Rotational motion of a rigid body with the system IP–Coach

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Abstract

A correct understanding of basic physical quantities related to the rotational motion of a rigid body by undergraduates is important not only in mechanics but also for understanding of basic physical principles in other branches of physics. The rotational motion of a symmetric rigid body about an axis through its centre of mass and perpendicular to the Earth's surface can be examined in the experiment which is presented in this paper. An optical gate connected to the system IP-Coach makes it possible to record the angular displacement as a function of time from which the angular velocity and angular acceleration can be calculated at any instant of time. Using these kinematic dependences, the influence of the moment of inertia as well as the influence of resistance forces on the rotational motion of a rigid body can be evaluated. The proposed experiment can be included as a demonstration or laboratory experiment in the basic physics course in technical bachelor's degree programmes.

1. Introduction

Rotational motion of a rigid body about a fixed axis is difficult for students to grasp mainly from the point of view of understanding the quantities which are used to describe this motion. This can be improved by the demonstration of this motion using apparatus which makes it possible to record simultaneously the values of angular displacement and time. The apparatus consists of the rotation apparatus and the system IP-Coach (optical gate, CoachLab panel and computer with IP-Coach program). The advantage of using the system IP-Coach is immediate displayof the measured kinematic quantities – angular displacement (φ) and derivedquantities – angular velocity (ω) and angular acceleration (α) as a functions of time (t). These quantities are then used for elucidation of basic dynamic characteristics of rotational motion of a rigid body –torque (moment of force) (M) and moment of inertia (I). Moreover, the system IP-Coach when used in laboratory practicals makes it more effective and faster to processthe measured data.

The above-mentioned apparatus can be also used in the laboratory practicalsin the specialized technical study programmes since it enables to study the influence of frictional forces as well asto observe subtle effectof viscous resistance forces of surroundings (air) on the motion.

2.Experimental

Rotation apparatus (3B Scientific) used in experimentis shown in Fig. 1. Itconsists of plastic weight fastener (1), 100 g weight discs (2), 10 g slotted weights (3), string (4), 20 g slotted weight (5), hanger for slotted weights (6), 200 g weight discs (7), spindle (8), base and mount (9) and hollow aluminium rod (10).

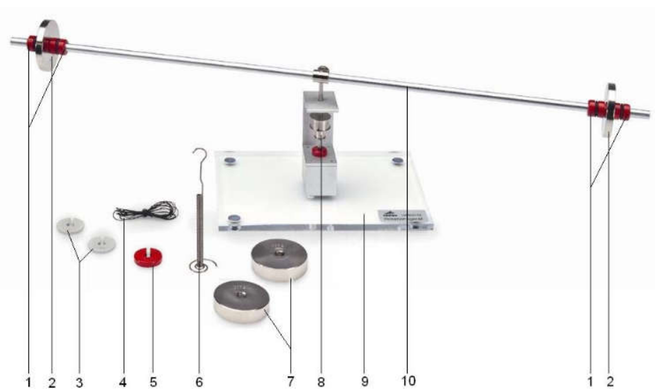

FIG. 1: Rotation apparatus. [https://www.3bscientific.com/rotation-apparatus-u8405715,p_853_18606.html]

The apparatus is set into motion by the weight of slotted weights (3,5) placed on a hanger (Fig. 2) fixed at the end of the string(4) wound around a spindle (8) with diameter of 1.8 cm. The stringruns over the deflection pulley of negligible mass (Fig. 2). The weight discs (2,7) can be placed on the crossbarin various positions using weight fasteners (1) changing the moment of inertia of the crossbar and in this way also the characteristics of the rotational motion [1].

FIG. 2: Rotation apparatus with a weight placed on a weight hanger.

The CoachLab panel (Fig. 3) with the optical gate connected (CMA Photogate code0662I) makes it possible to record the dependence of the number of half-revolutions on time during the rotational motion and to display the datasimultaneously on the screen as a graphandas a table[2].

FIG. 3: Rotation apparatus with the system IP – Coach: 1 – CoachLab panel (interface), 2 - optical gate.

3. Results and discussion

3.1 Uniformly accelerated rotational motion – kinematics

Generally, if a rotational motion takes place in plane, the following equations hold

$$\omega = \frac{d\varphi}{dt} \tag{1}$$

$$\alpha = \frac{d\omega}{dt} \tag{2}$$

Uniformly accelerated rotational motion is performed by the body rotating about fixed axis with constant angular acceleration α. Angular velocity ω and angular displacement φ as functions of time can be expressed by the following equations

$$\omega = \omega_0 + \alpha t \tag{3}$$

$$\varphi = \varphi_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \tag{4}$$

whereω_0 is initial angular velocity and φ_0 is initial angular displacement.

The measured experimental dependence of angular displacement on time is depicted in Fig. 4. In our experiment, the weight discs with mass of 200 g were placed symmetrically at the distance of 15 cm from the axis of rotation (labelling I2in graphs) and apparatus was set into motion by the slotted weights with mass *m* = 30 g.

TheIP-Coach programme makes it possible to record and to analyse the measured dependence of angular displacement on time. Using the option “Analyse” the function $y = ax^2 + bx + c$ can be fitted to the data. As can be seen, this function describes the measured dependence satisfactorily. Based on the comparison with the theoretical function (4) it can be assumed that the apparatus performed uniformly accelerated motion and the values of parameters a, b, ccorrespond tothe values of angular acceleration $\alpha = 2a$, initial angular velocity $\omega_0 = b$ and initial angular displacement $\varphi_0 = c$. The values of $\alpha, \omega_0, \varphi_0$obtained from the fitting procedure are listed in the inset in Fig. 4.

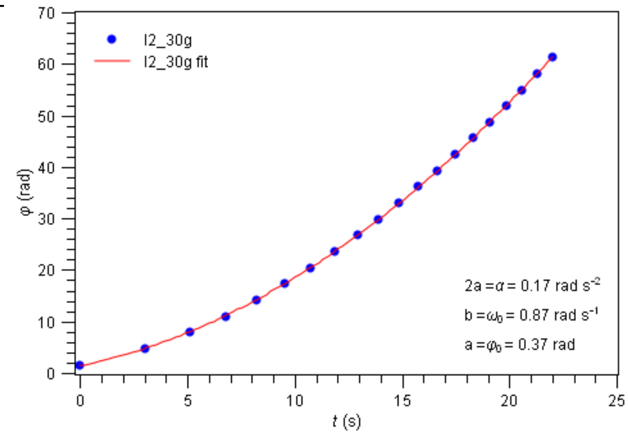

FIG. 4: Angular displacement φas a function of time $t$for uniformly accelerated rotational motion: blue dots – experimental data, red line –fittedfunction (Eq.(4))

Derivative of the experimental dependence of angular displacement with respect to time calculated using the option “Processing” in the IP-Coach programmeprovides the dependence of angular velocity on time (Fig. 5). It can be seen that angular velocity increases nearly linearly with time which is in agreement with the assumption of uniformly accelerated rotational motion for which equation (3) holds.

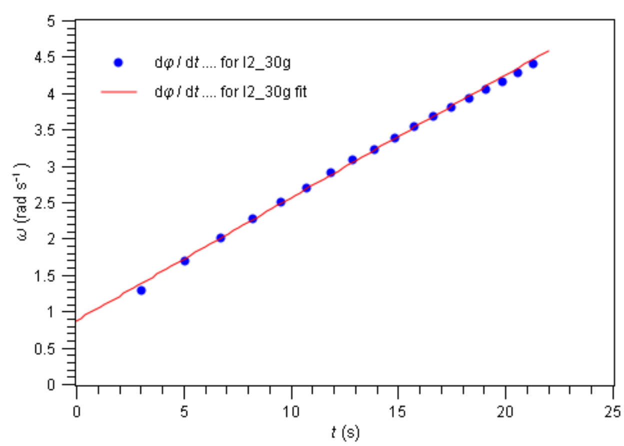

FIG. 5: Angular velocity ωas a function of time for uniformly accelerated rotational motion obtained by numerical calculation of the first derivative of angular displacement with respect to time: blue dots – values from experimental data, red line –Eq. (3)

The first derivative of angular velocity (Fig. 5), i.e. the second derivative of angular displacement,with respect to timeprovides constant angular acceleration as was already assumed (red line in Fig. 6, i.e., $\alpha = 2a = 0.17\ \mathrm{rad.s^{-2}}$), the obtained dependence is displayed in Fig. 6. However, the first derivative of the experimental data of angular velocity with respect to time results in a more complex dependence of angular accelerationon time with the values whose dispersion is approximately 20 % with respect to the above-mentioned value of angular acceleration α. This dispersion is brought about by repeated calculations of the derivatives of experimental dependences since they increase uncertainties of calculated values. A slight deviation of the values of the first derivative of experimental dependence of angular displacement with respect to time from a straight line (Fig. 5) results in a small negative slope of a trend line of the second derivative of angular displacement with respect to time (green line in Fig. 6). The more detailed discussion concerning this issue will be presented in section 3.3.

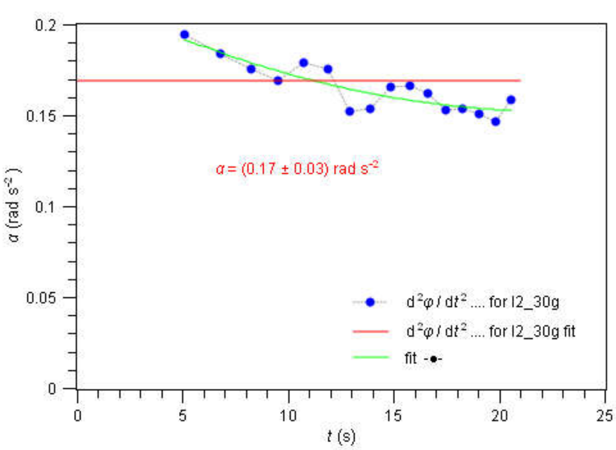

FIG. 6: Angular acceleration as a function of time for uniformly accelerated rotational motion obtained by the calculation of derivatives: blue dots –the second derivative of angular displacement with respect to time, green line –function polynomial fit (cubic)to the calculated values (to the set of blue dotsin this graph), red line – $\alpha = 2a$.

3.2Uniformly accelerated rotational motion – dynamics

Rotational motion of a rigid body about a fixed axis from the point of view of dynamics (acting forces) can be described by the following equation of motion

$$M = I\alpha \qquad (5)$$

where$M$ is torque (sum of acting moments of forces), I is moment of inertia of the body and α is angular acceleration. This equation says that the change in angular acceleration of rotational motion can be achieved by the change in torque$M$ at constant moment of inertia I or by the change in moment of inertia Iat a constant torque M[3].

It has to be noted that the equation of motion of the rotation apparatus is more complex than equation (5) because we deal with a system of rigid bodiesin motion:rotating crossbar with weight discs (a rigid body with the moment of inertia I)and a ‘falling’ slotted weight of mass m. The equations of motion for this system of bodies provides the following equation

$$M = (I + mr^2)\alpha \qquad (6)$$

where

$$M = mgr + M_{other} \qquad (7)$$

where$r$is the radius of the spindle and torque M_{other} in our experiment comes from resistance forces acting on the rotating apparatus (M_{other}can be considered approximately constant for our equipment, see discussion in Sec. 3.3).

Since for the used rotation apparatus it holds $I \gg mr^2$, equation (5) can be considered to be correct.

Torque $M(7)$ in our experiment can be changed by placing the slotted weightswith different masses $m$on a hanger fixed at the end of the string. The mass increaseΔm leads to anΔmgr increase of M.Theequation (5) implies that the increase in torque results in the increase in angular acceleration.The dependence of angular displacement on time will be steeper (Fig. 7).

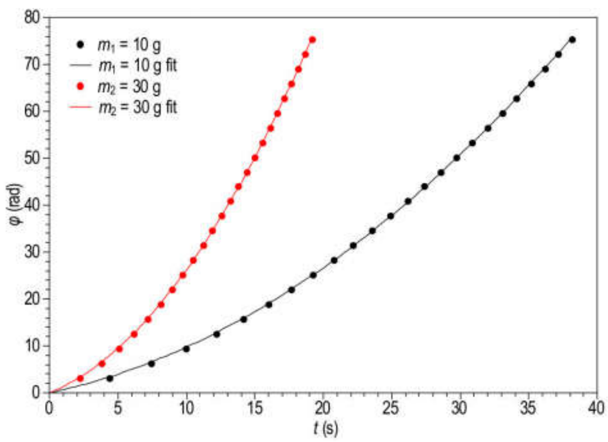

FIG. 7: Angular displacement as a function of time for two different values of torque at constant moment of inertia

Moment of inertia I can be markedly changed by changing the distance of weight discs placed on the crossbar from the axis of rotation. According to the equation (5) the increase in moment of inertia results in the decrease in angular acceleration and increase in angular displacement in time will be smaller. The smaller moment of inertia results inthe steeper dependence of angular displacement on time. The plots of angular displacements versus time for two different moments of inertia $I_1 > I_3$ are displayed in Fig. 8, torque was produced by the weight of the slotted weight with mass of 30 g. Moment of inertia was changed by the changing the positions of weight discs on the crossbar, which were 21 cm and 11 cm from the axis of rotation for $I_1$and I_3, respectively.

As can be seen, the increase of angular displacement in time is slower for I_1 than forI_3.

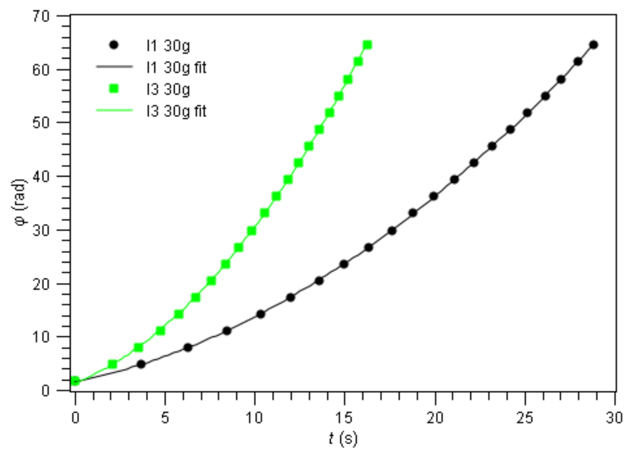

FIG. 8: Angular displacement as a function of time for two different values of moment of inertia ($I_1 > I_3$) and constant torque.

3.3.The influence of resistance forces on rotational motion

In sections 3.1 and 3.2 the influence of resistance forces on experimentally obtained data has not been discussed yet. The so far presented experimental procedure and data processing can be used in the basic physics course in which the knowledge of basic quantities and equations related to rotational motion should be deepened.

If this experiment is used in physics courses for specialized technical programmes, it is necessary to include the influence of resistance forces (frictional and viscous resistance forces) in the description of rotational motion in order to evaluate dynamic quantities in a correct way.

If frictional forces are taken into account then equation of motion of rotational motion of a rigid body (5) will have the following form

$$I\alpha = M - M_f \tag{8}$$

wherethe torque of frictional forcesM_f is explicitly expressed now.

We will assume that torque of frictional forces is constant for our apparatus since the weight of slotted weights with $m$acts on the axle in perpendicular direction with respect to the axle axis and it does not increase the force which acts on the agate bearings and it does not increase the torque of frictional forces M_f. Its value can be determined in an experiment without driving weight:$m = 0, M = 0$. The crossbar will be set into motion manually by a stroke. The motion will be retarded due to the frictional forces between vertical axleand agate bearings. Torque of frictional forces M_f can be then determined from experimental dependence of the number of half-revolutions on time (Fig. 8).

Fitting function $y = ax^2 + bx + c$ to the experimental data and taking into account equation (4), the value of angular deceleration ($\alpha = \pi \cdot 2a$) can be obtained which is $\alpha = -0{,}0106\,\mathrm{rad.\,s^{-2}}$ for the data plotted in Fig. 9. Moment of inertia can be calculated using the following expression

$$I = I_0 + 2m_d R^2 \tag{9}$$

where$I_0$is moment of inertia of the crossbar ($I_0 = \frac{1}{12}ml^2$) and $2m_d R^2$is moment of inertia of weight discs placed at distance R from the axis of rotation.

If the mass of weight discs is $m_d = 200\,\mathrm{g}$ and their distance from axis of rotation is $R = 20\,\mathrm{cm}$, the mass of the crossbar is $m = 0.0433\,\mathrm{kg}$, its length is $l = 0.6\,\mathrm{m}$,then moment of inertia of rotating apparatus can be calculated using (9) and it is $I = 0{,}0173\,\mathrm{kgm^2}$.

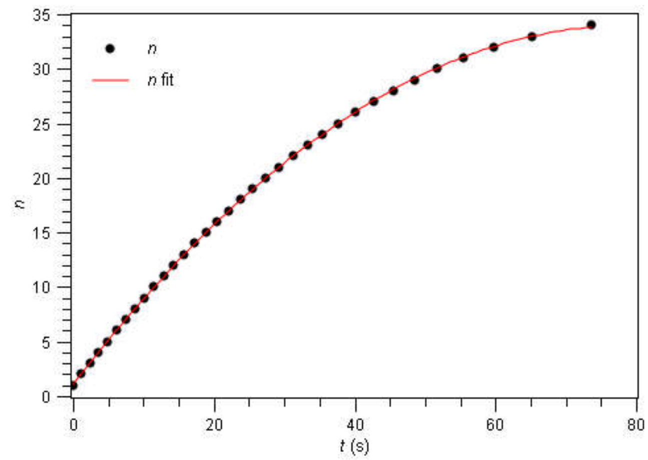

FIG. 9: Number of half-revolutions as a function of time for uniformly decelerated rotational motion: dots – experimental data, red line –correspond to function (Eq. (4))divided by πfitted to the measured data.

Using experimental values for angular deceleration and moment of inertia, the moment of frictional forces can be calculated using equation (6) for $M = 0$ ($m = 0$)

$$M_f = -I\alpha \tag{7}$$

The experimental data provided the value $M_f = 1{,}834.10^{-4}\mathrm{Nm}$.

It has to be noted that the rotational motion of rigid body is also slightly influenced by viscous resistance forces of the surrounding air which depends on the velocity of the body. This is the cause of slight deflection of the values of angular velocity from a straight line in Fig. 5. The dependence of angular acceleration on time (Fig. 6) is not constant but decreases in time (with increasing angular velocity). Decrease is successively smaller since the torque of viscous resistance forces is function of velocity which increases in time and more and more compensates the torque of driving weight.The total torque which accelerates the apparatus is gradually smaller and smaller. We do not discuss frictional and viscous resistance forces acting on the other parts of the system (pulley, weights of mass *m*)since theycan be considered negligibly small in comparison with resistance forces discussed above.

7. Conclusion

Rotational motion of a rigid body about a fixed axis can be easily demonstrated using the rotation apparatus and the system IP–Coach. The advantage of the proposed experimental setup is the possibility of immediate displayof basic kinematic quantities – directly measured angular displacement, and derived quantities -angular velocity and angular acceleration as functions of time. These quantities can be related to basic dynamic characteristics of the rotational motion of a rigid body through the equation of motion for rotational motion. The influence of torque and moment of inertia on kinematic quantities of rotational motion can also be demonstrated. The influence of frictional and viscous resistance forces can also be observed. The proposed experiment can be used not only in laboratory practicalsbut also as a video demonstration or live experiment during the lecture since students can observe directly uniformly accelerated and decelerated rotational motions and kinematic quantities related to this motion as functions of time.

Acknowledgements

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