Graphical Vector Method for Solving Relative Velocity & Dynamics Problems with Causality Visualization

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Abstract

A novel graphical vector method in the solving of the standard 2-object with pulley on an incline problem was devised and has been taught in the classroom setting. The case of pulley with inertia would not hinder the presented vector methodology in solving vector equations of motion in dynamics. The delivery plan would first prepare the class to use graphical vector method in solving relative velocity problems without the 90-degree velocity diagram convenience; and the visualization of causality through vector drawing has been emphasized. Good Physics Education delivery would be vital for doing Physics Education Research on the collected student performance data to assess student learning. Physics Education materials in spatial thinking training in relationship to the neuro-science learning research findings is discussed.

1. Introduction

Vector is a capstone concept in Mechanics and is usually taught in a College Physics I course. On the other hand, the 2-object incline plane problem is usually taught using the scalar equations in the x and y coordinates associated with Newton's 2^{nd} Law of motion F = ma, which was taught with force and acceleration as vectors. We have developed a set of graphical vector methods for solving these problems and they are presented in this report. The discussed topics include walking map, projectile velocity with gravity puling, relative velocity, forces in equilibrium, 2-object incline problem, rotational problem, and momentum. The goal is to merge vector into a student's mindset and reinforce the causality relationship. Like most college physics textbooks, the kinematics is a precursor to dynamics with newton's Law of Motion; we have used walking map and velocity vector diagram as preparatory materials for the full use of vector method in dynamics.

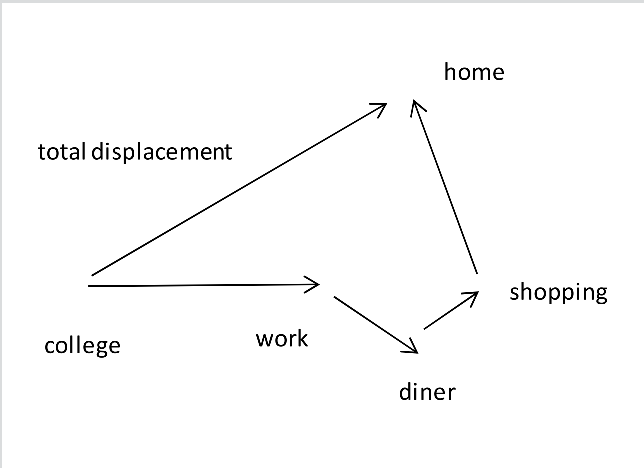

FIG. 1: A schematic view of the walking vectors.

The walking map in Figure 1 is usually a good introduction to the displacement vector concept without much demand on the math pre- requisite. A student would recognize the general polygon with its sides as vectors. The starting point connection to the final destination with an arrow-drawing would be the total displacement vector. When the teaching continues to projectile motion, the Figure 2 illustration could build on Figure 1 visualization with the realization that acceleration*time (9.8 m/s/s* time is a vector for gravity puling, which is the cause for the initial velocity v0 vector to become the final velocity vf vector.

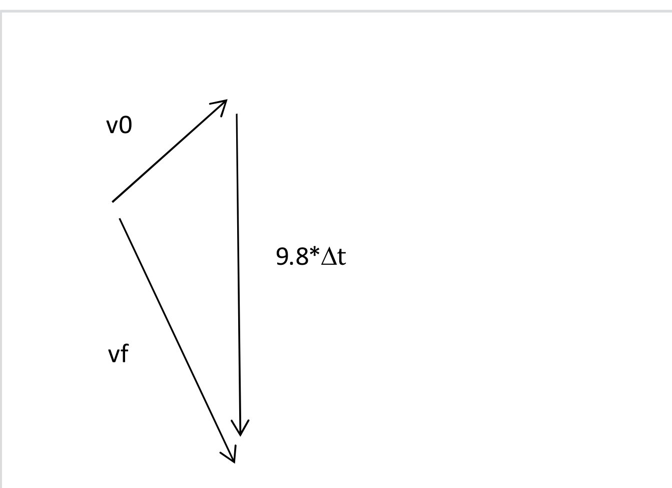

FIG. 2: A schematic view of the projectile velocity vectors.

The use of vector addition in relative velocity problems without the 90-degree velocity diagram convenience would follow, as illustrated in Figure 3. The water current would push the boat off course such that the arrival would be the targeted destination when the boat direction is set properly.

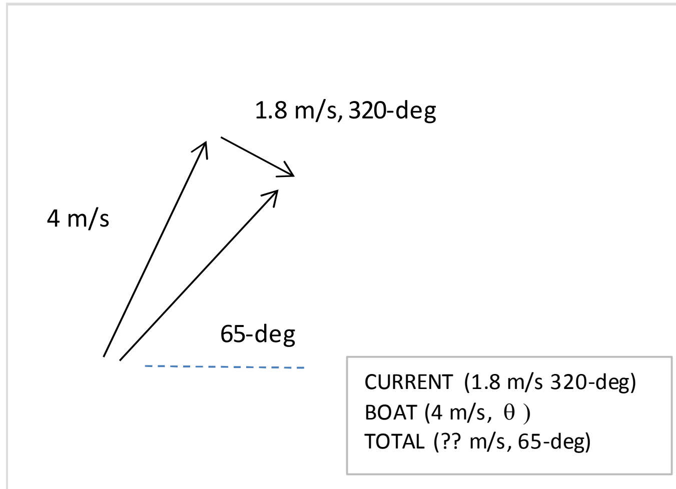

FIG. 3: A schematic view of the velocity vectors.

The graphical vector method would offer not only an easy visualization, but a guaranteed answer within drawing accuracy. The algebra method is not easy for every student as the steps involve trigonometry. In fact our experience showed that only about 30% students in a calculus physics class could solve the associated scalar equations in x and y coordinates within 30 minutes. As for our college physics class designed for the health sciences and pre- med majors, only about 10% could solve the scalar equations involving trigonometry terms. As for our technology physics class designed for students aiming for technician jobs, the percentage would drop to about 2%. But every student would understand the graphical vector solution presented in Figure 4. Here the concept of causality is reinforced because the sketch in Figure 3 dictates that the Boat vector must be drawn from the start-point of Current vector.

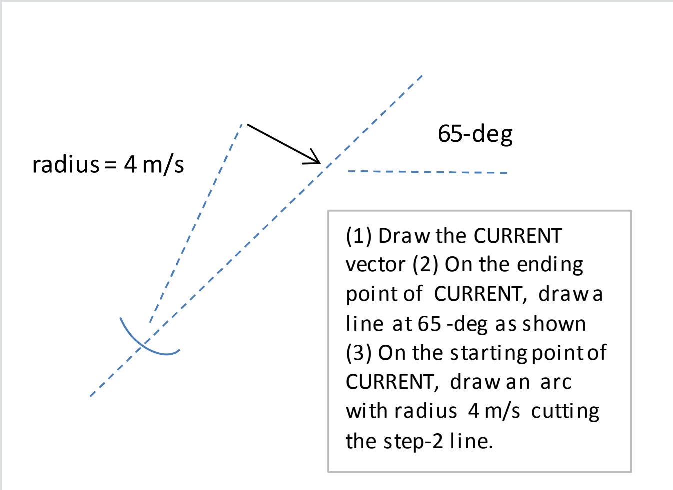

FIG. 4: An illustration of the construction of the velocity vectors in Figure 3..

2. Force Related Problems

The equilibrium condition is a typical problem in Mechanics. Figure 5 displays a situation where a horizontal force is keeping an object at rest on an rough incline plane. The friction could be any value below the maximum allowed by μ, the coefficient of static friction.. There is more than one answer and the problem can be considered as ill-posed. Nevertheless physics is about concept learning, and the fact that there could be several answers would not mean poor teaching technique.

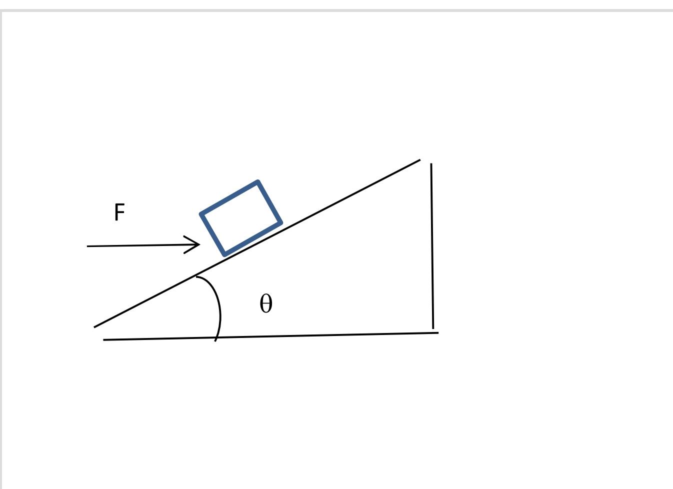

FIG. 5: A schematic view of the applied force.

The forces are shown in Figure 6 and the solution is shown in Figure 7 for the case of maximum horizontal force.

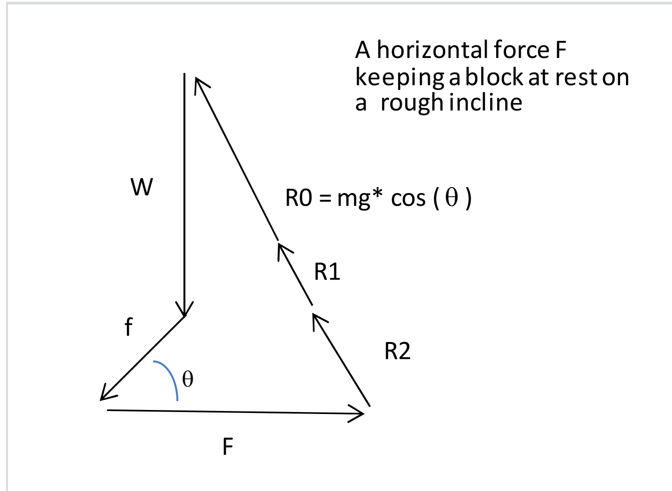

FIG. 6: A schematic view of the force vectors in Figure 5

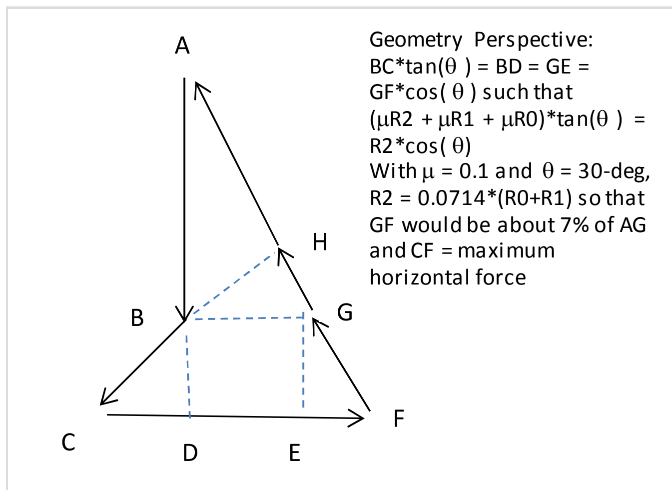

FIG. 7: An illustration of the construction of the force vectors in Figure 6.

The corresponding minimum force situation is shown in Figure 8.

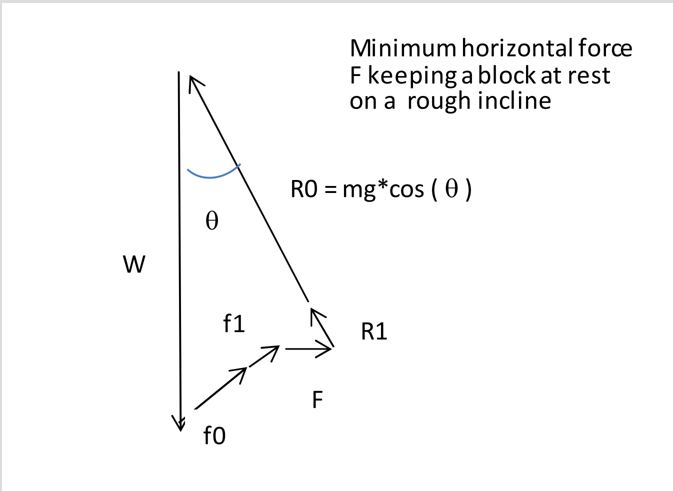

FIG. 8: A schematic view of the force vectors for the minimum horizontal force case where f0 = μ * R0 and f1 = μ *R1.

The graphical vector method is shown in Figure 9.

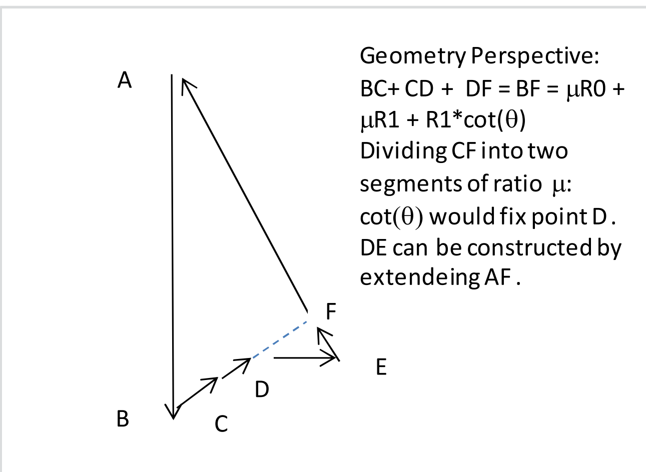

FIG. 9: An illustration of the construction of the force vectors in Figure 8

A standard 2-object on incline problem is shown in Figure 10.

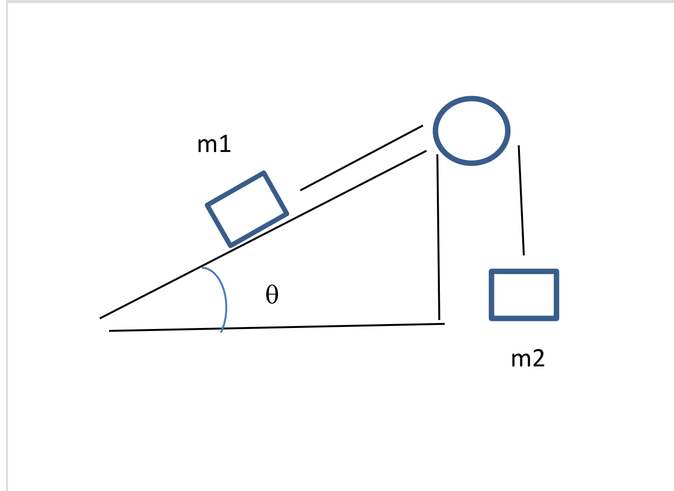

FIG. 10: A schematic view of the 2-object on an incline problem.

The free body force diagrams for the m2 object driving the acceleration are displayed in Figure 11.

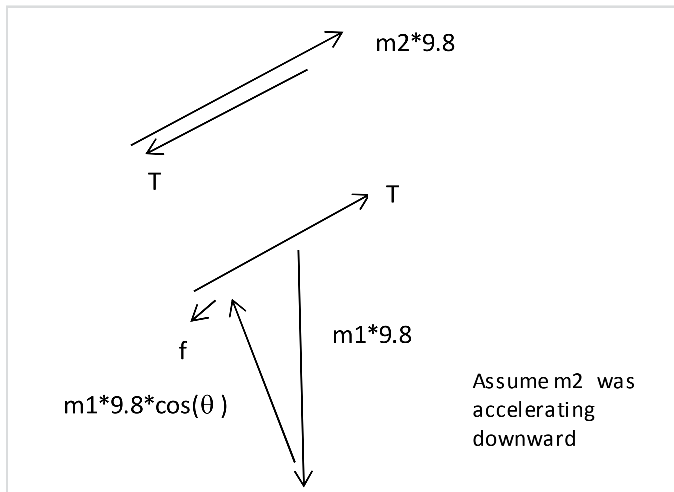

FIG. 11: A schematic view of the force vectors in Figure 10 when the vertically hanging object is dropping..

The graphical solution is displayed in Figure 12. The Segment AC would need to be divided into two portions in a ratio of m1 to m2.

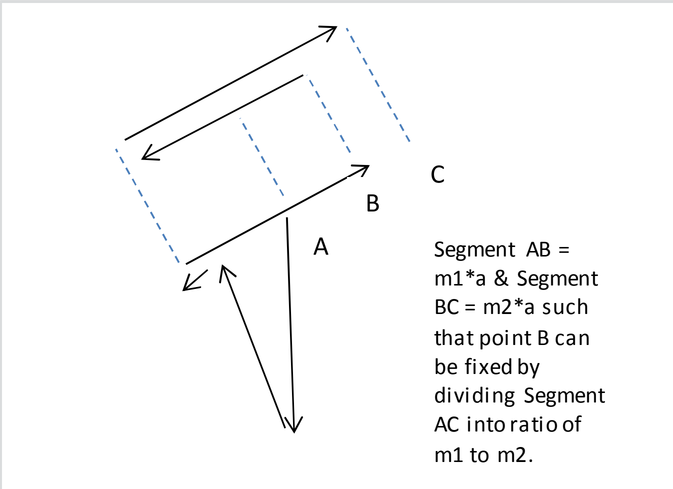

FIG. 12: An illustration of the construction of the force vectors in Figure 11 when the vertically hanging object is dropping.

The free body force diagrams for the m1 object driving the acceleration are displayed in Figure 13.

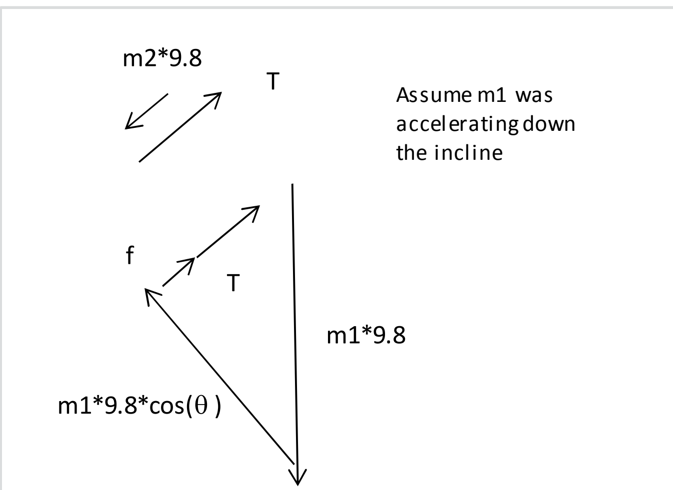

FIG. 13: A schematic view of force vector in Figure 10 when the inclined object is sliding downward.

The graphical solution is displayed in Figure 12. The Segment BD would need to be divided into two portions in a ratio of m2 to m1.

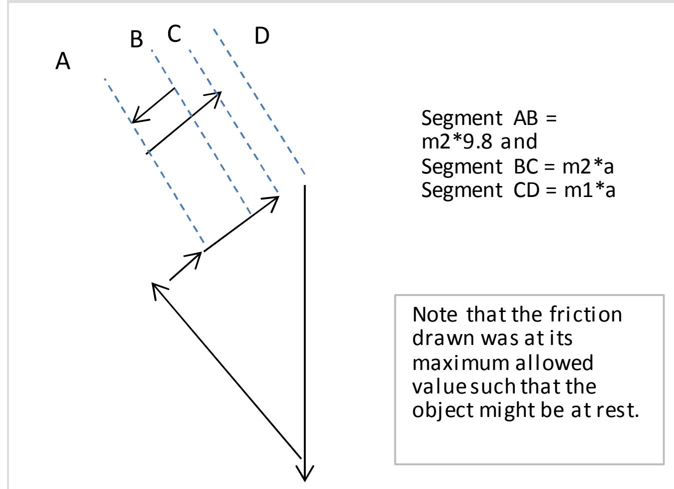

FIG. 14: A An illustration of the construction of the force vectors in Figure 12 when the inclined object is sliding downward..

When the pulley has inertia, the free body force diagrams for m1 and m2 is shown in Figure 15, assuming that m2 falling s driving the acceleration. Note that the T1 pulling up the m1 object would be different from the T2 preventing m2 going a free fall.

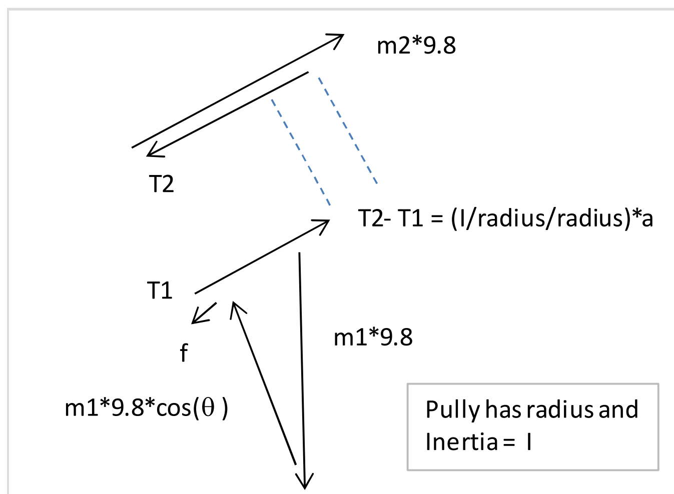

FIG. 15: A schematic view of the force vectors when the pulley has an inertia value.

The graphical solution is displayed in Figure 16. The Segment AD would need to be divided into three portions in the ratio proportion of m1: I/radius/radius: m2.

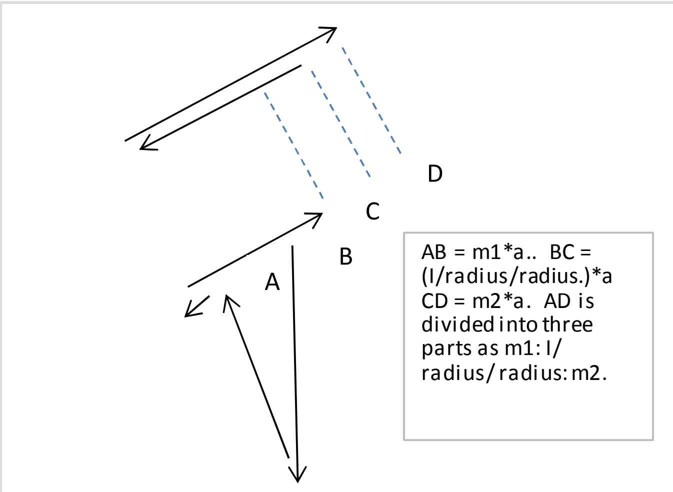

FIG. 16: An illustration of the construction of the force vector when the pulley has an inertia value in Figure 15..

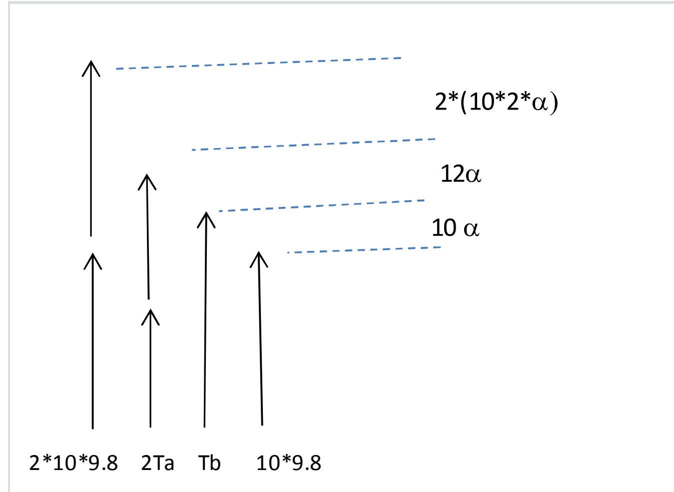

FIG. 18: An illustration of the construction of the force vectors in Figure 17.

It is interesting to point out that the graphical vector method can also be employed when the torque equation has two different moment-arm distances. A double-pulley with two handing masses is shown in Figure 17. The torque equation has two forces with two different moment-arm distances.

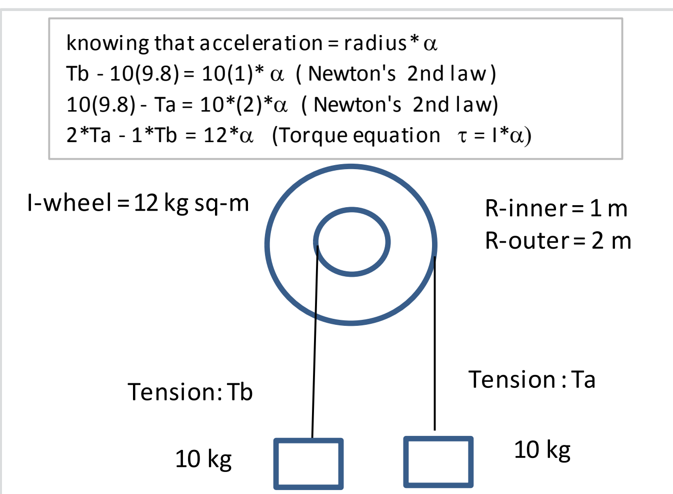

FIG. 17: A schematic view of applied weights on a double-pulley system.

The graphical solution is displayed in Figure 18. Note that the m2 equation of motion is multiplied by a factor of 2 to facilitate the vector drawing solution.

The standard two-car intersection collision problem would also carry a graphical vector solution. The total momentum P-total would equal to the wreckage momentum P-wreckage, with no external force or impulse. The students would recognize that each momentum vector has different mass value. This momentum vector diagram echoes the projectile velocity vector diagram shown in Figure 2 where each vector can be multiplied by the projectile mass resulting in a momentum vector diagram. The mass*9.8*Δt would be the external impulse induced by gravity pulling.

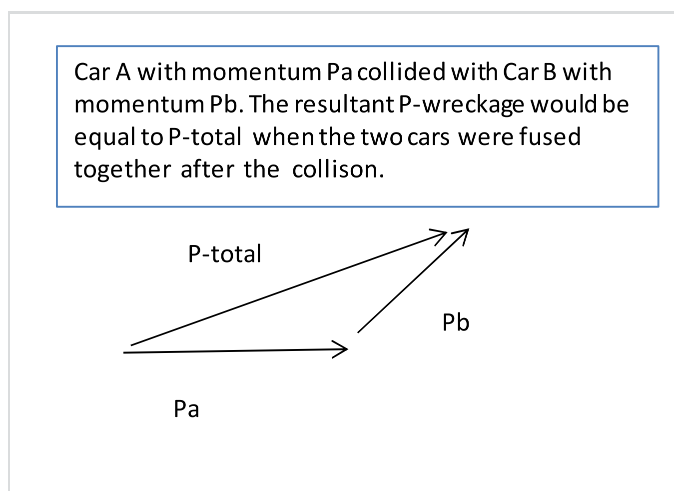

FIG. 19: A schematic view of the momentum vectors in a collision.

3. Discussion

A literature search on vector teaching showed a few references where vectors crossing each other in drawing [1] and would create confusion for below average students in our teaching experience, or no graphical construction method was presented in the writing [2], or the published numerical examples are on single-object situations [3]. Our above examples are aimed to improve vector teaching without lowering the syllabus.

Good Physics Education delivery would be vital for doing productive Physics Education Research with the collected student performance data to assess student learning. The inclusion of the graphical vector methods presented above would be consistent with the vector nature of force as put forward by Newton. A directional change can be visualized easily unlike scalar equation approach in the solving of the vector F = ma equation. Force is the cause and the observed acceleration is the effect. Extension to the impulse vector concept with the knowledge of time duration would help the teaching of momentum as a vector concept in dynamics. In fact, in the teaching of Schrödinger equation in modern physics, a typical course after the first year calculus physics course, the momentum operator is taught readily without any mathematical expression of a quantum force.

A sound collection of physics education materials should also be consistent with neuro-science findings. Handwriting where the letters must be free-form creations of the child herself/himself is an important tool to train a child's brain reading circuit [4], and that motor execution would play an essential part, not perceptual feedback [5]. By the same token, graphical vector drawing could strengthen that part of the brain neural circuit responsible for spatial thinking, a vital aspect of understanding vector in physics. Taking notes on laptops rather than in longhand has been reported as a less efficient method in classroom learning [6], and the graphical vector method would reinforce learning through tactile practice of drawing and longhand.

4. Conclusions

We have presented a novel graphical vector method in the solving of the standard 2-object on an incline problem. The case of pulley with inertia would not hinder the presented vector methodology in solving vector equations of motion. The graphical vector materials would serve as a platform for physics education research and neuro-science learning research in future studies.

Acknowledgements

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