# Neutrino Oscillation and Neutrino Mass

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**Abstract**

In the standard model (SM) of particle physics neutrinos are massless and chargeless spin ½ particles. But the discovery of neutrino oscillations has shown that neutrinos have mass. From neutrino oscillations, we know only differences of mass-squared but not the absolute masses of individual neutrinos. In this article, we discuss neutrino oscillation and neutrino mass briefly.

## 1. Introduction

According to the standard model (SM) of particle physics fundamental constituents of matter are of two types: quarks and leptons [1]. This model assumes three generations (or families) of quarks and three generations of leptons (Table 1). Quarks are called (up, down), (charm, strange), and (top, bottom). The leptons consist of three flavours of charged leptons, the electron $e^-$, muon $\mu^-$ and tau $\tau^-$, together with three flavours of neutrinos – the electron neutrino $\nu_e$, muon neutrino $\nu_\mu$ and tau neutrino $\nu_\tau$. All neutrinos are assumed to be massless and neutral.

Neutrinos are the second most abundant particles in the universe (photons are first). Neutrinos are very elusive and hardly interact with matter [2–7]. They do not enjoy electromagnetic and strong interactions but take part only in the weak interactions. They only interact with charged fermions and massive gauge bosons through the weak interaction. Neutrinos are copiously produced in the sun, in cosmic rays and even in laboratories. They are produced via the following processes:

(a) $(\nu_e, \bar{\nu}_e)$: Beta decay $(\bar{\nu}_e)$, Fission $(\bar{\nu}_e)$ and Fusion $(\nu_e)$ reactions.

(b) $(\nu_\mu, \bar{\nu}_\mu)$: Pion decay $(\pi^+ \rightarrow \mu^+ + \nu_\mu$ or the charge conjugate process).

(c) $(\nu_\mu, \bar{\nu}_e, \nu_e, \bar{\nu}_\mu)$: Muon decay $(\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$ or the charge conjugate process).

Solar neutrinos are produced through process (a) while atmospheric (i.e. cosmic ray) neutrinos come from (b) and (c). Accelerator neutrinos rely on (b); reactor antineutrinos result from fission reactions (a). There are other neutrino sources e.g. supernovae etc. Physicists detected the first neutrinos from a supernova in 1987 when a star collapsed some 150,000 light-years away in the Large Magellanic Cloud, the galaxy nearest to the Milky Way.

**Table 1: Three generations of Leptons and Quarks**

| First Generation | Second Generation | Third Generation |
|---|---|---|
| $\nu_e$ $e^-$ | $\nu_\mu$ $\mu^-$ | $\nu_\tau$ $\tau^-$ |
| u d | c s | t b |

## 2. Neutrino Puzzles and Neutrino Oscillation

In the 1960s, John Bahcall was trying to calculate what types of nuclear processes are occurring in solar fusion [2]. He predicted that the reaction $H^+ + H^+ \rightarrow He^{2+} + \nu + \text{(other)}$ generates around $7\times10^{10}$ neutrinos/(cm$^2$.s) on earth. Around 100 billion solar neutrinos are passing through our body every second. But they interact so weakly with other matter that remarkably little is known about them. The fusion reactions that take place in the sun only produce electron neutrinos. In order to detect these neutrinos he teamed up with experimentalist Ray Davis. Ray Davis and his team built a tank to hold 380,000 litres of perchloroethylene in the Homestake Gold Mine in South Dakota. They detected the solar electron neutrino flux at earth which was about 1/3 of the theoretical value. This was known as "*solar neutrino puzzle*". A similar discrepancy was also seen in atmospheric neutrinos. Atmospheric neutrinos are created as a consequence of cosmic ray protons from space hitting earth's atmosphere (which contains protons and neutrons). High energy proton/proton or proton/neutron collisions produce charged pions. These charged pions decay into muons and muon neutrinos. Then muons decay into an electron, an electron neutrino and a muon neutrino. Thus atmospheric neutrinos predict that for every electron neutrino there should be two muon neutrinos. But from IMB and Kamiokande experiments it was observed a ratio of one to one. This was "*atmospheric neutrino puzzle*".

In 1996, the SuperKamiokande detector was built in a zinc mine under 1,000 meters of solid rock in Japan. It was filled with 50,000 tons of ultra-pure water (not heavy water) and was designed to detect atmospheric neutrinos [7]. These neutrinos interact with atomic nuclei in the water to produce electrons, muons or tau leptons. Atmospheric neutrinos are mostly muon neutrinos. In 1998 [8], SuperKamiokande collaboration discovered that muon neutrinos converted or oscillated to tau neutrinos as they passed through the earth. The SuperKamiokande collaboration announced the first evidence for neutrino mass. Neutrinos oscillate in flavour because they have mass [9,10]. The SuperKamiokande was also used to study solar neutrinos. The fusion reactions that take place in the sun only produce electron neutrinos. But these neutrinos can subsequently oscillate into both muon neutrinos and tau neutrinos. Though the experiment was able to detect the solar neutrinos, it was unable to distinguish different neutrino types. Meanwhile, the Sudbury Neutrino Observatory (SNO) was constructed in a nickel mine under more than 2,000 meters of rock in Canada. Its tank was filled with 1,000 tons of heavy water. It was designed to study solar neutrinos. The SNO [11,12] could identify the electron neutrinos because it was filled with 'heavy water', which contains hydrogen nuclei with an extra neutron. The combined data from SuperKamiokande and SNO determined how many muon neutrinos or tau neutrinos were incident at the detector. The SNO results also provided further evidence for neutrino mass and confirmed that the total number of neutrinos from the sun agreed with theoretical calculations.

Takaaki Kajita was the team leader of the SuperKamiokande collaboration and Arthur B. McDonald directed the Sudbury Neutrino Observatory. On 6th October 2015, the Royal Swedish Academy of Sciences has announced to award the Nobel Prize in physics for 2015 jointly to *Takaaki Kajita*, University of Tokyo, Kashiwa, Japan and *Arthur B. McDonald*, Queen's University, Kingston, Canada "*for the discovery of neutrino oscillations, which shows that neutrinos have mass*". Their work has been published in an international reputed journal – Physical Review Letters [8,11,12].

## 3. Quantum Mechanics of Neutrino Oscillation

B. Pontecorvo [13] in 1958 and Z. Maki, M. Nakagawa and S. Sakata [14] in 1962 proposed that neutrino oscillation is a quantum mechanical effect. Neutrinos must have some mass for oscillations to occur. If neutrinos have finite masses, each flavour eigenstate ($\nu_e$, $\nu_\mu$ and $\nu_\tau$) can be expressed by a combination of mass eigenstates $\nu_1$, $\nu_2$ and $\nu_3$ with mass $m_1$, $m_2$ and $m_3$ [15–17]. For simplicity let us discuss two flavour neutrino oscillation for example, between $\nu_e$ and $\nu_\mu$. Let $\theta_{e\mu}$ be the $\nu_e - \nu_\mu$ mixing angle. If a neutrino were produced as an $\nu_e$ at the source and travelled a distance L, the probability that it oscillated into a $\nu_\mu$ is given by:

$$P\left(\nu_e \to \nu_\mu\right) = \sin^2 2\,\theta_{e\mu}.\sin^2\left(\frac{1.27\ \Delta m^2.L}{E_\nu}\right) = A.\sin^2\left(\frac{\pi L}{\lambda}\right) , \qquad (1)$$

where $\lambda = \frac{\pi E_\nu}{1.27\,\Delta m^2}$ acts as an oscillation length and $A = \sin^2 2\theta_{e\mu}$ as the amplitude of oscillation; $\Delta m^2 = \left|m_2^2 - m_1^2\right|$ in $\mathrm{eV}^2$ measures the difference of mass squares between the neutrinos, $E_\nu$ is the neutrino energy in GeV, and L in km is the distance of the detector from the neutrino source. Basically there are two variables: $\Delta m^2$ and $\theta$. From equation (1) it is clear that

(a) The ideal distance of the detector from the source for observing the oscillations is $L = \lambda/2$, so that $\sin^2\left(\frac{\pi L}{\lambda}\right) = 1$.

(b) $\Delta m^2$ is dependent on (E/L); for small values of $\Delta m^2$ one needs small values for (E/L) to see the oscillations.

Here, we discuss oscillation between only two neutrino flavours $\nu_e$ and $\nu_\mu$. These flavour states can be expressed as the superposition of the mass eigenstates $\nu_1$ and $\nu_2$:

$$\begin{aligned} \left|\nu_e\right\rangle &= \left|\nu_1\right\rangle c + \left|\nu_2\right\rangle s \ ; \\ \left|\nu_\mu\right\rangle &= -\left|\nu_1\right\rangle s + \left|\nu_2\right\rangle c \end{aligned} \quad , \qquad (2)$$

where $c = \cos\theta$ and $s = \sin\theta$. For two flavours a single angle, $\theta$, suffices to completely specify one basis in terms of the other. Consider now the state vector of an $\nu_e$ produced at t = 0. Thus, initially $\left|\psi(0)\right\rangle = \left|\psi_{\nu_e}\right\rangle = c\left|\psi_1\right\rangle + s\left|\psi_2\right\rangle$. (3)

If the stationary states $\left|\psi_1\right\rangle$ and $\left|\psi_2\right\rangle$ correspond to energies $E_1$ and $E_2$ respectively, then at a later time the state vector will be:

$$|\psi(t)\rangle = c|\psi_1\rangle\exp(-iE_1t) + s|\psi_2\rangle\exp(-iE_2t) . \quad (4)$$

The probability, $P(\nu_e, 0; \nu_\mu, t)$ of the state $|\psi(t)\rangle$ (originating as a $\nu_e$ at t = 0) appearing as a $\nu_\mu$ is $\left|\langle\psi_{\nu_\mu}|\psi(t)\rangle\right|^2$ and is seen to be:

$$P(\nu_e, 0; \nu_\mu, t) = c^2 s^2 \left|-\exp(-iE_1t) + \exp(-iE_2t)\right|^2 . \quad (5)$$

The neutrinos are expected to have small masses, $m_i$, and are in the ultrarelativistic regime $\left(E_i \cong p + \frac{m_i^2}{2p}\right)$ where p ( >> $m_i$ ) is the magnitude of the neutrino momentum. In this situation, we can rewrite the probability given in eq. (1):

$$P(\nu_e, 0; \nu_\mu, t) = \sin^2 2\theta_{e\mu} \sin^2\left(\frac{\pi L}{\lambda}\right), \quad (6)$$

In the right hand side, the first factor is a consequence of the "mixing" while the second factor leads to the "oscillatory" behaviour. For vacuum oscillations, the former, dependent on the mixing angle $\theta_{e\mu}$, is a constant but in the Mikheyev-Smirnov-Wolfenstein (MSW) [18] matter effect, it changes with the matter density. From eq. (6)

$$P(\nu_e, 0; \nu_e, t) = 1 - P(\nu_e, 0; \nu_\mu, t)$$

$$= 1 - \sin^2 2\theta_{e\mu} \sin^2\left(\frac{\pi L}{\lambda}\right). \quad (7)$$

From eq. (7) it is seen that $P(\nu_e, 0; \nu_e, t)$ is less than unity. The essential ingredients for this are twofold:

(i) The neutrinos must be massive and non-degenerate ($\Delta m^2 \neq 0 \Rightarrow \lambda$ is finite).

(ii) The mass eigenstates of the neutrinos $\nu_1, \nu_2$ must be different from the flavour eigenstates $\nu_e, \nu_\mu$ ($\sin 2\theta_{e\mu} \neq 0$).

An important theme of neutrino flavour change is the MSW effect, which is a matter-enhanced neutrino oscillation; in this case, the conversion $\nu_e \rightarrow \nu_x$ results from interaction between $\nu_e$ and solar electrons as the neutrinos travel from the centre of the sun. This effect originates from the additional interactions of a neutrino in a medium. It is well-known that interactions increase the inertia and mass is a measure of inertia. Thus, interactions in a medium result in a varying neutrino mass. While the solar neutrinos produced in the interior are coming out they pass through dense regions of the sun and experience the MSW effect. They interact with the solar electrons $(\nu + e^- \rightarrow \nu + e^-)$ giving to a contribution to the effective mass. From eq. (1) it is clear that the oscillation in flavour depends not on the mass of any particular neutrino type, but rather on the mass-squared difference between the flavours.

## 4. Neutrino Mass

The origin of neutrino masses is one of the biggest puzzles in particle physics today [6,7,19,20]. Although there are strong evidences for neutrino masses, till date we do not know the mechanism responsible for the generation of neutrino masses. Experiments have determined that neutrinos $\nu_1$ and $\nu_2$ have similar mass with $\nu_1$ being lighter than $\nu_2$. But till today we do not know whether $\nu_3$ is much higher in mass ("normal hierarchy") or much lower in mass ("inverted hierarchy"). The absolute masses of neutrinos are not known, but a wide variety of experiments and theoretical models are setting their limits. A few of them are discussed below.

**(i) Beta decay:**

A nucleus with an overabundance of neutrons can transform to a more stable nucleus by emitting an electron and an antineutrino. This kind of process is known as beta ($\beta$) decay. The mass of the neutrino can be determined from the endpoint of the $\beta$-spectrum. The upper limit on the absolute scale of the electron neutrino mass is obtained from the tritium beta decay [21] as $m(\nu_e) < 2$ eV.

**(ii) Neutrinoless double beta decay:**

Double beta decay $(\beta\beta)$ is a nuclear transition (Z, A) $\rightarrow$ (Z+2, A) in which two neutrons bound in a nucleus are simultaneously transformed into two protons plus two electrons (there may be some light particles also). (i) In the two-neutrino double beta decay mode $(2\nu\beta\beta)$, there are $2\overline{\nu}_e$ emitted together with $2e^-$. The lepton number is conserved for this mode and this mode of decay is allowed in the standard model of electroweak interaction. (ii) In the neutrinoless double beta decay mode $(0\nu\beta\beta)$, only the $2e^-$ are emitted and nothing else. This neutrinoless double beta decay occurs when the two antineutrinos, instead of manifesting themselves as real states, "annihilate". This can only occur if neutrinos are their own antiparticles. This mode violates the law of lepton number conservation and is forbidden in the standard model. Hence its observation may lead to a signal of "new physics". The lepton number violation can generate a lepton asymmetry in the early universe, which will be able to explain the present baryon asymmetry of the universe.

Neutrinoless double beta decay is the only experiment that can probe the Majorana nature of the neutrino (i.e. the neutrino and antineutrino are identical) [22–25]. The values of the neutrino mass-squared differences are known, but the absolute values of neutrino masses are elusive. The observation of neutrinoless double beta decay would not only reveal the neutrinos are Majorana fermions, but would also provide information regarding the absolute values of the neutrino masses. The two-neutrino double beta decay has already been experimentally observed. There is possible evidence of neutrinoless double beta decay in the Heidelberg-Moscow experiment [22] but so far, neutrinoless double beta decay has not yet been observed conclusively [25].

Assuming Majorana nature of neutrino, a strong limit on the mass eigenstate of $\nu_e$ is obtained as $m_{\nu 1} \leq 0.4 - 0.5$ eV from neutrinoless double beta decay experiments with Germanium [26,27] and Tellurium [22,28]. Furthermore, the search for the $0\nu\beta\beta$ decay is the only way to probe the Majorana nature of neutrinos and one of the most promising ways to search for lepton number violation.

**(iii) Neutrino oscillations:** In this method, neutrino mass squared differences $\Delta m_{ij}^2 = m_i^2 - m_j^2$ are determined. The two different $\Delta m^2$ values are $\left|\Delta m_{atm}^2\right| = (1.9 - 3.0) \times 10^{-3}\,\mathrm{eV}^2$ and $\Delta m_{sol}^2 = 8.0_{-0.3}^{+0.4} \times 10^{-5}\,\mathrm{eV}^2$. This range and indicated error bars show the present sensitivity. This mass determination is independent of the charge conjugation properties of neutrinos.

**(iv) Cosmological observations:** From cosmic microwave background and large scale structure data, the size of fluctuations is observed at different scales. Since the light neutrinos would have smeared out fluctuations at small scales, the power spectrum at small scales is sensitive to the neutrino mass. Although the absolute mass of the neutrinos have not yet been determined, there is an upper bound on the sum over all neutrino masses from cosmological observations [29]: $\sum_{i=e,\mu,\tau} m_{\nu_i} \leq 0.61$ eV, which are to some extent model- and analysis dependent [30]. This mass determination is independent of the Majorana or Dirac nature of neutrinos.

## 5. Conclusion

The electron-neutrinos produced from the sun were measured to be less than what was predicted by the standard solar model and experiments with the atmospheric neutrinos demonstrate that there was a depletion of atmospheric muon-neutrinos while there was no depletion of electron-neutrinos. One possible explanation for the observed solar neutrino deficit is that the $\nu_e$ produced in the centre of the sun could convert itself to another type, i.e., $\nu_e \to \nu_x$ with $x = \mu$ or $\tau$, during its passage to the earth via a process called *neutrino oscillation*. Similarly, the atmospheric muon-neutrino deficit could be due to the conversion of $\nu_\mu$ to $\nu_\tau$. In this way, "atmospheric neutrino puzzle" and "solar neutrino puzzle" were resolved by neutrino oscillations in 1998 and 2001(2) respectively. $\Delta m^2_{23}$, $\sin^2 2\theta_{23}$, $\Delta m^2_{12}$ and $\sin^2 2\theta_{12}$ have been measured accurately by the present generation experiments assuming 2-flavour neutrino oscillations [15]. It is also possible that CP invariance can be violated in the lepton sector. Neutrino oscillation is a quantum mechanical effect. Neutrinos must have some mass for oscillations to occur. The Nobel Prize in physics for 2015 has been awarded jointly to *Takaaki Kajita*, University of Tokyo, Kashiwa, Japan and *Arthur B. McDonald*, Queen's University, Kingston, Canada "*for the discovery of neutrino oscillations, which shows that neutrinos have mass*". From neutrino oscillations, we know only differences of mass-squared but not their individual masses. Although the absolute masses of neutrinos are not known, a wide variety of experiments and theoretical models are setting their limits as discussed in section 4. Recently, Robertson [31] has discussed neutrino mass. According to him, neutrino oscillations set a lower limit of 0.02 eV and upper limit from measurements is 2.0 eV. Recently, Fritzsch [32] has calculated the masses of three neutrinos: $m_1 \approx 0.003\,\text{eV}$, $m_2 \approx 0.012$ eV, and $m_1 \approx 0.048\,\text{eV}$.

Neutrinos show up in precision cosmological observations: since they have a small mass they should cluster on sufficiently large scales. Neutrinos may also be messengers of dark matter annihilation in our galactic halo or in the core of the sun. Neutrinos are important for the study of the sun, stars, core-collapse supernovae, the origins of the cosmic rays, the large-scale structure of the universe, and big bang nucleosynthesis. These tiny neutrino masses are of great interest because they might arise from some fundamentally different mechanism to the way the masses of other particles are generated i.e. the Higgs mechanism. Although the SM is very successful to explain many low as well as high energy phenomena in particle physics but within the framework of this model it is not possible to realize the massive neutrinos. The existence of neutrino mass is one of the signatures of new physics beyond the SM [33–35].

There are still many things about neutrinos that we need to know. A few of them are: (i) absolute mass of neutrinos (ii) whether neutrinos are Majorana particles ($\bar{\nu}_i = \nu_i$) or Dirac particles ($\bar{\nu}_i \neq \nu_i$), (iii) What is the pattern of neutrino masses (normal mass hierarchy or inverted mass hierarchy)? (iv) Why neutrino masses are so small or why there is such a large gap between the neutrino and the charged fermion masses? (v) CP violation in neutrino (lepton) sector, etc. We hope the results from further experiments will provide us the answer to these problems in near future.

The Karlsruhe Tritium Neutrino Experiment (KATRIN) in Germany is taking data to make a very precise measurement of the electron energy spectrum from beta decay. We hope we can know the mass of neutrinos from KATRIN and astronomical surveys very soon. The current generation of oscillation experiments including Double Chooz, RENO, Daya Bay, T2K and NOvA, will try to resolve the neutrino mass hierarchy. From the ongoing and future neutrino experiments, we expect more surprises. Neutrinos have and will continue to provide important information on structure formation in the early universe, earth, solar and supernova physics, nuclear properties, and rare decays of charged leptons and hadrons [36]. The study of neutrino physics and the implications of the results connect many disciplines together, from particle physics to nuclear physics to astrophysics to cosmology. Thus, neutrino physics continues to be a very exciting field and may also bring us new surprises in this 21st century.


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## References


1. S. Adams, *Frontiers – Twentieth Century Physics*, Replika Press Pvt. Ltd., Delhi, pp. 168 – 189 (2000).
2. D. Lincoln and T. Miceli, *The Physics Teacher*, **53**, 331-338 (2015).
3. P. Langacker, J. Erler and E. Peinado, *J. Phys. Conf. Ser*. **18,** 154 – 187 (2005) [arXiv: hep-ph/0506257].
4. G. Altarelli, arXiv: hep-ph/ 0611117 (2006).
5. D. P. Roy, *Physics News–Bulletin of Indian Physics Association*, **39**(3), 51 (2009) [arXiv: 0809.1767 [hep-ph]].
6. S. F. King, *Contemp. Phys*. **48**(4), 195 – 211 (2007).
7. H. Murayama, *Phys. World*, **15**(5), 35–39 (2002).
8. Y. Fukuda, T. Kajita et al. [Super-Kamiokande Collaboration], *Phys. Rev. Lett.* **81**(8), 1562 (1998).
9. E. Hecht, *The Physics Teacher*, **41**, 164 (2003).
10. M. Goswami and S. Sahoo**,** *Science Reporter*, **45**(4), pp. 33 – 34 (2008).
11. Q. R. Ahmad, A. B. McDonald et al. [SNO Collaboration], *Phys. Rev. Lett.* **87**(7), 071301 (2001).
12. Q. R. Ahmad, A. B. McDonald et al. [SNO Collaboration], *Phys. Rev. Lett.* **89**(1), 011301 (2002).
13. B. Pontecorvo, *Sov. Phys. JETP* **6**, 429 (1958); **7**, 172 (1958).
14. Z. Maki, M. Nakazawa, S. Sakata, *Prog. Theor. Phys*. **28**, 870 (1962).
15. T. Kajita, *Int. J. Mod. Phys. A*, **24**, 3437–3446 (2009).
16. R. N. Mohapatra and A. Y. Smirnov, *Ann. Rev. Nucl. Part. Sci.* **56,** 569 (2006).
17. R. N. Mohapatra et al., *Rept. Prog. Phys.* **70,** 1757-1867 (2007)[arXiv:hep-ph/ 0510213].
18. M. Mikheyev, A. Smirnov, *Sov. J. Nucl. Phys*. **42**, 913 (1986); L. Wolfenstein, *Phys. Rev*. **D17**, 2369 (1978), ibid **D20**, 2634 (1979).
19. S. Sahoo, *Indian J. Pure & Appl. Phys.* **48**(10), 691–696 (2010).
20. S. Sahoo, A. Pathak and A. Mondal, *ARPN Journal of Science and Technology*, **4**(6), 345 – 348 (2014).
21. E. W. Otten and C. Weinheimer, *Rept. Prog. Phys*. **71**, 086201 (2008). [arXiv:0909.2104 [hep-ex]].
22. H. V. Klapdor-Kleingrothaus, A. Dietz, H. L. Harney and I. V. Krivosheina, *Mod. Phys. Lett. A* **16**, 2409 (2001) [hep-ph/0201231].
23. D. R. Artusa et al. (CUORE Collaboration), *Eur. Phys. J. C* **74**(10), 3096 (2014) [arXiv: 1404.4469 [nucl-ex]].
24. S. Sahoo and B. Sahoo, *IAPT Bulletin*, **2**(2), 37–41 (2010).
25. J. J. Gómez-Cadenas & Justo Martín-Albo, arXiv:1502.00581 [hep-ex] (2015).
26. C. Weinheimer et al., *Phys. Lett. B*, **460,** 219 (1999).
27. V. M. Lobashev et al., *Phys. Lett. B*, **460,** 227 (1999).
28. C. E. Aalseth et al. [IGEX Collaboration], *Phys. Rev. D***, 65,** 092007 (2002).
29. E. Komatsu et al. [WMAP Collaboration], *Astrophys. J. Suppl*. **180,** 330 (2009) [arXiv: 0803.0547].
30. S. Hannestad et al., *J. Cosmol. Astropart. Phys*. **08,** 015 (2007).

31. R. G. H. Robertson, *Nuclear and Particle Physics Proceedings*, **265–266**, 7–12 (2015) [arXiv: 1502.00144 [nucl-ex]].
32. H. Fritzsch, *Mod. Phys. Lett. A* **30**(16), 1530012 (2015) [arXiv: 1503.01857 [hep-ph]].
33. G. Senjanovic, arXiv: 0911.0029 [hep-ph] (2009).
34. I. Bhattacharya, *Fundamental Journal of Mathematical Physics,* **3,** 23–31 (2013) [arXiv: 0910.5801 [hep-ph]].
35. A. Y. Smirnov, arXiv: 0910.1778 [hep-ph] (2009).
36. A. de Gouvea et al., arXiv: 1310.4340 [hep-ex] (2013).