How High Could a Mountain Be?

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Abstract

Mount Everest is the world's highest mountain and it has a height of about 9Km. The estimation of the maximum of height of a mountain on earth using basic physics is done by famous Physicist victor Weisskopf (1908 – 2002). According toWeisskopf, the height of a mountain on earth cannot be more than 44Km. If the height of a mountain is more than this critical value, the mountain would start sinking. During the sinking processes, the directionality of the bonds between the atoms of the solid rock below breaks due to the weight of the mountain. As a result, solid rocks below melt and flow aside. This value was estimated by taking the solid rock below the mountain as SiO_2 and the ratio of liquefaction energy to binding energy of the solid rock was supposed to be same as that of ice. Actually, this maximum value of the height of a mountain is less than 44Km because the rock below is warm and needs less energy to liquefy. Weisskopf has also derived a formula in terms of fundamental constants for calculating the critical height of a mountain on any planet with same material structure as earth, if the number of nucleons in the planet is known.

1.Introduction

Mount Everest is the world's height peak in the eastern end of Himalayas and it has a height of about 9Km. Mountains on earth cannot grow above 9Km. The answer is if a mountain is very high, it would start sinking due to its own weight.

Victor Weisskopf (1908 -2002) was a famous physicist. He had made significant contributions in nuclear physics and quantum electrodynamics. Weisskopf served as professor in physics at MIT from 1946 to 1960. He was the director of CERN from 1960 to 1965. After leaving CERN, Weisskopf returned to MIT in 1965. At MIT he became in1967 head of department of physics, a post he held until 1973.Every summer CERN organizes visits for local high school students. The estimation of the maximum height of a mountain is one of the lectures given to high school students by Weisskopf. According to him, the maximum height of a mountain on earth is around 44Km. Weisskopf emphasized the importance of using basic knowledge in physics as a means to make reasonable estimates.

Weisskopf has also derived a formula in terms of fundamental constants for calculating the critical height of a mountain on any planet with same material structure as earth, if the number of nucleons in the planet is known.

The rest of the paper is organized as follows. The estimation of the height of a mountain on earth by Weisskopf is given in Section 2. The height of a mountain in terms of fundamental constants is presented in Section 3. Conclusion is given in Section 4.

2. Estimation of the height of a mountain on earth by Weisskopf

The following derivation is from the lecture notes(1969) of Victor Weisskopf.

If a mountain is very high, it would start sinking due to its weight. The force due to the weight of the mountain is sufficient to break the directionality of the bonds between the atoms in the rock. This makes the underlying rock melt and flow aside so that the mountain sinks.

The shape of the mountain is taken as shown in Fig.1. Let the mass of the mountain be M and its height be h. Let it sink by a distance x.

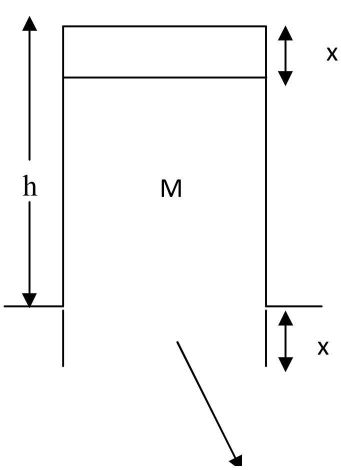

Figure1. Mountain of height h

Loss in gravitational energy when the mountain sinks by x

$= Mgx$ (1)

This loss in gravitational energy is used to liquefy the rock below. If the mountain has to sink by a distance x, a layer of thickness x below the mountain must melt and flow away.

The energy required to liquefy a layer of thickness x

$= E_{liq}(n\,x\,a)$ (2)

Where

E_{liq} = Liquefaction energy (i.e. latent heat of melting) per molecule

$n =$ Number of molecules per unit volume of the rock

$a =$ Area of cross-section of the base of the mountain

Equating (1) and (2), we get

$Mgx = E_{liq}(n\,x\,a)$

$\Rightarrow Mg = E_{liq}\,n\,a$ (3)

Now we conclude that if the mass M of the mountain is less than that given by (3), the mountain will not sink. Hence the masses of stable mountains is given by

$M \leq \frac{E_{liq}\,n\,a}{g}$ (4)

Let m denote the mass of a molecule of the rock, and A is the number of protons and neutrons in a molecule. If m_p denotes the mass of a proton, then $m \approx A\,m_p$, taking the mass of a neutron approximately equalto the mass of a proton. Then the mass M of the mountain can be written as

M = volume of the mountain $\times$ number of molecules per unit volume $\times$ mass of a molecule

$= ha \times n \times m$

$= h\,a\,n\,A\,m_p$ (5)

Substitution of (5) in (4) gives

$$h \le \frac{E_{liq}}{Agm_p} \quad (6)$$

The above expression shows that the height of a mountain must be less than the critical value $h_c = E_{liq}/Agm_p$ for the mountain not to sink into earth.

When a solid melts, the whole bonds between the atoms are not broken, just the directionality of the bonds are broken. This enables a liquid to flow andthe energy necessary to break the directionality of bonds, i.e to liquefy, is less than the binding energy. To calculate E_{liq}, Weisskopf has compared the latent heat of melting of ice (⊔ 80 cal/gm) with that of latent of vaporisation of water (⊔ 540cal/gm). The latent heat of melting of ice is approximately one-seventh of latent heat of vaporisation of water. Weisskopf has applied this ratio for ice to solids at the baseof the mountain. The binding energy of a solid is the energy applied to a solid to tear it completely into separate atoms. The binding energy of a solid is more than the energy required to tear a liquid into separate atoms. It would be a good estimate to assume that for a solid the melting energy is about one-tenth of binding energy. So Weisskopf takes

Energy of melting ~1/7 energy of vaporisation

~1/10 Binding energy of solid

Hence we have

$$E_{liq} = \beta B \quad (7)$$

where $\beta = \sim 0.1$ and B = binding energy of the solid per molecule.

Substituting (7) in (6), we obtain

$$h \le \frac{0.1B}{Agm_p} \quad (8)$$

B isabout 2.7ev for SiO_2,the main constituent of the rock. The mass number of silicon atom is 28 and that of oxygen is 16. So, $A = 28 + 2 \times 16 = 60$ for SiO_2.

$m_p = 1.672 \times 10^{-27}$ Kg and $g = 9.8\ m/S^2$

Using the above values in (8), we get

$$h \le \frac{0.1 \times 2.7 \times 1.6 \times 10^{-19}}{60 \times 9.8 \times 1.672 \times 10^{-27}} m$$

$\le 43941 m$

$\Rightarrow h \le 44\ Km$

Therefore, the height of a mountain on earth must be less than 44Km to be supported by rock at its base. Actually, the upper limit is smaller than 44Km because the rock at the base of the mountain is warm and needs less energy to liquefy. The height of world's top five mountains is given in *Table* 1.

Table1. Five world's highest mountains

Rank	Mountain	Height in Km
1	Everest	8.8498
2	K2(Mount Godwin Austen)	8.6106
3	Kanchenjunga	8.5859
4	Lhotse	8.5161
5	Makalu	8.4630

Mount Everest is the world's highest mountain and it has height of about 9Km. The highest mountain on moon is Mons Huygens, which has a height of 5.5Km. The highest mountain Maxwell Montes on the planet Venus is about 11Km. Boosaule Montes is the highest mountain on Jupiter's moon IO and has a height of 17.5Km. The highest mountain on Mars is Olympus Mons, which has a height of about 22Km. Olympus Mons is about 2.5 times higher than Everest. Hence, the estimation given by Weisskopf for the maximum height of a mountain on earth seems to be valid for other planets as well.

3.Height of a mountain in terms of fundamental constants by Weisskopf

Equation (6) shows that on other planets the maximum height (critical height) of a mountain would be different because the acceleration due to gravity g changes and the planet may be made of different material. So, Weisskopf has expressed (8) in terms of fundamental constants eliminating g .

The force of attraction between the mass m_1 and the earth is given by

$$m_1 g = \frac{G m_1 M_E}{R_E^2}$$

$$\Rightarrow g = \frac{G M_E}{R_E^2} \quad (9)$$

where M_E = mass of the earth

R_E = radius of the earth

G=gravitational constant=6.67×10^{-11} m^3/Kg-S^2

Let N_E = number of nucleons in the earth

Taking the mass of a proton approximately equal to the mass of a neutron, we can write

$$M_E = N_E m_p \quad (10)$$

The earth consists mostly of SiO_2 (A = 60) and iron (A = 67). The SiO_2 molecule and the iron atom have approximately the same value of A.

Therefore, the number of molecules in earth = N_E / A

The radius of SiO_2 molecule is taken approximatelyequal to that of iron atom. The radius of both SiO_2 and iron is written as fa_o with f□ 4.

Where a_o = Bohr radius = 0.528 Å

The radius of the hydrogen atom in the ground state is called Bohr radius.

Volume of earth = number of molecules in earth × volume of a molecule

$$\Rightarrow \frac{4}{3}\pi R_E^3 = \frac{N_E}{A} \times \frac{4}{3}\pi \left(fa_o\right)^3$$

$$\Rightarrow R_E = \left(\frac{N_E}{A}\right)^{1/3} \left(fa_o\right) \quad (11)$$

Using (10) and (11) in (9), we get

$$g = G N_E m_p \left(\frac{A}{N_E}\right)^{2/3} \left(\frac{1}{fa_o}\right)^2 \quad (12)$$

The *fine structure constant for gravity* is defined as

$$\alpha_G = \frac{G m_p^2}{\hbar c} = 5.9\times10^{-39} \quad (13)$$

This constant is a dimensionless constant.From (13) we get

$$G = \frac{\alpha_G \hbar c}{m_p^2} \qquad (14)$$

Where, $\hbar = \dfrac{h}{2\pi}$

h = Planck's constant =6.62×10^{-34}J-S

c = speed of light in vacuum

Using (14) in (12), we obtain

$$g=\left(\frac{\alpha_G\hbar c}{m_p}\right)A^{2/3}N_E^{1/3}\left(\frac{1}{fa_o}\right)^2 \quad (15)$$

Substituting (15) in (8), the critical height h_c of the mountain is given by

$$h_c=\frac{0.1\times B}{\hbar c}f^2\left(\frac{1}{\alpha_G}\right)\left(\frac{1}{N_E}\right)^{1/3}\left(\frac{1}{A}\right)^{5/3}a_o^2 \quad (16)$$

The binding energy B of the rock can be written as

$B=\eta$ Rydberg

1Rydberg (Ry) = $\frac{m_e e^4}{2\hbar^2}$ = 13.6 ev

Bohr radius = $a_o=\frac{\hbar^2}{m_e e^2}$

The *fine structure constant* α is defined as

$$\alpha=\frac{e^2}{\hbar c}\approx\frac{1}{137}$$

Where,

m_e = mass of an electron

e = charge of an electron

Using the above values in (16), we obtain

$$\frac{h_c}{a_o}=0.1\eta f^2\left(\frac{\alpha}{2\alpha_G}\right)\left(\frac{1}{N_E}\right)^{1/3}\left(\frac{1}{A}\right)^{5/3} \quad (17)$$

Where, $\eta\approx 1/5$ for SiO_2 .

The above expression (17) gives h_c/a_o in terms of dimensionless constants and it can be used to calculate the critical height h_c of a mountain on any other planet, if the number of nucleons in that planet is known and that planet has same material structure as earth. We cannot use (17) for a celestial object with a material structure different from that of earth as E_{liq} in that case would be different.

4.Conclusion

The maximum height of a mountain on earth according to the estimation of Victor Weisskopf using basic physics is about 44Km. Actually, the maximum height of a mountain is less than 44Km because the rock at the base of a mountain is warm and needs less energy to liquefy. On other planets, the maximum height of a mountain would be different because the acceleration due to gravity g changes and the planet may be made of different material. Weisskopf has also derived a formula in terms of fundamental constants for calculating the critical height of a mountain on any planet with same material structure as earth, if the number of nucleons in the planet is known. Mount Everest is the world's highest mountain and it has a height of about 9Km.The highest mountain on Mars is Olympus Mons, which has a height of about 22Km. So, the estimation given by Weisskopf for the maximum height of a mountain on earth seems to be valid for other planets as well.

References

[1] G.Venkataraman, *Why Are Things The Way They Are?* , (Universities Press,1992).