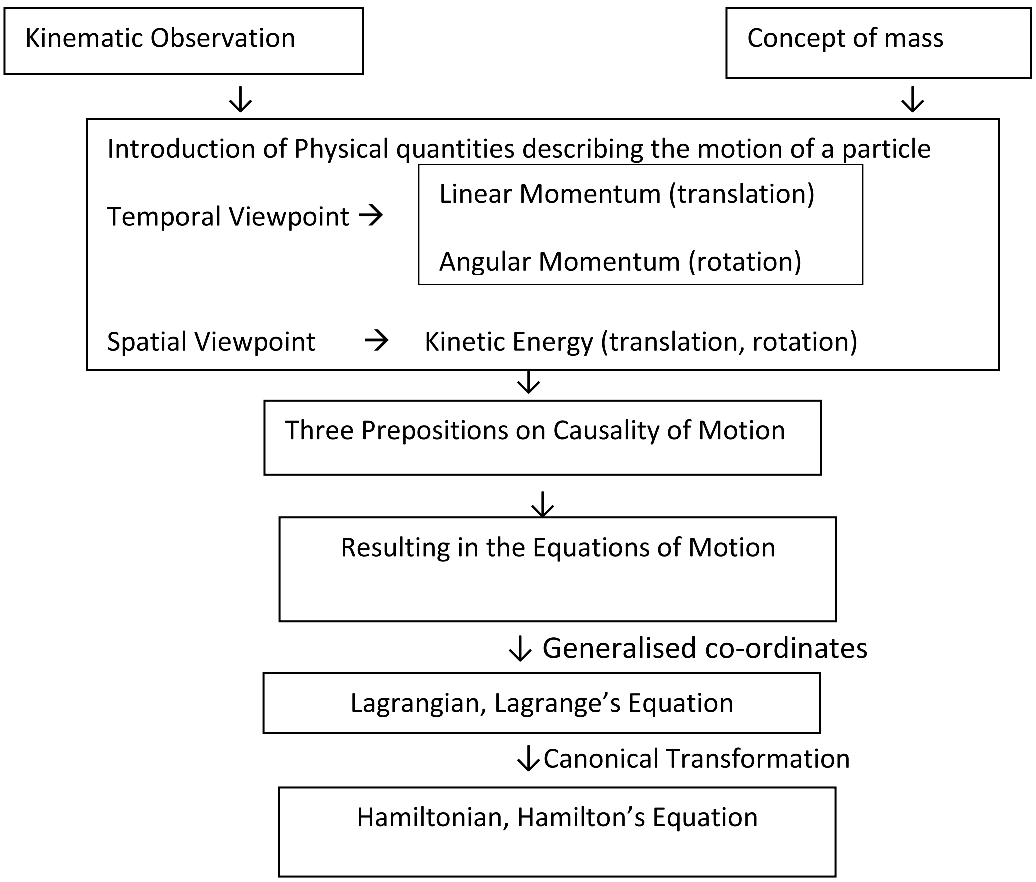

Chart: Construction of elementary mechanics and its expansion to analytical mechanics.

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