Revisiting the Concept of 2-D Bravais Lattices

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Abstract

In this paper, the concept of 2-Dimensional (2-D) Bravais Lattices is being arrived at using a constructivist approach, which is similar to the Socratic method of inquisitive questioning followed up with analysis and activity to obtain a comprehensive understanding of the idea. By posing the question as to how to construct a lattice by repeating a fundamental unit, we have created an activity that involves playing with triangles and quadrilaterals and figuring out which of them could easily tessellate to cover the 2-D chart without leaving any gaps. This lead to the construction of the five possible 2-D Bravais Lattices.

1 Introduction

2-D Bravais lattices were elementary and unrealisable theoretical constructs till the realisation of Graphene in 2004 [1], which was the first 2-D crystalline structure that is stable and has shown tremendous promise in a very short time. Various patterns on wallpapers [2] have exploited the possibility of periodic or symmetrical arrangements in 2 dimensions. Now many more 2-D crystalline structures such as NbS_2, MoS_2 etc. are being realised [3] [4].

On questioning students, it was found that recalling the five 2-D Bravais Lattices, after certain lapse of time post examinations is difficult and putting together all the 14 3-Dimensional (3-D) Bravais Lattices is definitely a daunting task at the undergraduate level.

There is a new found enthusiasm in the Physics community to innovate and explore ways of teaching fundamental concepts, wherein, one tries to systematically identify the alternative conceptions and gaps in learning and develop strategies to overcome them.This process has come to be known as Physics Education Research (PER) [5]. Further, new paradigms in education pedagogy such as Constructivism [6] and Constructionism [7] have paved way for learning concepts (in Physics) through a process of discovery by the students based on their previous knowledge, interaction among the peers and the guidance of the teachers. In this paper, an effort has been made to use PER strategies to develop modules which would help students to construct their own knowledge of 2-D Bravais Lattices, a preliminary concept in Solid State Physics.

Based on Bloom's taxonomy, Sapna et. al. [8] have devised learning/instructional objectives for Bravais Lattices in the context of Crystal Structure.The very first ideas of lattice and basis which form the foundation of crystallography require previous knowledge of symmetry and periodicity. 2-D and 3-D Bravais Lattices are abstractions that result from the classification of all the possible lattice structures.

Here, we are proposing a simple activity to realise the following learning objectives :

1. **Which is the smallest unit that can cover all space to form a lattice?**
2. **What are the distinct types of lattices in 2-D?**
3. **Which is the most convenient way to obtain them?**

2 Methodology

In this paper, we use the constructivist approach wherein we would want to arrive at the learning objectives through a strategy similar to Socratic method of questioning why, what and how followed by analysis, activity and comprehension in a repetitive manner till we gain complete clarity of the desired concept.

The implementation strategy for constructing the knowledge of 2-D Bravais Lattices includes the following steps:

1. firstly capturing the interest of the students by showing them some interesting pictures from the current science,

2. arousing their curiousity by posing a challenge and then raising questions to make them inquisitive,

3. putting them to activity to explore for themselves,

4. encourage them to share their findings with each other,

5. putting together the outcomes, analysing the findings along with their consensus and

6. finally arriving at the comprehensive understanding of the topic and applying it to solve the posed challenge.

2.1 Formulating the Question

Keen observation is the key to asking appropriate questions. We take advantage of current progress in science & technology and propose to look at Scanning Tunneling Microscope (STM) picture of Graphene as shown in Figure 1(a). It is interesting to note that Graphene has a hexagonal structure which is not one of the standard 2-D Bravais Lattices. Posing it as a challenge in the classroom and asking the students to identify the underlying 2-D Bravais Lattice and basis of Graphene, can prove to be a fruitful activity to probe student understanding. This paves way for starting the discussion with the question: ***Why is it that the hexagonal lattice not one of the five 2-D Bravais Lattices?***

A lattice is a periodic array of points which

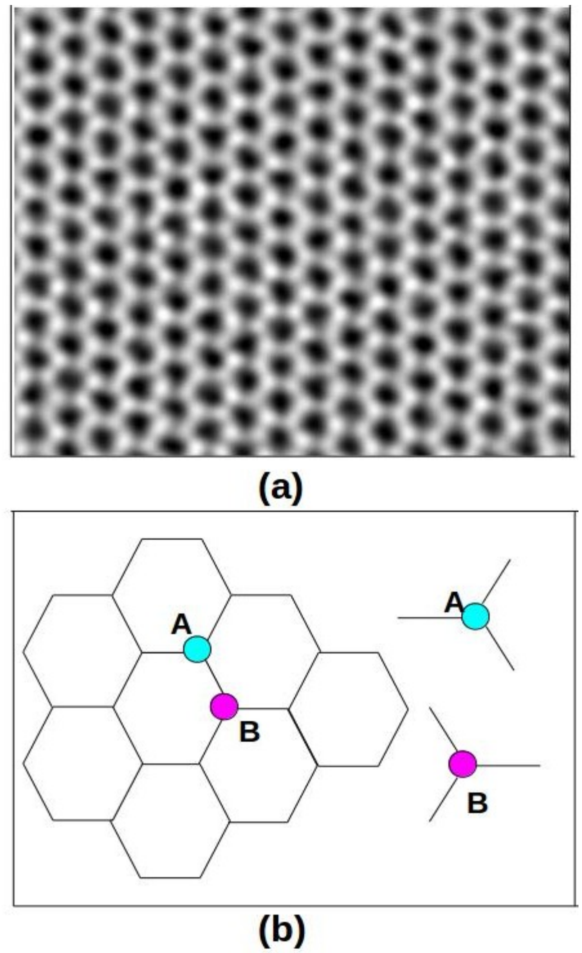

Figure 1: (a)STM image of Graphene. (*source:http://arxiv.org/pdf/1009.4714.pdf*) (b)In the structure of Graphene, shown are two points A and B. The honeycomb lattice observed is not a Bravais Lattice as the environment around points A and B differ by an angle of 60^o [9] [11]

ensures a similar environment from any given point all around [9].The vertices of such fundamental units which can build all possible periodic arrangements have come to be known as Bravais Lattices, named after the person who deduced them [10].

The Graphene structure shown in Figure 1(b), clearly depicts that the environment around points A and B is not same, thus ruling out a hexagonal structure to be a possible Bravais Lattice [11]. This brings us to the following questions:

1. **What are the possible periodic arrangements in 2-D which preserve the property of having exactly same environment at each location?**

2. **What are the various Bravais Lattices, how to obtain them and where do we begin?**

2.2 Building the Activity

The clue is to identify the smallest unit area that can be enclosed by the minimum number of lattice points, which when repeated covers all 2-D space without leaving any gap. Minimum three non-collinear points are required to cover an area in 2-D and joining them gives us a triangle, which can be chosen as the smallest unit for the purpose. From previous knowledge, we know that there are only seven different types of triangles possible as shown in column 1 of Figure 2. The activity proposed here involves trying to cover a chart paper using these 7 types of triangles by a combination of reflection, rotation and/or translation operations. The entire class can be divided into various groups, each of them working with either 1 or

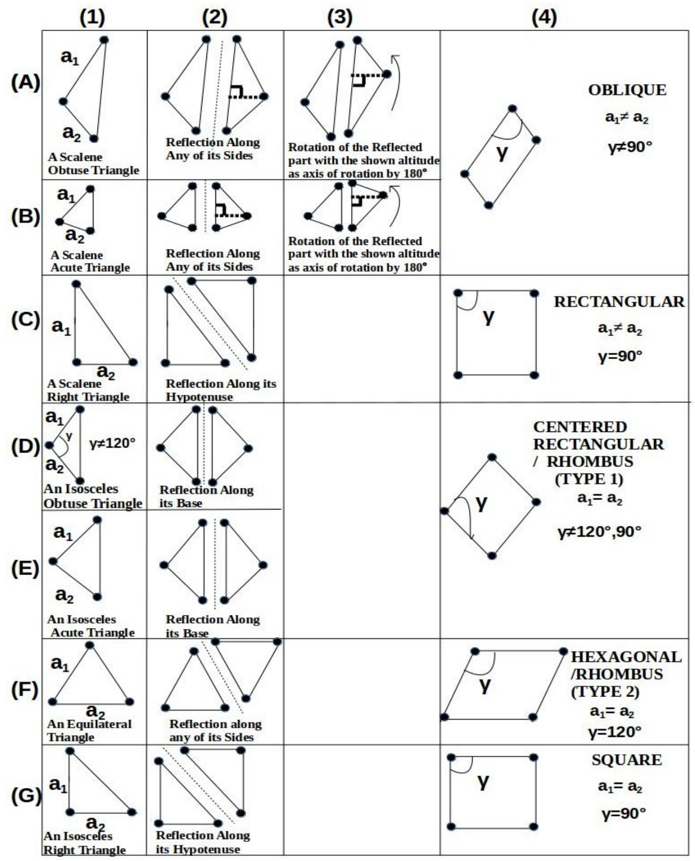

Figure 2: Depicting how five 2-D Bravais Lattices can be obtained using various types of triangles (classified on the basis of sides and angles). Though there may be many other possibilities of obtaining the lattices through triangles, but the (apparent) first intuitive method which incorporates basic symmetry operations (reflection/rotation around a convenient axis or both) has been employed. The dots (or the vertices of the triangles) depict the position of the lattice points in the lattice they constitute. While columns depict the type of triangles and the basic transformations employed, the rows constitute how various symmetry operations on different types of triangles lead to the formation of five 2-D Bravais lattices.

2 types of triangle cut-outs to fill the charts provided to them. The outcome of the activity can then be discussed in the classroom.

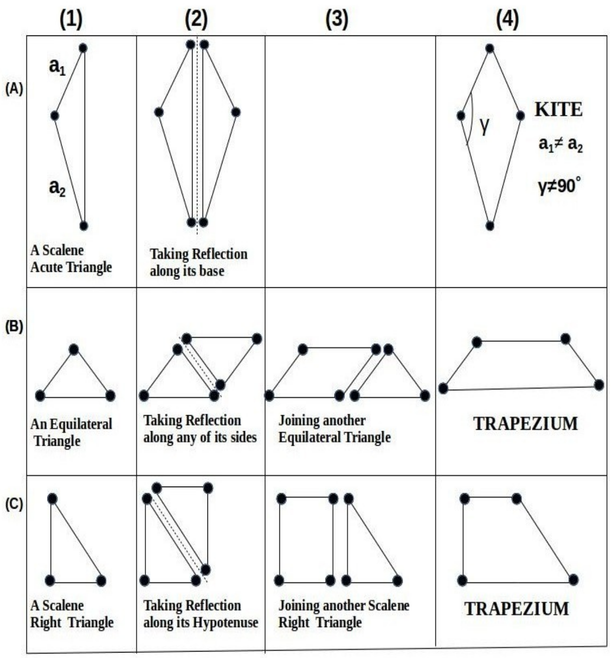

Figure 3: Depicting how triangles can form quadrilaterals which are not Bravais lattices (i.e. when they are tessellated over a given space, they will not be able to cover it wihout leaving gaps)

3 Discussion of Results

Triangles meet the requirement of the first learning objective as they are the smallest unit that can cover all space to form a lattice. Covering all space using triangles required all three operations of reflection, rotation and translation. This is a tedious process. On looking carefully at the output charts, we find that quadrilaterals are the ones which can be tessellated(i.e. translating a unit of area in order to cover a given space without leaving any gaps) along two directions to cover all space. Column 4 of Figure 2 shows those quadrilaterals that can be tessellated to cover all space. Those that do not satisfy the requirement of covering all space without gaps are given in column 4 of Figure 3.

4 Comprehension

The activity shows that there are only five possible quadrilaterals

1. **Oblique** ($a1 \neq a2, \gamma \neq 90^o$),
2. **Rectangular** ($a1 \neq a2, \gamma = 90^o$),
3. **Rhombus,Type-1 (obtained using isosceles obtuse/acute triangles giving** $a1 = a2, \gamma \neq 120^o/90^o$ **)**
4. **Rhombus,Type-2 (obtained using equilateral triangle giving** $a1 = a2, \gamma = 120^o/60^o$ **)**
5. **Square** ($a1 = a2, \gamma = 90^o$),

These are the appropriate fundamental units which when tessellated would cover all the space and could be thought of as primitive unit cells in 2-D. The two rhombuses can be alternatively visualised as centered regular hexagonal lattice and centered rectangular lattice as shown in Figure 4. In standard textbooks [9] [12] though, the connotation used for Rhombus, Type-2 is hexagonal

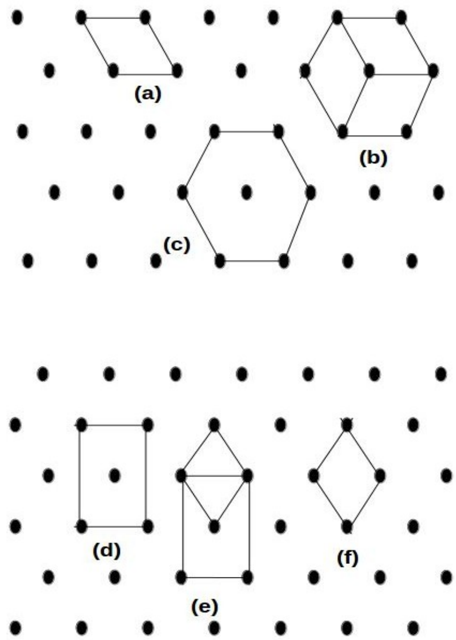

Figure 4: Figures showing equivalence conditions for two of the Bravais Lattices.
(a)shows a Rhombus, Type-2 (adjacent angles 60^o and 120^o) amidst the lattice
(b)shows how the Rhombus, Type-2 is reflected and rotated to obtain the centered regular hexagonal structure
(c)shows the Centered Regular Hexagonal amidst the lattice.
(d)shows a centered rectangle amidst the lattice.
(e)shows how the Rhombus, Type-1 is also inherent in the centered rectangle.
(f)shows the Bravais Lattice, Rhombus, Type-1.

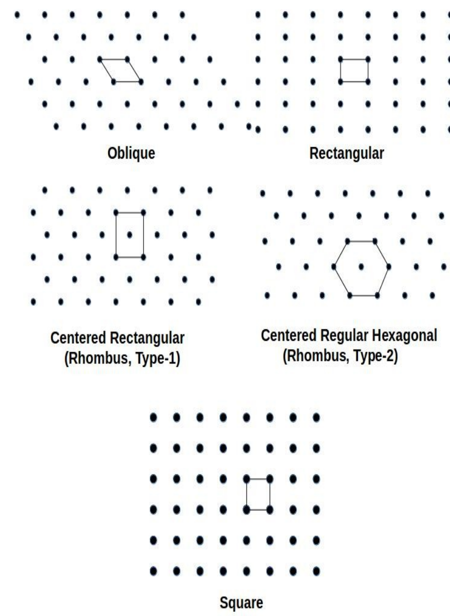

Figure 5: The five 2-D Bravais Lattices.

which is not correct and can confuse students. These are non-primitive unit cells but are more conventionally employed and are typically part of the five 2-D Bravais Lattices as they are known and are shown in Figure 5 for sake of completion. Now, coming back to Graphene, we notice that it is a **Rhombus, Type-1** ($a1 = a2, \gamma \neq 120^o/90^o$) lattice with two C-atoms forming a basis as shown in Figure 6.

5 Conclusion

We have developed an activity based strategy involving symmetry oopreations on triangles which are the smallest unit that could cover all space. Based on the obseravtions and analysis of the outcomes, quadrilaterals were found to be more convenient as they involve only tessellation.

The combination of all seven possible triangles that resulted in various quadrilaterals

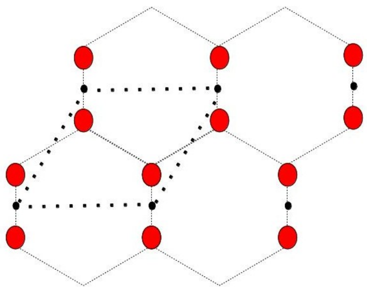

Figure 6: The dotted parallelogram, Rhombus Type-1 shows the underlying Bravais Lattice of Graphene structure with C-C basis(filled circles) at each of its vertices, the lattice points.

were tabulated into two groups based on the criteria whether the given quadrilateral tessellate to cover all space without leaving any gaps. The group of quadrilaterals that satisfy the criteria form the five primitive cells that generate all possible lattices in 2-D. We have shown that primitive cells consisting of Rhombus, Type-1 and Rhombus, Type-2 are equivalent to the non-primitive more conventionally used unit cells of centered rectangular and centered regular hexagonal lattices respectively and thus arrived at the five 2-D Bravais lattices as they are known today.

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