Estimating the Pressure of a Gas in a Balloon and Motion through Resistive Media

Sanjay Harrison[1] and Sindhu Vincent [2]
Northern Deanery. UK
47 Annand Way, Newton Aycliffe, Co Durham, DL5 4ZD, United Kingdom
quibit21@hotmail.com[1], sindhuvincent@gmail.com[2]

(*Submitted* June 13 2012)

Abstract

A simple method to estimate the pressure of a gas (P) inside a balloon is described. The basis of this method is derived from the principle of floatation – a principle most high school students would be familiar with. In this article we also describe the dynamics of the gas filled balloon as it traverses the depth of the liquid in the vessel. The dynamics of the balloon is described first by neglecting the effects of resistance through the liquid. The effect of resistance or drag is taken into account in subsequent sections of this paper. Expressions for the pressure of the gas in the balloon, the velocity (v) and acceleration (a) at a given instant and the terminal velocity ($v_{terminal}$) of the balloon are derived.

Introduction

Simple phenomena can very often hide some very interesting physics and studying such phenomena can lead to a greater appreciation of how seemingly disparate branches of physics come together and give rise to the phenomena in question. This paper elucidates the physics of linear motion through a resistive media and demonstrates how it can be used to determine estimates of gas pressures. The simple experiment described brings together kinematics and thermodynamics. It is believed that the demonstration of such connections would lead to greater appreciation amongst younger readers in particular of general physical principles.

Case 1 : Resistance due to Medium Ignored

The balloon containing the gas whose pressure is to be determined is immersed in a liquid (eg water) whose density is known (d_w) and held there with the help of an angled thin rod. The depth to which it is immersed (h) is measured and the moment it is released, a stopwatch is started and the time (t) the balloon takes to reach the surface when released is measured.

In calculating the pressure of the gas within the balloon, the following simplifying assumptions are made;

- The resistance experienced by the balloon as it makes its way to the surface of the liquid is negligible. This is justified by noting that the resistive force is proportional to the velocity of the balloon at any given instant. As the balloon only travels the depth of the vessel, the velocity acquired would be small.
- The weight of the balloon is negligible compared to the up thrust (U). This can be easily seen by noting that the numerical value of the volume of the balloon is much greater than the mass of the balloon.
- The gas within the balloon behaves as an ideal gas. This assumption is primarily for the purpose of calculational ease. It has to be emphasised that most gases only exhibit ideal behaviour at high temperatures and low pressures. However, as is evident

from the title of this paper, the experimental method described provides an 'estimate' of the pressure of the gas within the balloon. While this experimental method may not provide an exact value for the pressure, we believe that it in no way lessens the educational value it provides to the reader by connecting seemingly disparate areas of physics.

The mass of the gas (M) in the balloon will experience an upward force when immersed in the water containing vessel giving it an upward acceleration (a). Acceleration due to gravity is represented by 'g'. Once the relevant measurements have been made, the density of the gas (d_g) can be estimated as follows;

$$M\,a = V\,d_w\,g - M\,g$$

$$V\,d_g\,a = V\,d_w\,g - V\,d_g\,g$$

$$a = \frac{d_w}{d_g} - 1 \;\; g$$

Applying the second equation of motion to the balloon travelling the depth of the vessel (h);

$$h = \frac{at^2}{2}$$

$$h = \frac{gt^2}{2} \;\; \frac{d_w}{d_g} - 1$$

$$d_g = \frac{gt^2}{2}\,\frac{d_w}{2h+gt^2} \qquad (1)$$

Once the density of the gas has been calculated from the above equation, the ideal gas equation is modified as follows (P = pressure, V = volume, n = number of moles, M = mass of gas, m = molar mass of gas, T = temperature of gas, R = gas constant);

$$P\,V = n\,R\,T$$

$$\frac{PM}{d_g} = \frac{MRT}{m}$$

$$P = \frac{d_g}{m}RT \qquad (2)$$

Substituting equation 1 in equation 2 we obtain;

$$P = \frac{gt^2}{2} \cdot \frac{d_w}{2h+gt^2} \cdot \frac{RT}{m}$$

The temperature of the gas is assumed to be the same as the room temperature. The method described above can be used even if the gas filled balloon is suspended in a denser gas (eg helium filled balloon in air).

Case 2 : Resistance of liquid taken into account

This part of the theory has an extra level of complexity to it and it will be shown that an expression for the acceleration is only obtained in a transcendental form. Therefore a modification of the experimental method is required. We start by looking at the forces acting on the balloon as it travels through the liquid. We obtain,

$$M\,a = U - Mg - \frac{v^2 d_w AC}{2} \qquad (3)$$

The last term in equation 3 is the drag or resistance force experienced by the balloon as a result of its motion through the liquid. This expression for the drag is known as the drag equation[1] where 'A' is the cross sectional area of the balloon (which is taken as circular) and 'C' is the drag coefficient (0.47). It must be noted that the drag is proportional to the square of the velocity and therefore when the balloon is stationary the drag is equal to zero. This fact is used to modify the experimental setup.

The balloon is immersed fully in the liquid, but this time a sensitive spring balance is inverted and attached to the base of the balloon. The reading obtained would be equal to the upward force exerted on the balloon when it is stationary or just before it is released. Equation 3 then reduces to;

$$M\,a = U - M\,g \qquad (4)$$

Substituting the volume and respective densities into the above equation we get;

$$V\,d_g\,a = V\,d_w\,g - V\,d_g\,g$$

This simplifies to;

$$a = \frac{d_w}{d_g} - 1 \;\; g$$

As this is the acceleration of the balloon, the force acting on the balloon which can read off the spring balance is;

$$F = Vd_g \;\; \frac{d_w}{d_g} - 1 \;\; g$$

From this equation, an expression for d_g can be obtained as;

$$d_g = d_w - \frac{F}{Vg}$$

This expression for the density of the gas can be substituted into equation 2 to obtain an expression for the pressure of the gas in the balloon. ie

$$P = d_w - \frac{F}{Vg} \frac{RT}{m} \qquad (5)$$

Dynamics of the Balloon with Resistive Forces:

The equation of motion of the balloon is given by equation 3;

$$M\,a = U - Mg - \frac{v^2 d_w AC}{2}$$

Replacing the mass terms with the product of volume and the respective densities and expressing the volume of the balloon in terms of its cross sectional area (A), subsequent rearrangement results in;

$$a = \frac{d_w}{d_g} - 1 \;\; g - \frac{3v^2 C d_w}{8rd_g}$$

where 'r' is the radius of the spherical balloon. As acceleration is defined as the rate of change of velocity, the above equation can be simplified to give;

$$\frac{dv}{dt} = \frac{d_w}{d_g} - 1 \;\; g - \frac{3Cd_w}{8rd_g} \;\; v^2 \qquad (6)$$

This differential equation is of the form;

$$\frac{dv}{dt} = A - Bv^2 \qquad (7)$$

Where $A = \frac{d_w}{d_g} - 1 \;\; g$ and $B = \frac{3Cd_w}{8rd_g}$.

If we define $p^2 = \frac{A}{B}$ and given that v = 0 at t = 0, equation 7 can be solved by the method of partial fractions to give;

$$v = \frac{1 - e^{-2Bpt}}{1 + e^{-2Bpt}} \;\; p \qquad (8)$$

This expression gives the velocity of the balloon at any instant of time. Differentiating this equation with respect to time we obtain the acceleration;

$$a = \frac{2Bp^2 e^{-2Bpt}}{1 + e^{-2Bpt}} - \frac{2Bp^2 e^{-2Bpt} \; 1 - e^{-2Bpt}}{1 + e^{-2Bpt} \; ^2} \qquad (9)$$

The terminal velocity is the constant velocity acquired by the balloon when the resistive forces equal the upward acting force on the balloon. The terminal velocity can be obtained from equation 8 by taking the limit of the expression as 't' tends to infinity. This would give;

$$v_{terminal} = p$$

From the definition of p, we obtain;

$$v_{terminal} = \sqrt{\frac{8rg\,(d_w - d_g)}{3Cd_w}}$$

Taking the limit of equation 9 as 't' tends to infinity we note that

$$a = 0$$

This is accordance with the balloon acquiring a 'constant' terminal velocity.

Conclusion:

This paper describes a simple experimental setup that can be easily replicated in the classroom for determining the pressure of a gas inside a balloon. The experiment is described and adapted for when resistive forces are taken into account. In addition, we work out the dynamics of the balloon as it moves through the resistive medium and obtain expressions for its velocity, acceleration and terminal velocity. Since the theory of the method described ties together seemingly disparate areas of physics such as floatation, kinematics and gas laws, it can be used to illustrate to students, the wide applicability of physics and how it can be used to solve problems with very basic equipment. This could potentially stimulate interest among students and promote learning.

Reference

1. Batchelor, G.K. (1967). *An Introduction to Fluid Dynamics*. Cambridge University Press. ISBN 0521663962.