Vol 30 No. 1 Jan - Mar 2014 ISSN 0970-5953

INDIAN ASSOCIATION OF PHYSICS TEACHERS
IAPT

PHYSICS EDUCATION

Aurora

Pekka Parviainen

www.physedu.in

Volume 30, Number 1

In this Issue

EDITORIAL

It is a pleasure to publish issue 30.1 of Physics Education. This issue deals with a wide range of problems, from wave packet propagation to physics of Gulli- Danda. It has papers on Clausis- Mossotti equation, solar flares and on laboratory issues involving thin films and optics. A paper developing some quantitative measures of students' involvement in a laboratory may be interesting in the context of educational methods and their evaluation. I think it is much more interesting to read these papers than following any description of them.

I wish you a happy reading!

Pramod S. Joag.
Chief Editor, Physics Education
Chief-editor@physedu.in,
pramod@physics.unipune.ac.in

The Time evolution of a Square Wave Packet and a Triangular Wave Packet

Simon Dahal[1], Sai Smurti Samantaray[1] and B.A. Kagali[2]

[1]Department of Physics
Sri BhagawanMahaveer Jain Center for Post-Graduate Studies
Bangalore 560001, India
dahal.simon@gmail.com
smruti.samantaray89@gmail.com
[2]Department of Physics
Bangalore University
Bangalore 560001, India
bakagali@gmail.com

(Submitted: 31-08-13)

Abstract

In this article, we discuss the time evolution of a square wave packet and a triangular wave packet. The approach followed in this study is to express a square wave packet and a triangular wave packet as a sum of several Gaussian wave packets. Specifically, the time evolution of a square wave packet has been derived here with three and five Gaussian wave packets; then the time evolution of a triangular wave packet has been derived with three Gaussian wave packets. Their evolution with time has been plotted using MatLab™ over appropriately chosen time intervals. The results are compared with those of a Gaussian wave packet.

Keywords:- Wave packet, Schrödinger equation, Gaussian wave packet, Triangular Wave Packet, Time Evolution, MatLab™.

1.Introduction

Wave packets are superposition of plane waves used in representing a particle. According tode Broglie's matter-waves hypothesis, material particles such as photons and electrons exhibit wave nature and show wave phenomena such as interference and diffraction. For a localized particle, the superposition of many plane waves results in a function called the wave function ψ. The wave packets are decomposed by Fourier Transformation and their time evolution is found which is of physical interest. In this article, the time evolutions of non-Gaussian wave packets such as the ones mentioned in the abstract are found. The time evolutions of square wave packet and the triangular wave packet are of interest as they are often encountered in wave analyses. Using Green's function approach, Mita (2007) shows that the probability amplitude of any non-Gaussian wave packet approximately becomes a Gaussian as it disperses [1].Here we obtain the same result using a simpler approach of approximating a square wave packet and a triangular wave packet as a sum of several Gaussian wave packets. Mita (2007) points out the following advantages of using a Gaussian wave packet:

a) A Gaussian function is easy to analyze in closed form

b) The Fourier Transform of a Gaussian is also a Gaussian and

c) The Gaussian wave packet gives rise to a minimum uncertainty product at time t=0. [1]

In order to study the time evolution, we use the following form of the Gaussian wave packet as given by Greiner, W (2004) [2]. At time t=0,

$$\psi(x,0) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2} + ik_0x\right] \tag{1}$$

where the wave function depends on the position and time coordinates, σ is the standard deviation from the mean μ, the term $\frac{1}{\sigma\sqrt{2\pi}}$ is the amplitude of the wave packet and k_0 is the wave number.

For the sake of comparison, we write the expressions or the time evolution and the probability distribution of the Gaussian wave packet. We have also plotted the time evolution of the Gaussian wave packet using MatLab™ for the sake of comparison. The evolution of the Gaussian wave packet at time t is given by

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}\sigma\left(1+\frac{i\hbar t}{m\sigma^2}\right)^{\frac{1}{2}}} exp\left(\frac{2\sigma^2 ik_0x - \frac{i\hbar t k_0{}^2\sigma^2}{m} - \frac{2\hbar t k_0\mu}{m} - (x-\mu)^2}{2\sigma^2\left(1+\frac{i\hbar t}{m\sigma^2}\right)}\right) \tag{2}$$

The probability distribution is given by

$$|\,\psi(x,t)|^2 = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi\sigma^2} exp\left(-\frac{\left\{(x-\mu)-\frac{\hbar t k_0}{m}\right\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right) \tag{3}$$

where $\hbar = \frac{h}{2\pi}$; h is the Planck's constant. This is also a Gaussian distribution with width $\sigma\sqrt{1+\left(\frac{\hbar t}{m\sigma^2}\right)^2}$. We have assumed k_0 to be zero in order to simplify the calculation implying that the wave packet is at rest. The following values were used to plot the expression (3) in MatLab™:

Planck's constant $h = 6.6X10^{-34}$ Js

Mass of electron $m = 9.1X10^{-31}$kg

Mean value for the Gaussian wave μ = 0

Standard deviation of the wave σ = 0.5.

The following graph was obtained when the expression (3) was plotted with the above mentioned numerical values was plotted for different values of time t.

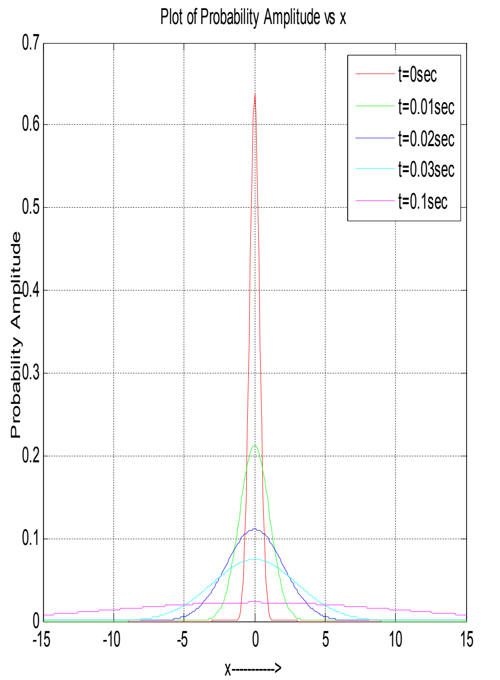


Figure 1: Time Evolution of a Gaussian wave function

2.Square Wave Packet

Consider a square wave packet with amplitude A and width Δx as shown below.

Figure 2: A square wave packet with amplitude A and width Δx

If we try to find its time evolution by the standard method, the integrations encountered are hard to solve. Hence, in order to simplify the calculations, the square wave packet is expressed as a sum of Gaussian wave packets of same width and amplitude as shown below. In order to find the time evolution of the approximated square wave packet, we find the time evolution of the system of Gaussian wave packets. Let us assume that the square wave packet is comprised of three Gaussian wave packets. Let their wave functions be ψ_1, ψ_2 and ψ_3; let their mean values be μ_1, μ_2 and μ_3 and let σ be the standard deviation. The forms of ψ at time t=0 and at a later time t for a single Gaussian wave packet is given by equations (1) and (2) respectively. Thus, the square wave packet is expressed as

$$\Psi(x,0) = \psi_1(x,0) + \psi_2(x,0) + \psi_3(x,0)$$

$$\Psi(x,0) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(x-\mu_1)^2}{2\sigma^2} + ik_0x\right] + \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(x-\mu_2)^2}{2\sigma^2} + ik_0x\right] + \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(x-\mu_3)^2}{2\sigma^2} + ik_0x\right] \quad (4)$$

and at a later time t>0, the square wave packet is expressed as

$$\Psi(x,t) = \psi_1(x,t) + \psi_2(x,t) + \psi_3(x,t)$$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}\sigma\left(1+\frac{i\hbar t}{m\sigma^2}\right)^{\frac{1}{2}}}\left\{exp\left(\frac{\begin{matrix}2\sigma^2 ik_0x - \frac{i\hbar t{k_0}^2\sigma^2}{m} - \frac{2\hbar tk_0\mu_1}{m}\\ -(x-\mu_1)^2\end{matrix}}{2\sigma^2\left(1+\frac{i\hbar t}{m\sigma^2}\right)}\right) + exp\left(\frac{\begin{matrix}2\sigma^2 ik_0x - \frac{i\hbar t{k_0}^2\sigma^2}{m} - \frac{2\hbar tk_0\mu_2}{m}\\ -(x-\mu_2)^2\end{matrix}}{2\sigma^2\left(1+\frac{i\hbar t}{m\sigma^2}\right)}\right) + exp\left(\frac{\begin{matrix}2\sigma^2 ik_0x - \frac{i\hbar t{k_0}^2\sigma^2}{m} - \frac{2\hbar tk_0\mu_3}{m}\\ -(x-\mu_3)^2\end{matrix}}{2\sigma^2\left(1+\frac{i\hbar t}{m\sigma^2}\right)}\right)\right\} \quad (5)$$

The probability distribution P(x,t) of the system of three Gaussian wave packets is given by

$$P(x,t) = |\Psi|^2 = |\psi_1 + \psi_2 + \psi_3|^2$$

$$|\Psi|^2 = {\psi_1}^2 + {\psi_2}^2 + {\psi_3}^2 + 2Re(\psi_1{\psi_2}^*) + 2Re(\psi_1{\psi_3}^*) + 2Re(\psi_2{\psi_3}^*) \quad (6)$$

where the asterisk indicates complex conjugate. The probability distribution for a single Gaussian wave packet is given by equation (3). Therefore, we can write for ${\psi_1}^2$, ${\psi_2}^2$ and ${\psi_3}^2$ in equation (6) as

$$\psi_1{}^2 = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi\sigma^2} exp\left(-\frac{\left\{(x-\mu_1)-\frac{\hbar t k_0}{m}\right\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)$$

$$\psi_2{}^2 = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi\sigma^2} exp\left(-\frac{\left\{(x-\mu_2)-\frac{\hbar t k_0}{m}\right\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)$$

$$\psi_3{}^2 = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi\sigma^2} exp\left(-\frac{\left\{(x-\mu_3)-\frac{\hbar t k_0}{m}\right\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)$$

For the term $2Re(\psi_1\psi_2{}^*)$ in equation (6), we write

$$2Re(\psi_1\psi_2{}^*) = 2\frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi\sigma^2} Re\ exp\left\{\left(\frac{2\sigma^2 ik_0x - \frac{i\hbar t k_0{}^2\sigma^2}{m} - \frac{2\hbar t k_0\mu_1}{m} - (x-\mu_1)^2}{2\sigma^2\left(1+\frac{i\hbar t}{m\sigma^2}\right)}\right) + \left(\frac{-2\sigma^2 ik_0x + \frac{i\hbar t k_0{}^2\sigma^2}{m} - \frac{2\hbar t k_0\mu_2}{m} - (x-\mu_2)^2}{2\sigma^2\left(1-\frac{i\hbar t}{m\sigma^2}\right)}\right)\right\}$$

Simplifying

$$2Re(\psi_1\psi_2{}^*) = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{\pi\sigma^2} exp\left\{\left(-\frac{\left((x-\mu_1)-\frac{\hbar k_0 t}{m}\right)^2+\left((x-\mu_2)-\frac{\hbar k_0 t}{m}\right)^2}{2\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)\right\}$$

Similarly, we can get$2Re(\psi_1\psi_3{}^*)$and $2Re(\psi_2\psi_3{}^*)$ as

$$2Re(\psi_1\psi_3{}^*) = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{\pi\sigma^2} exp\left\{\left(-\frac{\left((x-\mu_1)-\frac{\hbar k_0 t}{m}\right)^2+\left((x-\mu_3)-\frac{\hbar k_0 t}{m}\right)^2}{2\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)\right\}$$

and,

$$2Re(\psi_2\psi_3^{\ *}) = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{\pi\sigma^2} exp\left\{\left(-\frac{\left((x-\mu_2)-\frac{\hbar k_0 t}{m}\right)^2+\left((x-\mu_3)-\frac{\hbar k_0 t}{m}\right)^2}{2\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)\right\}$$

We assume k_0 to be zero and use the determined forms of the terms in the LHS of equation (6) and rewrite it in the final form as

$$|\Psi|^2 = \frac{\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)^{-\frac{1}{2}}}{\pi\sigma^2}\left[\frac{1}{2}\left\{exp\left(-\frac{\{(x-\mu_1)\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)+exp\left(-\frac{\{(x-\mu_2)\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)+exp\left(-\frac{\{(x-\mu_3)\}^2}{\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)\right\}+ exp\left(-\frac{(x-\mu_1)^2+(x-\mu_2)^2}{2\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)+exp\left(-\frac{(x-\mu_1)^2+(x-\mu_3)^2}{2\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)+exp\left(-\frac{(x-\mu_2)^2+(x-\mu_3)^2}{2\sigma^2\left(1+\left(\frac{\hbar t}{m\sigma^2}\right)^2\right)}\right)\right] \quad (7)$$

For the estimating with three Gaussian wave packets, a square wave function with some arbitrary amplitude within x=0 to x=2 and zero elsewhere was estimated. The Gaussian wave packets had a full width at half maxima equal to $\sigma = 2/10$. The three Gaussian wave packets had mean values at $\mu_1 = 1/3$, $\mu_2 = 1$ and $\mu_3 = 10/6$. For the purpose of approximation, a Gaussian wave packet of the form given in equation (1) was used with $k_0 = 0$. The plot thus obtained is as below

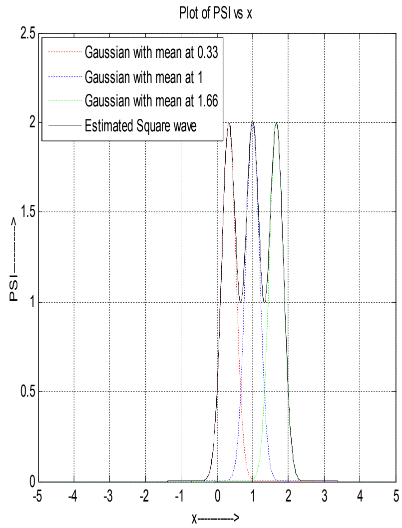


Figure 3: Approximation of a Square wave packet by three Gaussian wave packets

Now, with the same values of mean and standard deviation, the probability distribution of the approximated square wave packet given by equation (7) is plotted against x for different values of time'ť'. Here, again we use the same values of m and $\hbar$ as in section**1.** The plot obtained is as below.

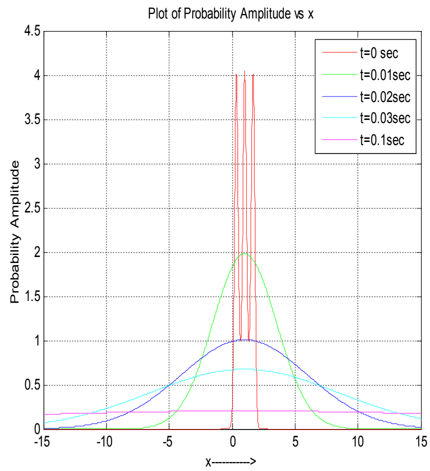


Figure 4: Time Evolution of a Square Wave packet approximated by three Gaussian wave packets

Thus, we see that as the square packet evolves with time, it spreads and approximately becomes a Gaussian.

The square wave packet was also approximated by five Gaussian wave packets with standard deviation $\sigma = 1/5$. The mean values for the Gaussian wave packets were taken to be $\mu_1 = 2/15$, $\mu_2 = 17/30$, $\mu_3 = 1$, $\mu_4 = 43/30$ and $\mu_5 = 28/15$. The width of the square wave packet was fixed to be a=2 and then the interval was divided into 5 parts a/15, 17a/60, a/2, 43a/60 and 14a/15. The resulting figure is shown below.

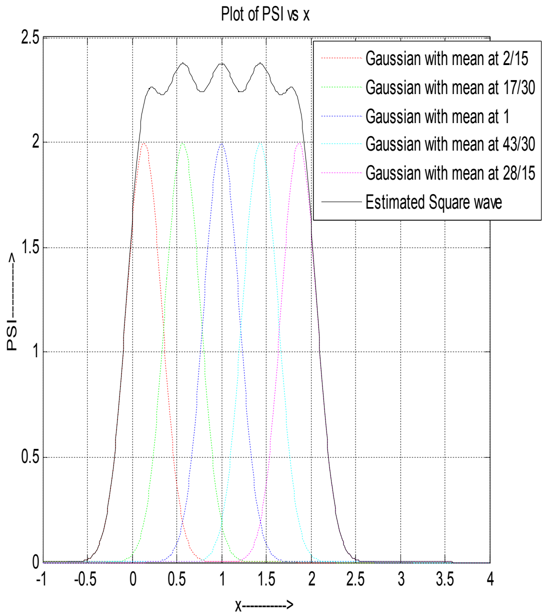


Figure 5: Estimation of Square wave packet by five Gaussian wave packets

The probability distribution of the Gaussian approximation of the square wave packet above is then plotted and is shown below.

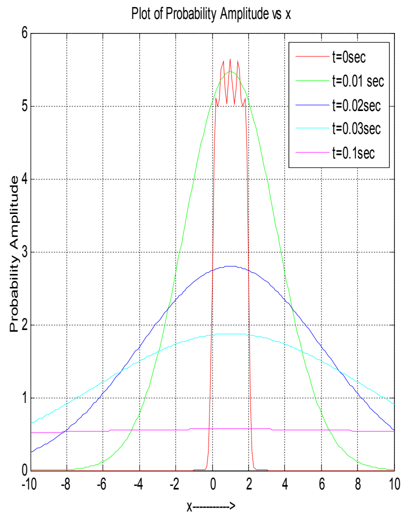


Figure 6: Time evolution of Square wave packet with five Gaussian wave packets

We note that the square wave packet estimated by five Gaussian wave packets gives a better approximation. See section **4** for a detailed discussion.

3. Triangular Wave Packet

In this section, we discuss the time evolution of a triangular wave packet expressed as a sum of Gaussian wave packets. Consider a triangular wave packet which is comprised of three Gaussian wave packets as shown below. Let the wave functions of the three Gaussian wave packets be ψ_1, ψ_2 and ψ_3 and let their mean values be μ_1, μ_2 and μ_3. Let the standard deviations of the three wave functions be σ_1 and σ_2. Here the two Gaussians on either side of the central Gaussian wave have the same standard deviation. The time evolution of a single Gaussian wave packet is given by equation (2). At time t=0, the wave function of the system of triangular wave packet resembles the form of equation (4). At a later time t>0, the wave function of the system is expressed as

$$\Psi(x,t) = \psi_1(x,t) + \psi_2(x,t) + \psi_3(x,t)$$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}\sigma_1\left(1+\frac{i\hbar t}{m{\sigma_1}^2}\right)^{\frac{1}{2}}}\left\{exp\left(\frac{2{\sigma_1}^2 ik_0x - \frac{i\hbar t{k_0}^2{\sigma_1}^2}{m} - \frac{2\hbar tk_0\mu_1}{m} - (x-\mu_1)^2}{2{\sigma_1}^2\left(1+\frac{i\hbar t}{m{\sigma_1}^2}\right)}\right)\right.$$

$$\left. + exp\left(\frac{2{\sigma_1}^2 ik_0x - \frac{i\hbar t{k_0}^2{\sigma_1}^2}{m} - \frac{2\hbar tk_0\mu_1}{m} - (x-\mu_3)^2}{2{\sigma_1}^2\left(1+\frac{i\hbar t}{m{\sigma_1}^2}\right)}\right)\right\}$$

$$+ \frac{1}{\sqrt{2\pi}\sigma_2\left(1+\frac{i\hbar t}{m{\sigma_2}^2}\right)^{\frac{1}{2}}} exp\left(\frac{2{\sigma_2}^2 ik_0x - \frac{i\hbar t{k_0}^2{\sigma_2}^2}{m} - \frac{2\hbar tk_0\mu_2}{m} - (x-\mu_2)^2}{2{\sigma_2}^2\left(1+\frac{i\hbar t}{m{\sigma_2}^2}\right)}\right)$$

The probability distribution of the system is given by

$$P(x,t) = |\Psi|^2 = |\psi_1 + \psi_2 + \psi_3|^2$$

$$|\Psi|^2 = {\psi_1}^2 + {\psi_2}^2 + {\psi_3}^2 + 2Re(\psi_1{\psi_2}^*) + 2Re(\psi_1{\psi_3}^*) + 2Re(\psi_2{\psi_3}^*) \tag{8}$$

The first three terms on the LHS of the above equation are

$${\psi_1}^2 = \frac{\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi{\sigma_1}^2} exp\left(-\frac{\left\{(x-\mu_1) - \frac{\hbar tk_0}{m}\right\}^2}{{\sigma_1}^2\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)}\right)$$

$${\psi_2}^2 = \frac{\left(1+\left(\frac{\hbar t}{m{\sigma_2}^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi{\sigma_2}^2} exp\left(-\frac{\left\{(x-\mu_2) - \frac{\hbar tk_0}{m}\right\}^2}{{\sigma_2}^2\left(1+\left(\frac{\hbar t}{m{\sigma_2}^2}\right)^2\right)}\right)$$

$$\psi_3{}^2 = \frac{\left(1+\left(\frac{\hbar t}{m\sigma_1{}^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi\sigma_1{}^2} exp\left(-\frac{\left\{(x-\mu_3)-\frac{\hbar t k_0}{m}\right\}^2}{\sigma_1{}^2\left(1+\left(\frac{\hbar t}{m\sigma_1{}^2}\right)^2\right)}\right)$$

In the above equation for $\psi_3{}^2$, σ_1 appears as we have assumed that the Gaussians on either side of the central Gaussian gave the same standard deviation i.e. $\sigma_1 = \sigma_3$. Also, the amplitude of ψ_2 depends on the slope of the triangle. The remaining terms in equation (8) are written as

$$2Re(\psi_1\psi_2{}^*)$$

$$= \frac{\left(1+\frac{\hbar^2t^2}{m^2\sigma_1{}^2\sigma_2{}^2}\right)^{-\frac{1}{2}}}{\pi\sigma_1{}^2\sigma_2{}^2} exp\left[-\frac{(\sigma_1{}^2+\sigma_2{}^2)\left(x-\frac{\hbar t k_0}{m}\right)^2 - 2(\mu_2\sigma_1{}^2+\mu_1\sigma_2{}^2)\left(\frac{\hbar t k_0}{m}-x\right)+\mu_2\sigma_1{}^2+\mu_1\sigma_2{}^2}{2\sigma_1{}^2\sigma_2{}^2\left(1+\frac{\hbar^2t^2}{m^2\sigma_1{}^2\sigma_2{}^2}\right)}\right]$$

And similarly

$$2Re(\psi_1\psi_3{}^*)$$

$$= \frac{\left(1+\frac{\hbar^2t^2}{m^2\sigma_1{}^2\sigma_3{}^2}\right)^{-\frac{1}{2}}}{\pi\sigma_1{}^2\sigma_3{}^2} exp\left[-\frac{(\sigma_1{}^2+\sigma_3{}^2)\left(x-\frac{\hbar t k_0}{m}\right)^2 - 2(\mu_3\sigma_1{}^2+\mu_1\sigma_3{}^2)\left(\frac{\hbar t k_0}{m}-x\right)+\mu_3\sigma_1{}^2+\mu_1\sigma_3{}^2}{2\sigma_1{}^2\sigma_3{}^2\left(1+\frac{\hbar^2t^2}{m^2\sigma_1{}^2\sigma_3{}^2}\right)}\right]$$

$$2Re(\psi_2\psi_3{}^*)$$

$$= \frac{\left(1+\frac{\hbar^2t^2}{m^2\sigma_3{}^2\sigma_2{}^2}\right)^{-\frac{1}{2}}}{\pi\sigma_3{}^2\sigma_2{}^2} exp\left[-\frac{(\sigma_3{}^2+\sigma_2{}^2)\left(x-\frac{\hbar t k_0}{m}\right)^2 - 2(\mu_2\sigma_3{}^2+\mu_3\sigma_2{}^2)\left(\frac{\hbar t k_0}{m}-x\right)+\mu_2\sigma_3{}^2+\mu_3\sigma_2{}^2}{2\sigma_3{}^2\sigma_2{}^2\left(1+\frac{\hbar^2t^2}{m^2\sigma_3{}^2\sigma_2{}^2}\right)}\right]$$

Once again, we assume the value of k_0 to be zero and since we have also assumed $\sigma_1 = \sigma_3$, we rewrite equation (8) in the final form as

$$\Psi^2 = \frac{\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi{\sigma_1}^2} exp\left(-\frac{\{(x-\mu_1)\}^2}{{\sigma_1}^2\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)}\right)$$

$$+\frac{\left(1+\left(\frac{\hbar t}{m{\sigma_2}^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi{\sigma_2}^2} exp\left(-\frac{\{(x-\mu_2)\}^2}{{\sigma_2}^2\left(1+\left(\frac{\hbar t}{m{\sigma_2}^2}\right)^2\right)}\right)$$

$$+\frac{\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)^{-\frac{1}{2}}}{2\pi{\sigma_1}^2} exp\left(-\frac{\{(x-\mu_3)\}^2}{{\sigma_1}^2\left(1+\left(\frac{\hbar t}{m{\sigma_1}^2}\right)^2\right)}\right)$$

$$+\frac{\left(1+\frac{\hbar^2 t^2}{m^2{\sigma_1}^2{\sigma_2}^2}\right)^{-\frac{1}{2}}}{\pi{\sigma_1}^2{\sigma_2}^2} exp\left[-\frac{({\sigma_1}^2+{\sigma_2}^2)x^2+2(\mu_2{\sigma_1}^2+\mu_1{\sigma_2}^2)x+\mu_2{\sigma_1}^2+\mu_1{\sigma_2}^2}{2{\sigma_1}^2{\sigma_2}^2\left(1+\frac{\hbar^2 t^2}{m^2{\sigma_1}^2{\sigma_2}^2}\right)}\right]$$

$$+\frac{\left(1+\frac{\hbar^2 t^2}{m^2{\sigma_1}^2{\sigma_1}^2}\right)^{-\frac{1}{2}}}{\pi{\sigma_1}^2{\sigma_1}^2} exp\left[-\frac{({\sigma_1}^2+{\sigma_1}^2)x^2+2(\mu_3{\sigma_1}^2+\mu_1{\sigma_1}^2)x+\mu_3{\sigma_1}^2+\mu_1{\sigma_1}^2}{2{\sigma_1}^2{\sigma_1}^2\left(1+\frac{\hbar^2 t^2}{m^2{\sigma_1}^2{\sigma_1}^2}\right)}\right]$$

$$+\frac{\left(1+\frac{\hbar^2 t^2}{m^2{\sigma_1}^2{\sigma_2}^2}\right)^{-\frac{1}{2}}}{\pi{\sigma_1}^2{\sigma_2}^2} exp\left[-\frac{({\sigma_1}^2+{\sigma_2}^2)x^2+2(\mu_2{\sigma_1}^2+\mu_3{\sigma_2}^2)x+\mu_2{\sigma_1}^2+\mu_3{\sigma_2}^2}{2{\sigma_1}^2{\sigma_2}^2\left(1+\frac{\hbar^2 t^2}{m^2{\sigma_1}^2{\sigma_2}^2}\right)}\right]$$

In our numerical analysis, the standard deviation of the central Gaussian wave packet i.e. σ_2 was chosen to be 0.5 and σ_1 and σ_3 were chosen to be 1. The mean values were chosen to be -5/2.5, 0 and 5/2.5. The probability distribution of the approximated triangular wave packet is plotted against x for different values of time t. The values of m and $\hbar$ were the same as the ones used in sections **1** and **3.**

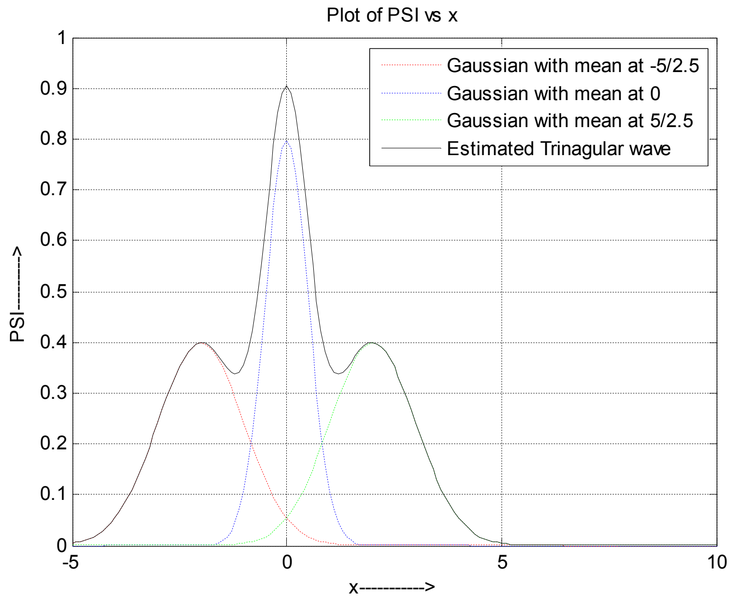


Figure 7: Estimation of Triangular wave packet by three Gaussian wave packets

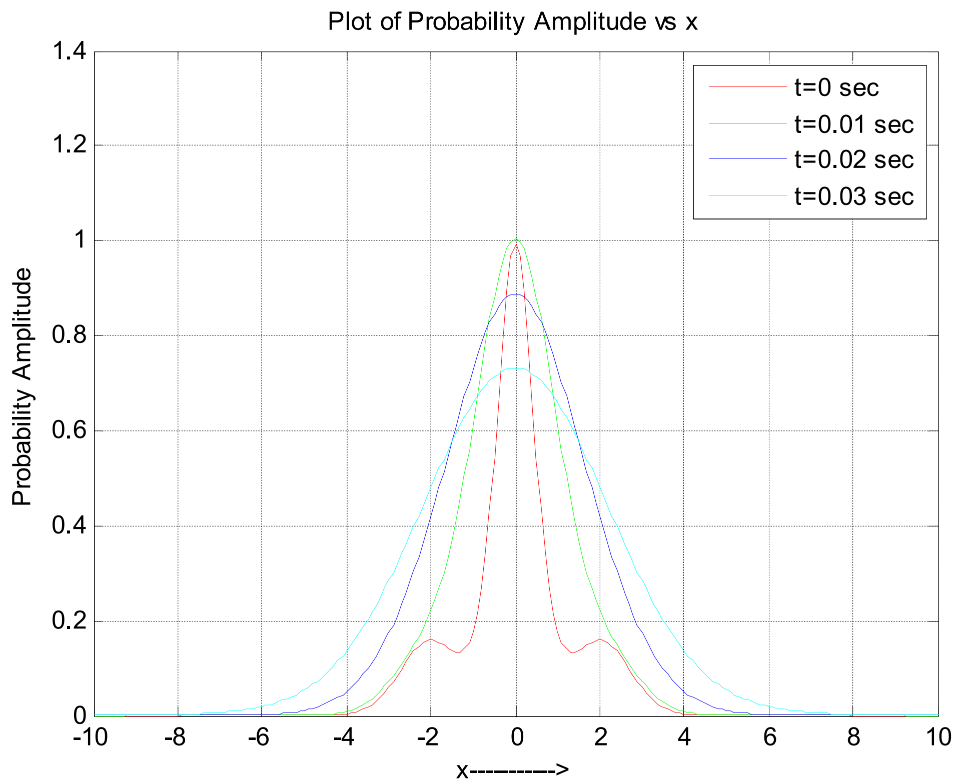


Figure 8: Time Evolution of Triangular wave approximated by 3 Gaussians

4. Results and Discussion

When the Square wave packet was approximated with three Gaussians, its amplitude was found to be 2 (fig. 4). Here the estimated square wave consisted of dips where the overlapping of Gaussian waves was not as significant. This is due to the intermediate terms in equation (7). Its probability amplitude was doubled, i.e. approx. 4 at time t=0, which is as expected. The probability amplitude at time t=0 consisted of some irregularities in the peak. With time, as the Gaussian waves evolved, so did the estimated square wave and thus the probability distribution of the square wave became smoother, broader and assumed an almost Gaussian shape which is clearly visible in fig. 5. In the case of Square wave approximated with five Gaussians, the dip in the final wave form reduced considerably due to significant overlapping thus estimating the square wave packet better than the one with the three Gaussian wave packets. Thus we see that the accuracy increases when the number of wave packets is increased. The probability distributions at t=0 and at later times are assumed to have the same behavior as mentioned earlier.

Similar results were obtained in the case of a Triangular wave packet being approximated by three Gaussian wave packets.

In the method adopted, care must be taken while choosing the standard deviation and mean values for the approximating Gaussian waves. The advantage of this method is that it is applicable for any wave packet in principle. The number of iterations can be improved by computer programming since there is an increase in the number of wave functions and also the number of times they are added

5. Acknowledgements

We thank the library facilities provided by Sri BhagawanMahaveer Jain Center for Post-Graduate Studies. We thank Mr. Krishna Kumar Kowshik in aiding us with the write-up of this paper.

References :

[1] KatsunoriMita (October 2007) Dispersion of no-Gaussian free particle wave packets. *Am. J. phys.* Vol. 75, No.10, 950

[2] GRIENER,W. (2004).*Quantum Mechanics-An Introduction*. Springer-Verlag, fourth edition, first Indian reprint edn.2,167.

ABRAMOWITZ.M.&STEGUN,I.A(1965). *Handbook of Mathematical functions and formulas, graphs and mathematical tables*. Dover, New York. 31

***

Effect of Solar Flare and Coronal Mass Ejection in our Earth

Lalan Prasad, Beena Bhatt and Suman Garia

Department of Physics

M.B. Govt. P.G. College, Haldwani, Nainital - 263141, India

(*Submitted* 16-08-2013)

Abstract

As we know sun is a prime source of life for our earth so it is very important to know all activities of the sun and hazard occurred by it. Sun contains enormous amount of energy in the form of electromagnetic spectrum, particles, electrons, ions, gases, atoms and plasma. Time to time they emit tremendous amount of energy and matter into the interplanetary space which create space weather and affect the other planets in the interplanetary space. A number of phenomena are continuously occurring day and night which is the source of space weather such as prominence, solar flares, coronal mass ejection (CMEs). Sun continuously released energies, energetic particles and plasma in the interplanetary space in all the directions. Sun's atmosphere is divided into three main layer (1) photosphere (2) chromosphere (3) and corona. Many activities continuously occurring into these layers, as solar flare and CMEs are one of these. Although not all the solar flares are affecting our earth's atmosphere but only some of those whose intensities are high like M-class and X-class solar flares.

How Does the Solar flare and CME affect the Earth?

The Sun affects the Earth through two important mechanisms. The first mechanism is the sun's energy. The solar flare is a localized explosive release of energy, high energetic particles and protons that appears as a sudden, short lived brightening of an area in the chromosphere. Flares are rarely visible in white light that is emitted at the photospheric level but visible in EUV and X-ray wavelengths, i.e. emitted at the chromospheric level they display enormous release of energy and produce structural changes. In the radio region, (i.e.

at radio wavelengths) their effect is marked by various types of emissions. Solar flares release their energy mainly in the form of electromagnetic radiation and energetic particles. These X-ray and EUV waves travel at the speed of light, taking only 8 minutes to reach us here at Earth. The energy released in the process of solar flares increased the velocity and acceleration of solar wind which is affecting the space weather.

The second mechanism in which the Sun affects Earth is through the impact of matter from the Sun. Plasma, or matter in a state where electrons wander freely among the nuclei of the atoms, can also be ejected from the Sun during a solar disturbance. This "bundle of matter" is called a CME.

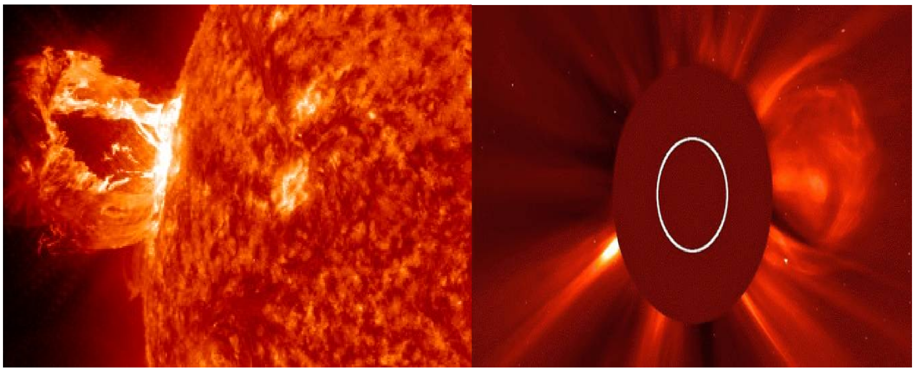


FIG. 1: Left:- solar flare, right:-Coronal Mass Ejection observed by LASCO/SOHO.

CMEs flow from the Sun at a speed of over 2 million kilometers per hour. It takes about 18 to 72 hours for a CME to reach us from the Sun. Since CMEs are large masses of ionic particles moving through interplanetary space, their energy is kinetic. The kinetic energy of a CME is around 10^{23} to 10^{24} Joule (Vourlidas et al., 2002).It has been shown that solar flare are not the necessarily drivers for CMEs (Kahler 1992). More powerful flares are generally associated with CMEs (Andrews 2003). CMEs do not necessarily need to correspond with flaring, although nearly all powerful (X or M class) flares have an associated CME (Andrews 2003, Yashiro et al. 2005). The larger flares tend to be associated with CMEs, but for even X-class flares about 10% of them lacked CME association (Gopalswamy et al. 2008). When a CME impacts the Earth's magnetosphere, it temporarily deforms the Earth's magnetic field, and inducing large electrical ground currents in Earth. Explosion powered by the sun's magnetic field (flares and CMEs) are the principal causes of space weather (Space Studies Board 2008).

Space weather affects our life in many ways such as they can change our environments, produce disturbances in our communication systems. As we know today we are totally depend on communication systems without it we cannot think about life. The source of communication system depends on satellites and earth's ionosphere where from high frequency wave propagation possible, both are influence by space weather. They may damage solar cell of satellites by solar flare protons, create plasma bubbles in earth's ionosphere affect astronaut safety, radio wave disturbance, signal scintillation, airline passenger radiation which goes through auroral region, electricity grid disruption, telecommunication cable disruption, earth current etc. (see figure 2.)

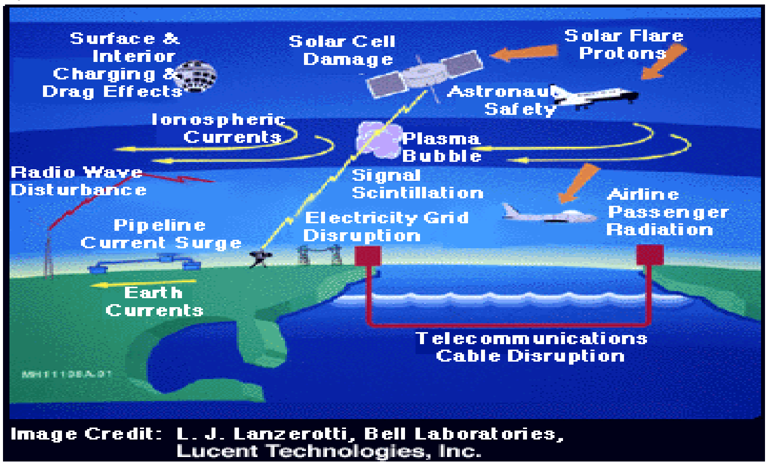


FIG. 2: space weather effect on our communication system

Here I try to explain some different types of hazards occurring by solar flare and CME on our earth.

Effect on magnetosphere

Our earth is a permanent magnet and their shield around the earth is termed as magnetosphere. The regions of magnetosphere shown in the figures in terms of their magnetic field lines originate from the northern hemisphere and terminality towards the southern hemisphere. Most of the solar wind speed is supersonic and superalfvenic. Solar wind exerts a pressure on earth's magnetosphere which compresses magnetic line of force as a result earth facing towards the sun developed a bow shock whereas on the other side also developed magnetotail.

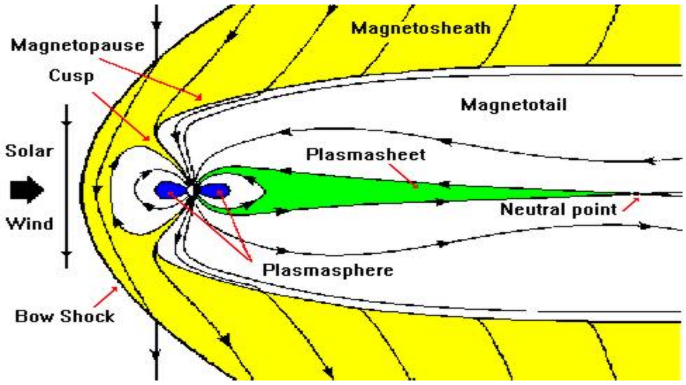


FIG. 3: Earth's Magnetosphere.

When the solar wind strikes on earth its kinetic energy converts into thermal energy. These energy flows behind the bow shock and make Magnetosheath. The area between the magnetosphere and magnetosheath is Magntopause (see figure 1). CME carry energetic particles, plasma and magnetic field of sun. When these CME pass near the earth the magnetic field of sun contained by CME is combined with magnetic field of earth. As a result both magnetic fields are joined together and this joining is called magnetic reconnection. This joining is very strong when field are antiparallel. This magnetic reconnection plays a key role for disturbance in earth's atmosphere. Plasma and other energetic particle inter through this reconnection and reach the earth's atmosphere and responsible for disturbances in ionosphere and communication media.

Effect on ionosphere and communication system

Ionosphere is the outermost layer of earth's atmosphere. It consists of several layers at different heights. At the day time it divided into D, E, F1 and F2 layer with different densities of ionization and at night F1 and F2 layer combined together and form a single layer F (see figure 4). As I said in my previous article (Prasad et al. 2006) each layer has its own properties, and the existence and number of layers change daily under the influence of the Sun. During the day, the ionosphere is heavily ionized by the Sun. During the night hours the cosmic rays dominate because there is no ionization caused by the Sun (which has set below the horizon).

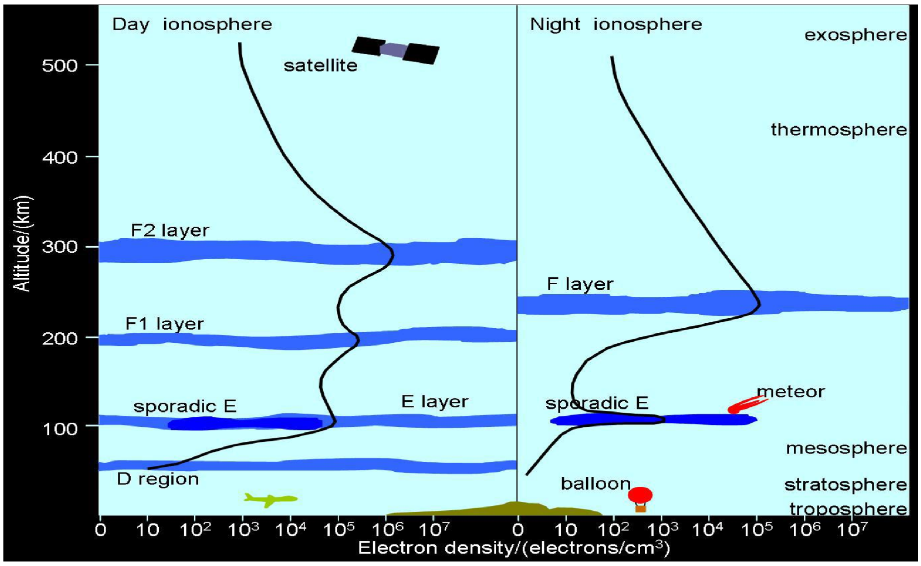


FIG. 4: The Earth's Ionosphere

Thus, there is a daily cycle associated with the ionizations. In addition to the daily fluctuations, activity by the Sun can cause dramatic sudden changes to the ionosphere. Wireless communications are used by military, police and other agencies in our India and other nations are also affected. The frequency used for wireless communication is kHz to MHz's. When energy from a solar flare or other disturbance reaches the Earth, the ionosphere becomes suddenly more ionized. Thus, changing the density and location of layers and free electrons in the ionosphere has a strong influence on the propagation of radio signals. Radio frequencies of very long wavelength (very low frequency or "VLF") "bounce" or reflect off these free electrons in the ionosphere thus, conveniently for us, allowing radio communication over the horizon and around our curved Earth. The strength of the received radio signal changes according to how much ionization has occurred and from which level of the ionosphere the VLF wave has "bounced" (Prasad et al. 2006).

Effect on Electric Grid

CMEs carry magnetic field of sun with him. When this magnetic fields move near the conductor such as a wire, a geomagnetically induced current (GIC) is produced in the conductor. The GIC is a direct current (DC). The main cause of GICs is the interaction of the geomagnetic field with the magnetic field carried by CMEs and magnetized solar wind. These GICs is the main cause of disturbance in power grid. Within the electric power system, GICs can cause transformers to operate in their nonlinear saturation range during half of the AC cycle. The consequence of half-cycle saturation includes distortions of the voltage pattern (reflected in the existence of harmonics to the primary frequency), heating within the transformers, or voltage-to-current phase shifts expressed as reactive power consumption in the system (Carolus et al. 2013). The most famous GIC event occurred in Hydro-Quebec power plant in Canada on March 13, 1989, the electrical supply was cut off to over 6 million people for 9 hours due to a huge geomagnetic storm (Kappenman & Albertson 1990).

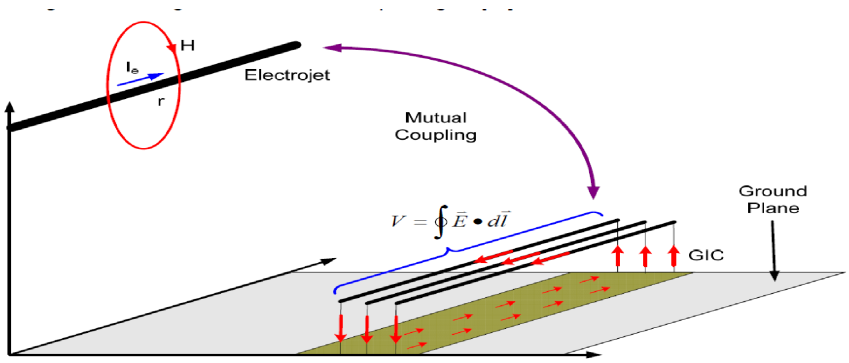


FIG. 5: Geomagnetic effects on power grid.

Effect on gas pipelines

They may also affect the gas pipelines. As we know gas pipelines are constructed by steels and coated by corrosion resistance coatings. The liquid or gases stored in it are under high pressure. For minimizing corrosions cathodic protection is used by keeping the negative potential with respect to the ground. GIC cause swings in the pipe-to-soil potential and increase the rate of corrosion during major GSs (Gummow 2002).

Effect on human body

The energetic particles can pass through the human body and causing biochemical damage. Due to the radiation risk increase the possibility of occurrence of cancer and some other deceases. Astronauts are mostly affected by this if they are not well protected.

Formation of aurora

FIG. 6: Aurora

When a charged particle emitted from the sun reached in earth's atmosphere they collides with gas particle of atmosphere as a result a coloured pattern is formed. Aurora is visible at high latitudes. The colour pattern is depending on flow of charged particles and magnetic field in it. Oxygen molecules produce green and red colours while nitrogen molecules produce purplish-red and blue colours pattern.

Acknowledgements

We are thankful to IUCAA, Pune for content page service and also thankful to ARIES, Nainital for providing library and computing facilities. Thankfully, Acknowledge the UGC, New Delhi for financial assistance.

References

[1] M. D. Andrews, solar phys., 218, 261 (2003)

[2] J. Carolus, Schrijver and sarah D. Mitchell, J.Space Weather Space Clim. 3(2013)

[3] R. A. Gummow, *GIC effects on pipeline corrosion and corrosion-control systems. J. Atmos,* Sol. Terr. Phys., 64(16), 1755–1764 (2002)

[4] N. Gopalswamy, S. Akiyama and S. Yashiro, Universal Heliophysical Processes, Proceeding IAU Symposium No. 257, 2008

[5] Fan-yiu HUNG, *Space Weather (Part II: Effects of Space Weather)*, (Source/Credits: Space Environment Center, Boulder, CO, National Oceanic and Atmospheric Administration, US Dept. of Commerce), 2004

[6] J. G. Kappenman, and V. D. Albertson, *Bracing for the geomagnetic storms*, IEEE Spectrum, March 1990, 27-33

[7] S. W. Kahler, ARAA, 30, 113 (1992)

[8] Prasad Lalan and Joshi V.K., *Physics education, solar flare and their effects*, Phy. Edu. 267 (2008)

[9] Space Studies Board, Severe space weather events-understanding societal and

economic impacts. National Academy Press, Washington, DC, USA, (2008)

[10] A. Vourlidas, D. Buzasi, R. A. Howaed and E. Esfandiari, *Mass and energy properties of LASCO CMEs, in solar variability: From Core to Outer Frontiers*, ESA SP-506, p. 9194, (2002)

[11] S. Yashiro, N. Gopalswamy, S. Akiyama, G. Michalek, and R. A. Howard, J. of Geophys. Rres., 110, A12S05 (2005)

Physics Education **Jan – Mar 2014**

On Magnetic analogue of Clausius-Mossotti equation

Ankita Niranjan and Bikash K. Padhi

Department of Physics, Indian Institute of Technology,
Delhi, New Delhi-110016, India

((Submitted 14-07-2013)

Abstract

In this paper the magnetic equivalent of the Clausius - Mossotti equation is re-derived for a modern reader. The equation relates the relative permeability of a diamagnetic substance to the magnetic atomic polarizability of the atoms. The historical background and importance of this equation and its relation to the Clausius-Mossotti equation is also discussed briefly.

1. Introduction

Due to the symmetric structure of the Maxwell equations there are several equations in electrostatics which have got their magnetic equivalents. For instance the electric and magnetic fields produced by an isolated electric charge and a hypothetical magnetic monopole, respectively, follow the inverse square law. Again the expression for the electric field and magnetic field produced by an electric dipole and a magnetic dipole, respectively, both vary according to the inverse cube law [1,2]. Motivated by such striking symmetries between the two, derivation of an equation which would be a magnetic analogue of the Clausius-Mossotti equation is carried here. This equation enables us to find the extent of magnetization of a diamagnetic substance in terms of a certain microscopic characteristic of it. The equation holds good only for diamagnetic substances.

Standard textbooks [1-3] on electromagnetism or solid state physics often discuss the derivation and applications of the standard Clausius-Mossotti equation. However, none seem to extend the idea to the magnetic case. However, it is the 'magnetic analogue' which was developed first by Poisson and then people like Faraday, Mossotti and Clausius extended this idea to the electric case. So in the last section we discuss this *misunderstood* history of this pair of equations in more detail.

The derivation presented here is much simpler for a modern reader to follow as compared to that done in the original works of Poisson and others in the beginning of 19th century. It is done simply by creating a model which is the magnetic equivalent of the electrostatic model, usually used to derive the Clausius-Mossotti equation [1-3]. For convenience the derivation is divided into two sections. In the first section the average value of the microscopic magnetic field produced by a tiny ideal magnetic dipole over a sphere of radius R is calculated. Then the expression for this average field is used in the second section to get to the desired equation. The reader may note that the derivation of average magnetic field over a sphere is somewhat unconventional.

2. Average magnetic field due to a dipole over a sphere

The average electric field inside a sphere ($\langle \mathbf{E} \rangle_{\text{sphere}}$) of radius R due to a tiny dipole of dipole moment $\mathbf{p}$ present *anywhere* inside it is given by [4, 5]

$$\langle \mathbf{E} \rangle_{\text{sphere}} = \frac{-\mathbf{p}}{4\pi\varepsilon_o R^3}. \qquad (1)$$

This may be deduced as follows. Consider a fictitious sphere of radius R centered at the origin. A tiny charge q is located at position $\mathbf{r}_0$ inside the sphere. The average field produced by it over the volume of the sphere, by using Coulomb's law is

$$\langle \mathbf{E} \rangle_{\text{sphere}} = \frac{q}{4\pi\varepsilon_o V} \iiint_{\text{sphere}} \frac{\mathbf{r}-\mathbf{r}_o}{|\mathbf{r}-\mathbf{r}_o|^3}\, dV. \qquad (2)$$

V is the volume of the sphere.

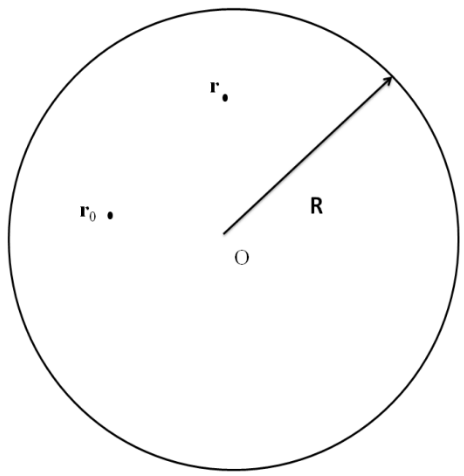


FIG.1: Average field over a sphere due to a point charge q at $\mathbf{r}_0$

Now the electric field at the point $\mathbf{r}_0$ (cf. Fig.1) due to a uniformly charged sphere, centered at the origin, of charge density ρ is

$$\mathbf{E}_{\text{sphere}} = \frac{\rho}{4\pi\varepsilon_o} \iiint_{\text{sphere}} \frac{\mathbf{r}_o-\mathbf{r}}{|\mathbf{r}-\mathbf{r}_o|^3}\, dV. \qquad (3)$$

A comparison of (2) and (3) reveals that the average field due to a charge q at $\mathbf{r}_o$ equals the field at $\mathbf{r}_o$ due to a uniformly charged sphere of charge density $\rho = -q/V$. The latter, however, using Gauss' divergence theorem turns out to be

$$\mathbf{E} = \frac{\rho}{3\varepsilon_o}\mathbf{r}_o. \qquad (4)$$

Thus,

$$\langle \mathbf{E} \rangle_{\text{sphere}} = -\frac{q}{4\pi\varepsilon_o R^3}\mathbf{r}_o. \qquad (5)$$

Now if we place another charge –q at $\mathbf{r}_o - \mathbf{d}$ inside the sphere, then the net average field inside the sphere due to the pair of charges (of dipole moment $\mathbf{p}=q\mathbf{d}$) can be obtained by using (1).

One may recall that the electric field at position $\mathbf{r}$ due to an ideal electric dipole of moment $\mathbf{p}$ located at the origin is [4] (in SI unit)

$$\mathbf{E}_{\text{dip}} = \frac{1}{4\pi\varepsilon_o r^3}\left(3(\mathbf{p}.\hat{r})\hat{r}-\mathbf{p}\right) - \frac{\mathbf{p}}{3\varepsilon_o}\delta(\mathbf{r}). \qquad (6)$$

Similarly the expression for magnetic field in case of a magnetic dipole $\mathbf{m}$ located at the origin is given by [5] (in SI unit)

$$\mathbf{B}_{\text{dip}} = \frac{\mu_o}{4\pi r^3}\left(3(\mathbf{m}.\hat{r})\hat{r} - \mathbf{m}\right) + \frac{2\mu_o\mathbf{m}}{3}\delta(\mathbf{r}). \qquad (7)$$

The delta function $\delta(\mathbf{r})$ terms that appear in equations (6) and (7) are of great importance (6, 7). If one carries out the volume average of $\mathbf{E}_{\text{dip}}$ over a sphere of radius R, the contribution due to the first term in (6) vanishes and one obtains the value of $\langle \mathbf{E}_{\text{dip}} \rangle$ the same as (1), which is due to the delta function term only. To put it mathematically,

$$\iiint_{\text{sphere}} \frac{\left(3(\mathbf{p}.\hat{r})\hat{r}-\mathbf{p}\right)}{r^3}\, dV = 0, \qquad (8)$$

and,

$$\langle \mathbf{E}_{dip} \rangle = \frac{1}{V} \iiint_{\text{sphere}} \frac{-\mathbf{p}}{3\varepsilon_o} \delta(\mathbf{r})\, dV$$

$$= \frac{-\mathbf{p}}{\left(\frac{4\pi R^3}{3}\right)3\varepsilon_o} = \frac{-\mathbf{p}}{4\pi\varepsilon_o R^3}. \qquad (9)$$

One can note that the coefficient of the delta term in (6) is nothing but the average electric field over a sphere multiplied by the volume of the sphere, or it is the volume integral of the electric field of the dipole over a sphere of any size.

For the magnetic case, a similar derivation of (8) can be carried out by using tiny circulating current loops in place of charge distribution.

Thus, the average value of the magnetic field produced by a magnetic dipole, inside a sphere of radius R can be calculated as,

$$\langle \mathbf{B} \rangle_{\text{sphere}} = \frac{1}{V} \iiint_{\text{sphere}} \mathbf{B}_{dip}(\mathbf{d})\, dV'. \qquad (10)$$

Invoking equation (7) and by using (8) with **p** replaced by **m**, we obtain

$$\langle \mathbf{B} \rangle_{\text{sphere}} = \frac{\mu_o}{2\pi R^3}\mathbf{m}. \qquad (11)$$

3. Derivation of the analogue equation

When an external magnetic field **B** is applied to a diamagnetic substance, the electronic orbits of the atoms get modified and they acquire a magnetic moment $\Delta\mathbf{m}$. Classical [8] or semi classical [9] derivations of induced magnetization in a diamagnetic substance upon application of an external magnetic field **B** give us a relation which tells us that induced dipole moment $\Delta\mathbf{m}$ and applied magnetic field **B** are linearly proportional, provided the field is not too strong.

$$\Delta\mathbf{m} = \alpha_m \mathbf{B}. \qquad (12)$$

If we consider the diamagnetic substance to be spatially *homogeneous* then a uniform external field applied on it can be assumed to magnetize every atomic dipole by the same amount. Thus a proportionality constant α_m can assigned to the *entire* diamagnetic substance, which may be called ‘magnetic polarizibility’, in analogy with electric polarizibility. Another advantage of spatial homogeneity is - a uniform applied field **B** will generate a uniform dipole moment per unit volume, **M** where **M** and **B** are related to **H** and μ_r by the macroscopic definitions (for linear materials)

$$\mathbf{M} = \chi_m \mathbf{H}. \qquad (13)$$

Or,

$$\mathbf{M} = \frac{\chi_m}{\mu_o(1+\chi_m)}\mathbf{B}. \qquad (14)$$

χ_m is the magnetic susceptibility of the diamagnetic substance. Microscopically, the magnetization vector can expressed as the cumulative sum of all the tiny atomic magnetic dipoles

$$\mathbf{M} = \sum_{i=1}^{N} \Delta\mathbf{m}_i. \qquad (15)$$

Here N is the number density of atoms in the substance (hence an integer). Eq. (15) can be rewritten as

$$\mathbf{M} = \sum_{i=1}^{N} \Delta\mathbf{m}_i = N\Delta\mathbf{m}_i \qquad (16)$$

From (12), (14) and (16) one may be tempted to conclude that $\frac{\chi_m}{\mu_o(1+\chi_m)} = N\,\alpha_m$.

However, the field vector **B** appearing in (12) is not the same field vector that appears in equation (14). The **B** appearing in (14) is the *total macroscopic field* in the medium and the **B** in (12) is due to everything *except* the particular atomic dipole under consideration. So (12) can be rewritten as

$$\Delta\mathbf{m} = \alpha_m(\mathbf{B} - \mathbf{B}_{self}) . \qquad (17)$$

Here $(\mathbf{B} - \mathbf{B}_{self})$ is the local field present in the vicinity of the atomic dipole that is located at the center of a fictitious sphere (cf. Fig. 1) and $\mathbf{B}_{self}$ is the average field over the sphere due to the dipole itself. It is reasonable to question why the average field over the sphere is taken instead of the field at the center of the sphere where the dipole is located. However, average magnetic field due to all dipoles outside the sphere is same as the field they produce at the center [10].
The radius of the sphere is related to the number density of atoms as

$$N\left(\frac{4\pi R^3}{3}\right)=1 \ . \qquad (18)$$

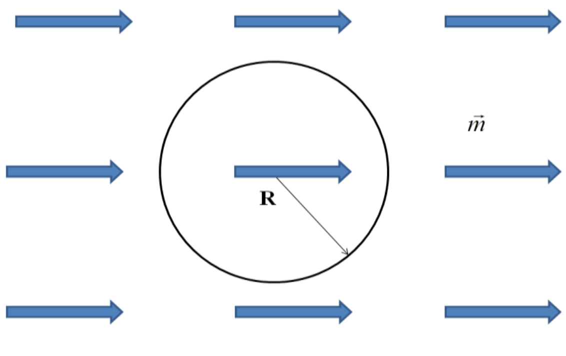


FIG. 2: The arrangement of dipoles. The sphere represents the fictitious boundary of each of the induced atomic dipoles **m** in the diamagnetic substance.

The average magnetic field produced by the dipole (moment) would be

$$\langle\mathbf{B}\rangle_{sphere} = \frac{\mu_o}{2\pi R^3}\Delta\mathbf{m}, \qquad (19)$$

Now, consider any atom, or magnetic dipole with index ‘i in the substance. $\mathbf{B}_{self}$ of this atom is then given by

$$\mathbf{B}_{self,i}= \frac{\mu_o}{2\pi R^3}\Delta\mathbf{m}_i \ . \qquad (20)$$

Plugging this value in (17), one gets

$$\Delta\mathbf{m}_i = \alpha_m(\mathbf{B} - \frac{\mu_o}{2\pi R^3}\Delta\mathbf{m}_i) . \qquad (21)$$

Rearranging the terms we get,

$$(1+\frac{\alpha_m\mu_o}{2\pi R^3})\ \Delta\mathbf{m}_i = \alpha_m\mathbf{B} . \qquad (22)$$

Now summation of both sides of (22) for N terms is taken and then the results of (18) is used to get

$$(1+\frac{\alpha_m\mu_o}{2\pi R^3})\ \mathbf{M} = \alpha_m N\mathbf{B} . \qquad (23)$$

Further invoking (14) to eliminate **B** and **M** from the previous equation we get

$$\alpha_m = \frac{3}{N\mu_o}\left[\frac{\chi_m}{3+\chi_m}\right] . \qquad (24)$$

Using $\chi_m = \mu_r - 1$

$$\alpha_m = \frac{3}{N\mu_o}\left[\frac{\mu_r-1}{\mu_r+2}\right] . \qquad (25)$$

Here μ_r is the relative permeability of the diamagnetic substance. The relation (25) is the *magnetic analogue* of Clausius- Mossotti equation.

For a diamagnetic substance, μ_r is less than unity. Hence, (25) tells us that α_m will always be negative. This reflects the fact that in a diamagnetic substance, the applied field **B** and the change in magnetic moment $\Delta\mathbf{m}$ are always antiparallel. To put it in another way, in the presence of an external magnetic field, each atom picks up a little extra dipole moment and these increments are all antiparallel to the field direction.

4. Brief historical survey of the equation

In this section we briefly discuss the historical development of the Clausius-Mossotti relation and its magnetic analogue which unfortunately lacks a different name for itself. The scientific figures responsible the development of his equation are S.D. Poisson (France), M. Faraday (England), J.C. Maxwell (Scotland), O.F. Mossotti (Italy), R. Clausius (Germany) and H.A. Lorentz (The Netherlands). We discuss their contributions chronologically.

The history of this equation begins in 1824, when Poisson presented his book [11] at a meeting of the French Academy in which he had carried out a detail mathematical analysis of the *problem of magnetic induction*. In his classic text on Electricity and Magnetism [12], Maxwell mentions that "the mathematical theory of magnetic induction was first given by Poisson…" In order to explain the phenomenon of magnetic induction Poisson hypothesized that an imaginary 'magnetic matter' or 'magnetic fluid' is *confined* to *certain* molecules of the magnetic substance. That molecule is magnetized in which the *two opposite kinds* of magnetic matter, which are present precisely in *equal* quantity, are separated towards *opposite poles* of the molecule. He called such molecules 'magnetic elements' of the substance and examined the particular case in which these elements are spherical and are uniformly distributed throughout the substance. He calculated the ratio 'K' called Poisson's Magnetic Coefficient - the ratio of the volume of magnetic elements to the whole volume of the substance. This turned out to be $\left[\frac{\mu_r-1}{\mu_r+2}\right]$, the factor that appears in the expression for magnetic polarizibility in Eq. (25). Maxwell ruled out the validity of such a hypothesis by using the experimental works of Thalen. However, he concludes [13], "… the value of Poisson's mathematical investigation remains unimpaired, as they don't rest on his hypothesis." Later on, to explain this phenomenon of magnetic induction Ampere hypothesized that the magnetism of a molecule is due to an electric current that already exists in it which constantly circulates in some closed path within the molecules of the magnet, and must not flow from one molecule to another. These are the two alternative pictures of a magnetic dipole.

Thus, using the hypothesis of Poisson and Ampere, and the magnetic analogue of Clausius-Mossotti equation, the problem of magnetic induction was completely resolved. Note all this happened about half a century *before* the development of the Clausius-Mossotti equation for dielectrics.

After a few years of Poisson's formulation, Faraday [14] for the first time [15] applied Poisson's idea to dielectrics. It was Mossotti who studied the problem in greater detail and presented it in his memoirs [16]. He introduced the 'cavity method' [17] which he later developed in his second book [18]. Meanwhile, Clausius was also studying the same problem [19]. For the first time, he explicitly wrote the formula of what is now famous as the Clausius-Mossotti equation, as called by Lorentz. It may be noted that all of them attacked the same problem using different approaches.

Coming back to our derivation, the approach that is followed in this paper (use of local field or Lorentz field) significantly departs from that used by Poisson yet resembles the one used by H.A. Lorentz [20] and L.V. Lorenz [21] in their derivation of the Lorentz-Lorenz equation (used in optics).

So it's evident that historically, Poisson's equation plays a more fundamental role as compared to that by the Clausius- Mossotti equation.

5. Concluding remarks

In the derivation the use of the relation between magnetic field **B** and dipole moment $\Delta\mathbf{m}$ is independent of whether the scenario is quantum mechanical or classical. The only assumptions are that the substance must be a linear, homogeneous, isotropic diamagnetic substance for our equation to be valid. One can check the equation very easily by putting relative magnetic permeability = 1, which is true for non-magnetic substances. For this case, the polarizability comes out to be zero, which is quite obvious as a non-magnetic substance would not respond to an external magnetic field.

One must recognize that it is the self field term $\mathbf{B}_{self}$ which actually leads us to such an equation for diamagnetic substances. We may consider the case of paramagnetic materials to see whether the self field term leads us to any such equation or not. In this case, the atoms possess a net dipole moment even in the absence of an external magnetic field. When a magnetic field is applied, magnetization arises due to the reorientation of the atomic dipoles. The potential energy of a dipole of dipole moment **m** in presence of a magnetic field **B** is ϕ = - **m.B**. As we have said above the **B** here should be written as $\mathbf{B}$-$\mathbf{B}_{self}$. And using (11) and (18), $\mathbf{B}_{self}$ can be written as $\frac{-2\mu_0 N}{3}\mathbf{m}$. Hence

$$\phi = -\mathbf{m.B} + \frac{2\mu_0 N}{3}\mathbf{m}$$

$$= -mB\cos\theta + \frac{2\mu_0 N}{3}m^2 \quad . \qquad (26)$$

The energy distribution of atoms can be adequately described by the classical Maxwell-Boltzmann statistics. Thus, the number of atoms dN whose energy lies between ϕ and ϕ+dϕ is given by

$$dN = A\exp(\frac{-\phi}{K_B T})\, d\phi$$
$$= A\exp(\frac{2N\mu_0 m^2}{3})\exp(\frac{\mathbf{m.B}}{K_B T})mB\sin\theta\, d\theta. \qquad (27)$$

Here T is the temperature of the system. A is the normalization constant that can be found out by imposing the condition that the integration of dN must be equal to the total number of atoms N when θ goes from 0 to π.

The term arising due to the self field can be absorbed into the normalization constant; therefore the $\mathbf{B}_{self}$ seems to play no significant role in the case of paramagnetic substances.

The scope of the magnetic analogue equation is not as wide as that of the Clausius-Mossotti equation. In fact, many seem to be unaware of the very existence of such an equation. However it plays a key role in the study of magnetic fluids [22], permittivity and permeability studies of mixtures [23], and negative effective permeability [24].

Acknowledgement

The authors are thankful to Prof. V.K. Tripathi for useful discussions and comments and to Prof. R. Chatterjee for her valuable remarks.

References

[1] J.D Jackson, Classical Electrodynamics, 3rd ed. (Wiley, New York, 1975).
[2] David J. Griffiths, Introduction to Electrodynamics, 3rd ed. (Prentice Hall, Englewood Cliffs, NJ, 1989).
[3] C. Kittel, introduction to solid state physics, 7th ed. (John Wiley & Sons, Inc., New York, 1996)
[4] Reference 2, p. 149.
[5] Reference 2, p. 188.

[6] J.H. Hannay "The Clausius-Mossotti equation: an alternative derivation", Eur I. Phls. 4, 141-143 (1983).
[7] D.J. Griffith, "Hyperfine splitting in the ground state of hydrogen", Am. J. Phys. 50, 698-703 (1982)
[8] Reference 2, p.261
[9] S.L. O'Dell and R.K.P. Zia, "Classical and semi classical diamagnetism" Am. J. Phys. 54 (1), 32-35 (1986)
[10] Reference 1, p.253
[11] S.D. Poisson, M´emoire sur la th´eorie du magn´etisme, M´em. de l'Acad. roy. de France, V (1824), 247–338. (copy from "Archives de l'Acad´emie des Sciences, Paris.")
[12] J.C. Maxwell, A Treatise on Electricity and Magnetism (Vol. 2), Dover, New York (1954). (Republication of 3rd edition of 1891 ed.) p. 57
[13] Ibid, p. 58
[14] M. Faraday, Experimental researches on electricity, Phil. Trans. Roy. Soc. London, II Ser. (1838)
[15] K. Markov, L. Preziosi, "Heterogeneous media: Micromechanics modelling methods and simulations", 1st ed. Birkhauser (2000).
[16] O.F. Mossotti, Sur les forces qui regissent la constitution int´erieur des corps aper¸cu pour servir `a la d´etermination de la cause et des lois de l'action mol´eculaire, Ac. Sci. Torino, 22 (1836), 1–36
[17] Reference 15, p.125-128.
[18] O.F. Mossotti, Discussione analitica sul'influenza che l'azione di un mezzo dielettrico ha sulla distribuzione dell'elettricit`a alla superficie di pi`u corpi elettrici disseminati in eso, Mem. Mat. Fis. della Soc. Ital. di Sci. in Modena, 24 (1850), 49–74.
[19] Clausius, R., Die mechanische Behandlung der Elektricitat, Vieweg, Braunshweig (1879)
[20] H.A. Lorentz, The Theory of Electrons (2nd ed.) Dover publ. (1954), p. 138-145
[21] L. Lorenz, Uber die Refraktionskonstante, Ann.Phys. Chem. 11 (1880), p.70 .
[22] I. Brevik, "Fluids in electric and magnetic fields: Pressure variation and stability" Can. J. Phys., 60. 449 (1982).
[23] L. Lewin, "The electrical constants of a material loaded with spherical particles", Proc. Inst. Electr. Eng. 94, 65 (1947).
[24] M. S. Wheeler et al., "Three-dimensional array of dielectric spheres with an isotropic negative permeability at infrared frequencies", Phys. Rev. B 72, 193103 (2005).

Study of Factors Influencing Electrodeposition of Thin Film.

Anuradha B. Bhalerao[1*], B.G. Wagh[2]

[1]Department of Physics
K.K.WaghInstitute of Engineering Education and Research
Nasik, 422003, India.

[2]Department of Physics
K.K.Wagh Arts, Science and Commerce College
Pimpalgaon (B),Nasik, 422209, India.

(*Submitted* 03-03-2013)

Abstract

There are various techniques to deposit solid semiconducting materials in the form of thin layer called as thin film.Electro deposition is one of these techniques. It is the process of depositing substance by passage of electric current through the conducting medium called electrolyte, producing a chemical change.The property of electrodeposited film depends on electrolyte, pH of the electrolyte, electrode nature, Potential and current density. The article discusses effect of these parameters on formation mechanism of thin films along with basic study of the method.

1. Introduction

A wide variety of processing technologies are available for the deposition of thin films. The technologies differ to a large degree in their physical and chemical principles of operation and in commercially available types of equipment. Each processing technology has been developed because it has unique advantage over others. However, each processing technology has its limitations. In order to optimize the desired film characteristics, a good understanding of the process control and the advantages and restrictions applicable to each technology is necessary.

Electrodeposition, which is one of these deposition techniques, is as old as the electricity.Electrodeposition of metallic films (commonly known as electroplating) has been known and used for preparing metallic mirrors and corrosion-resistant surfaces.It is also used in electroforming, i.e., to give shapes to electrodeposits. In recent years, there has been considerable interest in the electrodeposition of semiconductor films for photovoltaic applications.In this article an attempt is made to focus electrodeposition technique for deposition of thin films due to following advantageous factors.

- Synthesis is possible at low temperatures with less instruments
- Doping of is possible during synthesis.
- Liquid materials can be easily converted into thin films with required quantity,different size and shapes.

2. Electrodeposition

Electrodeposition of metals and alloys involves the reduction of metal ions from aqueous, organic, and fused-salt electrolytes by the passage of an electric current. It is based on the study of reactions in which charged particles cross the interface between two phases of matter, typically a metallic phase (the electrode) and a conductive solution, or electrolyte. A process of this kind is known as an electrode process. The first quantitative formulation of electrodeposition was done by Michael Faraday, popularly known as Faraday's laws of electrodeposition.

2.1 Basics of Electrodeposition

i. Faradays First law

The total amount of chemical change produced by an electric current is proportional to the total charge passing through the electrolyte.

ii .Faradays Second law

When same quantity of electricity is passed through different electrolytes arranged in series, the weights of substances librated at the respective electrodes are directly proportional to their chemical equivalent weight.

W= (E/F) ct

Where E is equivalent weight and c, t are current and time respectively with F = 96485 C/mol. However in practice the problem is much complicated and factors like sticking of electrodeposited atoms, Joule heating affect the efficiency of the process.

iii.Nernst Equation

When a solid is immersed in a polar solvent or an electrolyte solution, surface charge will develop and the electrode attains a potential given by the Nernst equation [3]

$$E = E_0 + (RT/nF).\log_e (a_{ion})$$

whereE_0 is the potential difference between the electrode and the solution when the activity $a_{ion}$of the ions is unity, F is Faraday's constant, R is the gas constant, and T is the temperature. [3]

2.2 Components of Electrodeposition Technique

i. Electrode

At least two electrodes (cathode and anode) are needed for deposition process Fig.[1]. An applied electric field across these electrodes provides the main driving force for the ions. The positive M^+ and negative x^- ions deposit at cathode and anode respectively.

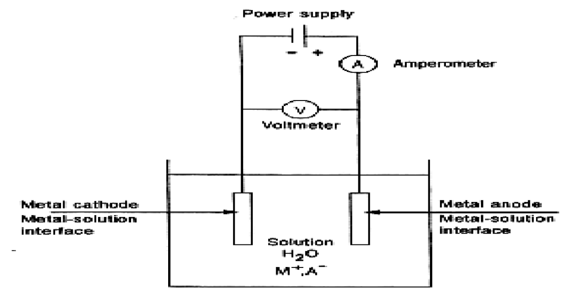
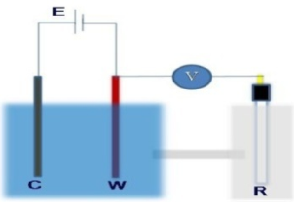


FIG.1: Two Electrode Cell

1) Cathodic deposition is more popular in electroplating because most metal ions are positive ions. 2) Anodic deposition gives poor stoichiometry and adhesion.

In most electrochemical experiments our interest is concentrated on only one of the electrode reactions. Since all measurements must be on a complete cell involving two electrode systems, it is common practice to employ a referenceelectrode as the other half of the cell.

Fig.2



C- Counter Electrode
R- Reference Electrode
W- Working Electrode

FIG.2: Three Electrode Cell

The major requirement of a reference electrode isthat, its potential must be stable. It is achieved by involving a saturated solution of an insoluble salt of the ion. The common reference electrode is the calomel electrode.[3]

ii. Electrolyte
The electrolyte or bath provides the ions to be electrodeposited. It has to be electrically conductive. It can be aqueous, non-aqueous or molten and it must contain suitable metal salts sometimes additive is included to improve the quality of electro deposit and to achieve uniform surface finish.
iii. Power Supply Unit
The power supply units can be of following types

a. Direct current at constant current which lead to galvanostatics deposition.
b. Direct current at constant voltage which lead to potentiostatic deposition.
c. Potentiostatproviding pulsed deposition, potentiodynamic deposition.

Overall mechanism due to supply of power is as follows:
A current or voltage pulse applies an electric field so that M^+ ionswould move to cathode.The elemental electro deposition process is as follows:

$$M^+ + e^- \rightarrow M$$

If electrolyte contain more than one ionic species that can be simultaneously deposited

$$M^+ + e^- \rightarrow M$$
$$N^+ + e^- \rightarrow N$$
$$M^+ + N^+ + 2e^- \rightarrow MN$$

As the electro deposition proceeds the ionic concentration in bath is depleted and has to be replenished by adding salt.[4]

2.3 Nature of Electrodeposition process

A common misconception about electrolysis is that "ions are attracted to the oppositely-charged - electrode." Ionic motion throughout the bulk of the solution occurs mostly by diffusion, which is the transport of molecules in response to a concentration gradient. Only the ions near to the interfacial region are likely to undergo migration.

2.4.1 Steps Involved in Electrodeposition Reaction

There are four steps that occur within 1-1000A^0 from substrate.
i.Ionic transportii.Discharge
iii. Breaking up of ion-ligand bond
iv.Incorporation of atoms on to the substrate followed by nucleation and growth

2.4.2 The various processes that occurs during electrodeposition:

i. Process that occur in electrolyte bulk
Ions that are sufficiently removed from the electrode surface can be move towards it under the influence of:
a. Potential gradient $d\Phi/dx$ leading to ion drift.
b. A concentration gradient dc/dx leading to diffusion of ions.
c. Density convective current $d\rho/dx$. ρ is the density of electrolyte due to consumption of ions at the electrode.

The three terms contribution of migration, diffusion and convective current starts conversion process to mass transport toward the electrode.

ii. The process that occur near the electrode but within the electrolyte

Ionic species in the electrolyte are normally surrounded by a hydration sheath or some other complex forming ion or ligand present in electrolyte they move together as one entity and arrive near the electrode surface. Where the ion ligand system either accept electron from the cathode or donate electron to anode. This ionic discharge reaction occurs in electrolyte between 10 and 1000A^0 from the electrode-electrolyte interfacial region.

iii.Process that occur on the electrode surface

The discharge ions arrive near the electrode where steps by step leading to the formation of new solid phase or growth of an electrodeposited film. The atoms thus deposited have a tender to form either an ordered conglomerate of crystalline phase or a disordered amorphous phase.

2.4 Influence of Cell Potential and pH on Deposition: (Stability diagrams):

In balancing redox equations, many electron-transfer reactions involve hydrogen ions and hydroxide ions. The standard potentials for these reactions therefore refer to the pH, either 0 or 14, at which the appropriate ion has unit activity. As, multiple numbers of H^+ or OH^- ions are often involved, the potentials given by the Nernst equation can vary greatly with the pH. Hence, it is useful to look at the situation, by considering what combinations of potential and pH allow the stable existence of a particular species. This information is most usefully expressed by means of an E-vs.-pH diagram, also known as a Pourbaix diagram.

Ex. Stability of water:

Water is subject to decomposition by strong oxidizing agents such as C_{l2} and by reducing agents stronger than H_2 with the parameters in shaded region in the fig. 3 below.[5]

The Reduction reaction: $2H^+ + 2e^- \rightarrow H_2$ (g)

OR

In neutral or alkaline solutions reduction reaction is$H_2O + 2\ e^- \rightarrow H_2(g) + 2\ OH^-$

These two reactions are equivalent and follow the same Nernst equation which, at 25°C and unit H_2 partial pressure reduces to

$E = E° - (.059/2) \times 2\ pH = –0.059\ pH$

TheOxidation Reaction: $H_2O \rightarrow O_2(g) + 4H^+ +2\ e^-$

This reaction is governed by the Nernst equation which similarly becomes E = 1.23 – 0.059 pH,

so the E-vs-pH plots for both processes have identical slopes and yield the stability diagram for water as shown in Fig.(4).

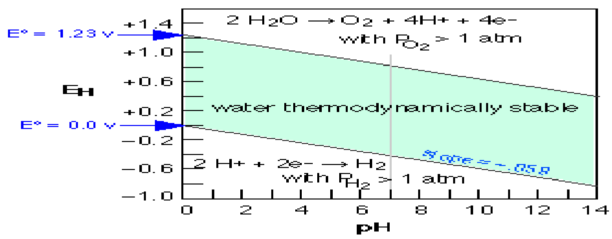


FIG.3Stability (Pourbaix) diagram for water

The two E° values shown at the left refer to "standard" conditions of unit H^+ activity (pH=0) and gas pressures of 1 atm. At combinations of pH and E that lie outside the shaded area, the partial pressures of O_2 or H_2 exceed 1 atm., signifying the decomposition of water. The unity partial pressures are arbitrary criteria; in a system open to the atmosphere.Water can decompose even at much lower H_2 partial pressures, and at oxygen pressures below 0.2 atm. Such diagram has relevance to electrochemical corrosion of metals. The metals above hydrogen in the activity series will tend to undergo oxidation (corrosion) by reducing H^+ ions or water.

Thus, in order to optimize the desired film characteristics, a good understanding ofpourbaix diagram, electrode process and the process control parameters is necessary.

3.Conclusion

Electrodeposition is a promising competitor in thin-film preparation because of several advantages, such as the possibility for large-scale production, minimum waste of components, and easy monitoring of the deposition process. The use of electrode process control parameters in aqueous electroplating method is extremely important. The potential benefits of this include: brightening the deposit, increasing the grain size, changing mechanical and physical properties, reducing stress, reducing pitting, and increasing the lifetime of deposition bath and deposits.

References

[1] Kasturi Chopra, *Thin Film Phenomena*, McGraw Hill Book Company, New York, (1969).

[2] Allen J. Bard,Larry R. Faulkner, *Electrochemical methods: Fundamentals and applications/ 2nd ed.*, A John Wiley &Sons, New York, (2001).

[3] M. VenkataKamalakar*,* *Synthesis, characterization and investigation of electrical transport in metal nanowires*and nanotubes,Jadavpur University, Kolkata(2009).

[4] R.K.Pandey., S.N.Sahu, Hand *book of semiconductor electrodeposition,* New York, (1996).

[5]Aqueous Stability Diagrams, M. Pourbaix, *Atlas of Electrochemical Equilibria in Aqueous Solutions,*Pergamon, London, (1966).

Physics of Gulli – Danda

Suresh C Joshi*

Head, Learning Resource Center and Department of Physics
Ahlcon International School, Mayur Vihar, Phase – 1, New Delhi – 110091, India
*e-mail: scjoshidat2012@gmail.com

(Submitted: 19-11-13)

Abstract

Using traditional games as instructional tools to understand physics concepts is a good idea and can play a vital role in developing activity based physics curriculum. Learners are able to relate with such tools more easily thus helping them develop pre-knowledge, logical reasoning, strategy making, concentration and nurture their inquiry-based scientific temperament epistemologically. Gulli – Danda[†], a native game in many parts of the world, is used here as an instructional tool to formalize metacognitive aspects of physics terms evolved during the demonstration of the game. A vast range of kinematic terms are involved with the demonstration of Gulli – Danda which are studied in the present paper in detail. Different forms of energy associated with specific movements of the game have also been discussed thoroughly which strongly supports the use of Gulli – Danda as a potential tool to understand physics terms associated with it.

Furthermore its use in explaining momentum conservation, trajectories of projectile motion, center of mass, moment of inertia, and elastic as well as inelastic collision has been demonstrated. We suggest that such traditional games and models, which are easily available the learner, can be effectively used as low cost pedagogical tools to develop instructional strategies in physics education. It was also observed that learners were able to make a connection with the seen and unseen world through demonstration of Gulli – Danda, which provides powerful evidence for a strong connection between the neuroscience of brain chemistry and play and joy. It also advocates using traditional native games as props and 'play' as the basic component of enquiry. An analysis of the post test results of different sets of audiences at different places revealed that no learner scored less than fifty percent marks on these tests. This is a clear indication of achieving average gain in conceptual understanding of learners towards physics concepts by using Gulli – Danda.

PACS No.'s: 01.40.E, 01.40.ek, 01.40.Fk, 01.40.gb, 01.40.Ha
Key Words: Traditional native games, brain compatible pedagogical approach, low cost demonstration tool, physics of Gulli-Danda.

† *It is a popular traditional game of India and is the national game of Nepal and is played in rural areas and small towns of India as well as Pakistan, especially in Punjab. In the United States, a similar game is called Pee-wee. It is also known as Tipcat in English, Dandi - Biyo in Nepali, Alak - Doulak in Persian and Kon - ko in Khmer (the Cambodian language).*

1. Introduction:

Physics, from its genesis, has been the branch of science that attempts to use the language of mathematics to describe how nature functions. Its scope of study encompasses not only the behavior of objects under the action of given forces, but the theories that provide us with some of our deepest notions of space, time, matter and energy. Its ultimate objective is the formulation of a few comprehensive principles that bring together and explain all such disparate phenomena. It also deals with describing why and how certain real-world phenomena occur and enables us to envisage what may transpire in certain circumstances. At its core, physics establishes itself as a comprehensible and an intriguing subject that demands a logical and systematic approach.

It is a fair assumption that the majority of students concur that physics proves to be a very strenuous and difficult subject studied at school. Student difficulties stem from physics concepts, the way in which a physics course is taught, and physics problems which are sometimes very vague. Perhaps it is the general lack of interpretation of what physics is, combined with the subject's inherent difficulty and reliance on mathematics, which tends to discourage a student from studying physics. Students find physics difficult because they have to contend with different aspects such as experiments, formulas and calculations, graphs and conceptual explanations at the same time. Thus, physics as a discipline requires learners to employ a variety of methods of understanding [1 – 3]. This makes learning physics particularly difficult for many students, and there are bound to be many hindrances before the students get a good grasp of the subject.

The purpose of this study is to measure the effectiveness of employing one of the Indian traditional games (Gulli – Danda) as a demonstration instrument to verify if it helps in drawing learners' attention and enhancing their conceptual understanding of physics. The participants for this study were drawn from middle and high school students, teachers and researchers both from India as well as the United States. This study presents a new pedagogical approach [4] which uses potential of traditional games [5 – 8], as tools, to motivate learners and help teacher tailor instructions [9, 10] to individual learners' needs and interest. For this I have used 'Gulli – Danda' (name of the game in Hindi, an Indian native language) as a demonstration tool for my presentations [4] to different sets of audiences at different places in the United States and India.

Section 2 of the paper is devoted to the familiarity and understanding of Gulli – Danda whereas section 3 demonstrates the physics associated with it, which is drawn from the established kinematic terms and I have tried to present a connectedness between these terms with the different movements of Gulli – Danda. Section 4 exhibits the use of Gulli – Danda as a tool, which I did with the different sets of audiences at physics education research groups and schools in the United States to test its effectiveness [4]. Physics terms and ideas evolved during the demonstrations are also given in this section with the observations during the conduction of the activities. In addition, questions framed by the participants are discussed along with the comparison of post test results at different places.

2. Familiarity and understanding of demonstration tool used (Gulli – Danda):

It is a popular traditional game in India and is the national game of Nepal [11]. It is played in rural areas and small towns in India as well as Pakistan, especially in the Punjab area. Its origin in India is found dated back to the "Maurya Dynasty". It is a four plus player game. In the United States, a similar game is called Pee-wee. It is also known as Tipcat in English, Dandi-Biyo in Nepali, Alak - doulak in Persian and Kon - ko in Khmer (the Cambodian language) [11].

The Gulli is a cylindrical wooden piece tapered at both ends, usually 4 to 5 inches in length and approximately 1 to 1.5 inches in diameter in the middle. The Gulli's ends are blunt. A Danda is a straight cylindrical stick usually between 15 and 18 inches in length and around 1.5 inches in diameter.

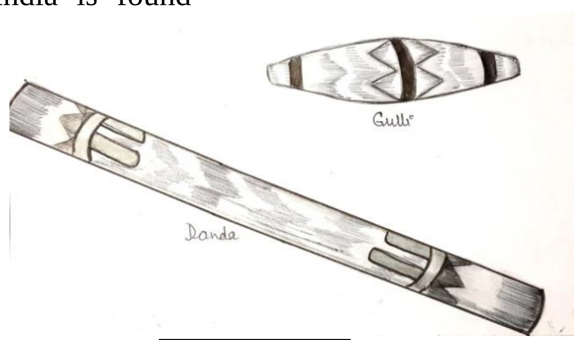


Diagram (1)

At a conceptual level, we can liken the Danda and Gulli to a bat and a ball, in the sense that a Danda is used as a bat to hit the Gulli. As this is a rural, native game with no recognition from any local or international sports bodies, there are no standards on sizes, shapes or type of materials to be used for the Gulli and Danda. People use whatever is convenient and available within their reach and do not really care about exact length, diameter and weight as long as the equipment serves its purpose. For that matter, there are no standard rules for the game as such, as each region, province and state follows its' own rules. Thus we have different variations of Gulli - Danda being played across different regions. However, certain general rules that are commonly followed while playing Gulli – Danda are as follows: First we need two teams with not less than four players on each team. After a coin toss, the winning team starts batting (or fielding, whatever they choose). A circle is drawn at one corner of the ground where the batsman will bat. This circle will have a small pit (groove) dug in the center for the Gulli to be placed in.

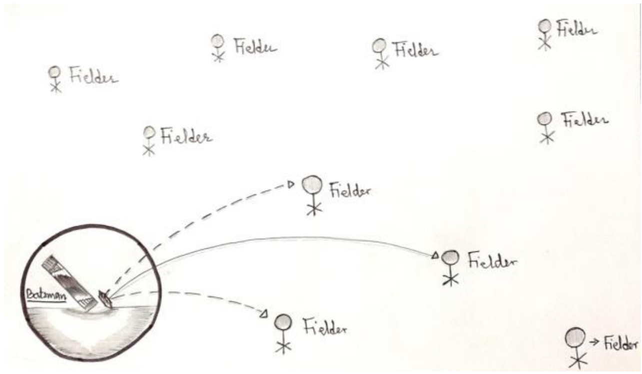


Diagram (2)

The first player from the batting team will use the Danda to bat, while the fielding team is spread out across the ground to field. The batsman gently hits the Gulli across one of the tapered ends with the Danda, and quickly strikes it hard as soon as it is lifted off its groove and when it is still in the air. It is a double motion strike from the batsman – somewhat like a "tip and whack". The idea behind the "tip" being gentle is to ensure that it lifts the Gulli off the ground to a reasonable height and reach for the batsman to hit it hard once again ("whack") while it is in the air. The Gulli remains airborne if the batsman strikes the Gulli with the Danda during the whack. If the batsman is unable to strike the Gulli in three continuous chances, the batsman is out. This is similar to a strikeout in Baseball.

If the batsman strikes the Gulli, and no fielder is able to catch it before it lands, the Danda is used to measure the distance from the center of the circle (from the batsman's batting spot) to the spot where the Gulli landed. Each Danda length adds one point to the batting team. If the batsman strikes the Gulli and a fielder catches it before it touches the ground, the batsman is out and then the next player of his team comes to bat. There is one other way that the batsman could be out. Once the batsmen hits the Gulli and no fielder catches it before it touches the ground, the batsman places the Danda within the small circle from where he was batting, horizontally on the ground, with its length facing the fielding team.

The fielding team picks up the Gulli from the ground and throws it at the Danda (after measuring the distance). If the Gulli strikes the Danda, the batsman is out. If it does not hit the Danda, the batsman does the "tip and whack" once again from the point where the Gulli has landed after the fielding team's throw at the Danda. In the event that the Gulli landed behind (beyond) the small circle, the start point for the batsman would be his original batting spot. The batsman repeats the same process of hitting the Gulli again until he is out. Repeat the above steps until all players of the batting team are out. The sum of all the points scored by the batting team becomes the target for the other team to beat. Now the fielding team will bat and the batting team will field. Repeat all above steps for the new batting team. The team with the highest score wins the match.

3. **Physics of Gulli – Danda:** The Gulli, which has conical ends, is initially at rest and the Danda, when raised to a height (h), contains potential energy given as,

$$U_D = Mgh.$$

Where M is the mass of the Danda and h is the height to which it is raised.

When the batsman brings the Danda down to h = 0, with velocity (V) to hit the Gulli, the energy stored in the Danda in the form of potential energy (U_D) gets converted into kinetic energy (KD) as the Danda loses the potential energy (neglecting air drag).

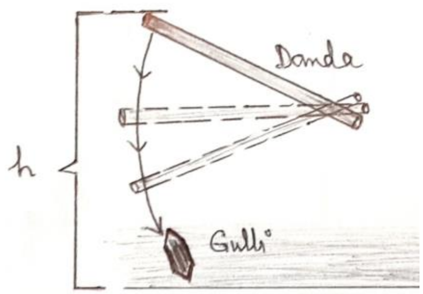


Diagram (3)

Since~~,~~ the lost potential energy is used to impart velocity to the Gulli, according to conservation of energy,

$$U_D = K_D,$$

$$Mgh = \frac{1}{2}\,MV^2. \qquad \ldots(1)$$

The velocity associated with the Danda is given as:

$$V = \sqrt{2gh}. \qquad \ldots(2)$$

In eq. (1) Mgh is the gravitational potential energy acquired by the Gulli and also the work done by the gravitational force expressed as,

Work done by the conservative force = – (change in potential energy).

The momentum in the horizontal direction will be conserved since there is no net external force in the horizontal direction.

Now,

$$MV = mu\,Cos\,\theta\,, \qquad \ldots(3)$$

where m is the mass of the Gulli and u is the velocity with which it is projected. θ is the angle made by the Gulli with the horizontal, known as angle of projection and is shown as,

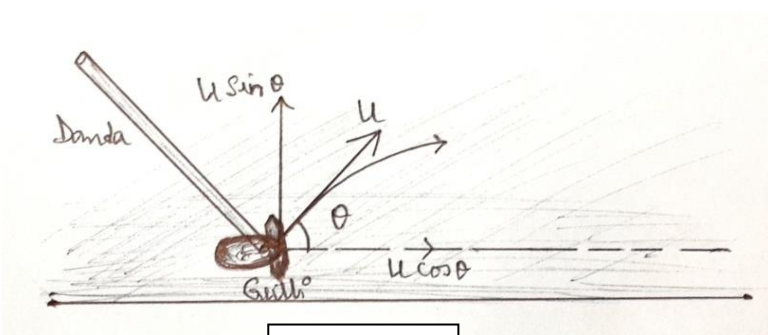


Diagram (4)

Let the batsman provide an impulse ‘I’ which makes the Gulli attain a velocity v at a height ‘h’. The impulse imparted is given as,

$$I = mv - mu = change\ in\ momentum. \qquad \ldots(4)$$

The path followed by the Gulli under the gravitational force is given by the equation,

$$y = x\tan\theta - \frac{gx^2}{u^2 Cos^2\theta}. \qquad \ldots(5)$$

Where x and y are the distances travelled by the Gulli with the velocity components in the X and Y axis respectively.

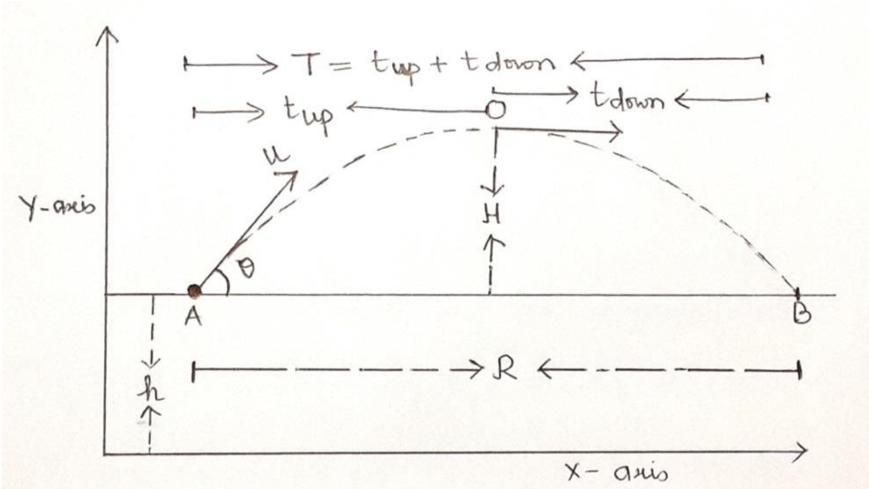


Diagram (5)

Equation (5) clearly shows that the trajectory followed by the Gulli is parabolic in nature. The center of mass of the Gulli also follows the same trajectory as that of the Gulli. To find out the center of mass of Gulli and Danda, we shall assume shapes of both the Gulli and the Danda as completely cylindrical and will be shown by the diagram:

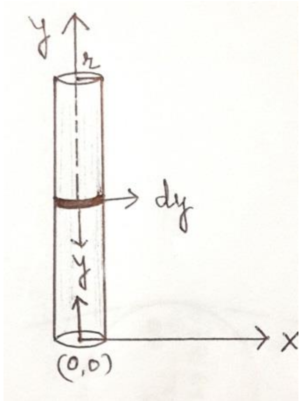


Diagram (6)

The position vector of the center of mass of the Gulli is given as,

$$\vec{r}_{CM} = \frac{\int \vec{r}\, dm}{\int dm},$$

where dm is the mass of a small segment of the Gulli. On putting the values of individual components in this equation, we get,

$$\vec{r}_{CM} = \frac{\int yj\frac{m}{h}dy}{\int dm} = \frac{\frac{m}{h}\int_0^h y\hat{j}\,dy}{\int dm} = \frac{h}{2}\hat{j}.$$ …(6)

The motion of the Gulli is best described as a combination of rotation and translation which gives the complete rotational aspect of the motion shown by the diagram as follows:

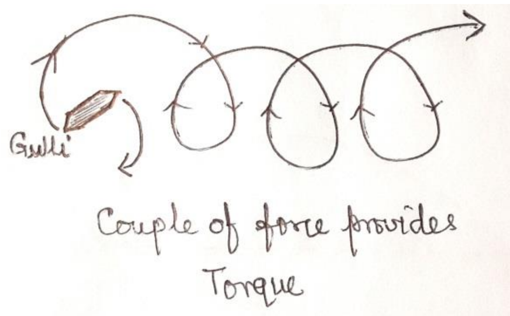


Diagram (7)

Kinetic energy of rotation of the system is given as,

$K_{rotational} = \frac{1}{2}I_{CM}\omega^2.$ …(7)

Where I_{CM} is the moment of inertia of the Gulli about an axis (of rotation) passing through the center of mass and ω is its angular velocity.

Angular momentum about the axis of rotation =

$m(\vec{v}\times\vec{r}) + I_{CM}\omega = [\vec{L}_{orbital} + \vec{L}_{spin}].$ …(8)

Just after the Gulli is hit, the power transferred to the Gulli by the Danda is given as,

$$P_g = \frac{\vec{F}.\overrightarrow{ds}}{dt} = mg.u\,Cos(90+\theta) = -umg\,Sin\theta.$$ …(9)

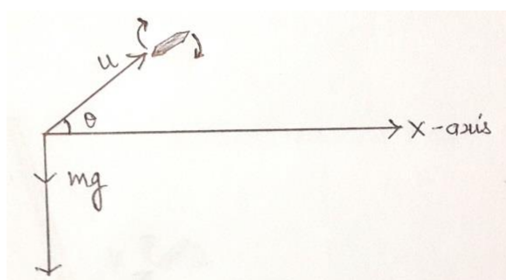


Diagram (8)

The radius of curvature just after the hit is shown as,

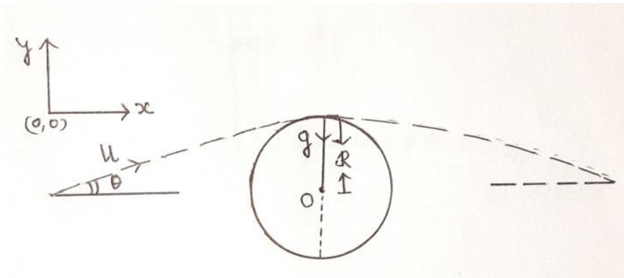


Diagram (9)

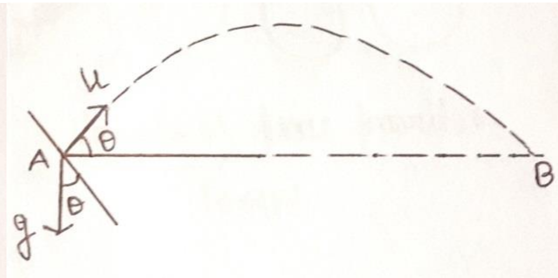


Diagram (10)

and is given as,

$$g\ Cos\ \theta = \frac{u^2}{R} \Longrightarrow R = \frac{u^2}{g\ Cos\ \theta}. \qquad \ldots(10)$$

If we analyze the motion in two dimensions, we get,

$$u_x = u\ Cos\ \theta, a_x = 0$$
$$u_y = u\ Sin\ \theta, a_y = -g$$

Time of flight can be expressed as,

$$T = \frac{2u\ Sin\theta}{g}. \qquad \ldots(11)$$

The distance travelled by the Gulli along the horizontal direction is the range of travel and is given as,

$$Range = u_x \times T = \frac{u^2\ (2Sin\theta\ Cos\theta)}{g} = \frac{u^2\ (Sin2\theta)}{g}. \qquad \ldots(12)$$

Since velocity u makes an angle θ with the horizontal, it takes the Gulli to the maximum height, after getting hit by the Danda, so the maximum height achieved by the Gulli during its motion in air is given as,

$$H = \frac{u^2\,Sin^2\theta}{2g}. \qquad \ldots(13)$$

The collision of Danda and Gulli will be perfectly inelastic, therefore the coefficient of restitution cannot be one, and it will have the value,

$$e = \frac{u}{v\ Cos\theta}. \qquad \ldots(14)$$

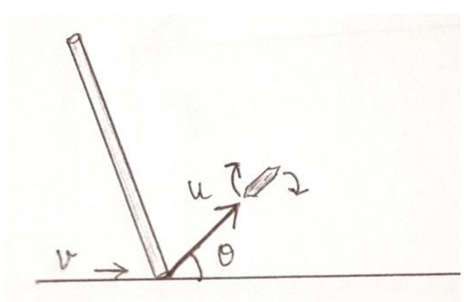


Diagram (11)

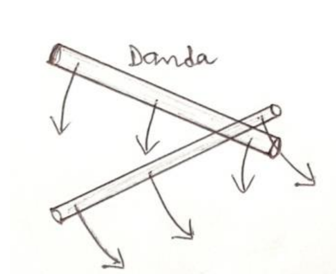


Diagram (12)

While hitting the Gulli, the motion of the Danda may be transitional because the trajectory followed by all the points of the Danda is the same.

4. Using Gulli – Danda as a demonstration tool:

Gulli - Danda was demonstrated to different sets of audiences at physics education research groups [4] and schools in the United States to test its effectiveness (without telling them about physics terms involved with the game). Presentations were focused on the use of traditional games as demonstration tools and their effectiveness to create active learning environments in the classrooms.

A video was also played with an illustration of 'How to play' Gulli - Danda (http://www.youtube.com/watch?v=JFtU9JnFVkU). After the video and demonstration, participants recorded their observations and responses in the form of physics terms and the ideas that evolved followed by discussion.

4.1 Physics terms / ideas evolved during the demonstration / presentation:

Force and effort, cylindrical shaped playing material, conical edges on the Gulli, angle of hitting, laws of motion, momentum conservation, speed, thrust, angle of projection, impulse, rotation, torque, elastic potential energy, gravity, superposition, linear and angular momentum, rotational motion, moment of inertia, acceleration, projectile motion, conservation of energy, vectors, center of mass, maximum horizontal range, friction, trajectory followed by the Gulli, gravity, displacement, action – reaction, angular velocity, free fall, angle of collision, momentum transfer, power, extended body vs. particle, air resistance, 2D kinematics, centripetal force, rotational energy, rotational vs. translation motion, elastic and inelastic collision, translation kinetic energy, fulcrum and elevation.

It is clear from this activity that learners come out with ideas and terms that they have come across in real life. They try to relate these terms with a particular activity or demonstration. The idea is that when an instructor demonstrates a particular game in class, he/she could take any term of the learners' choice forward to explain the concept further. Discussion could be taken to the next logical level subsequent to establishing the motivation and interest that learners would have developed during the initial part of the interaction. Once learners get the initial thrust of interest and motivation, the same could be used further by the teacher to develop and maintain learners' confidence throughout the interaction and also through the difficult topics.

Participants were given the opportunity to play Gulli-Danda to experience it first-hand. The activity was conducted at different places and results that were recorded from each of these places were very encouraging. Some of the observations are:

(i) Enough stimulation was seen amongst the participants because they were engaged in the activity.

(ii) Participants were encouraged to think independently. They were discussing different kinds of motion followed by the Gulli and predicting different forces used on the Danda to hit the Gulli. This motivated the learners to develop and design new models which nurtured capacity building and a scientific temperament in them.

(iii) There was no requirement of hi-tech machines to record the outcomes (it was all about evolving ideas).

(iv) A few physics ideas were picked for further deep study and detailed discussion (the instructor could choose any of the ideas or physics terms of the students' choice).

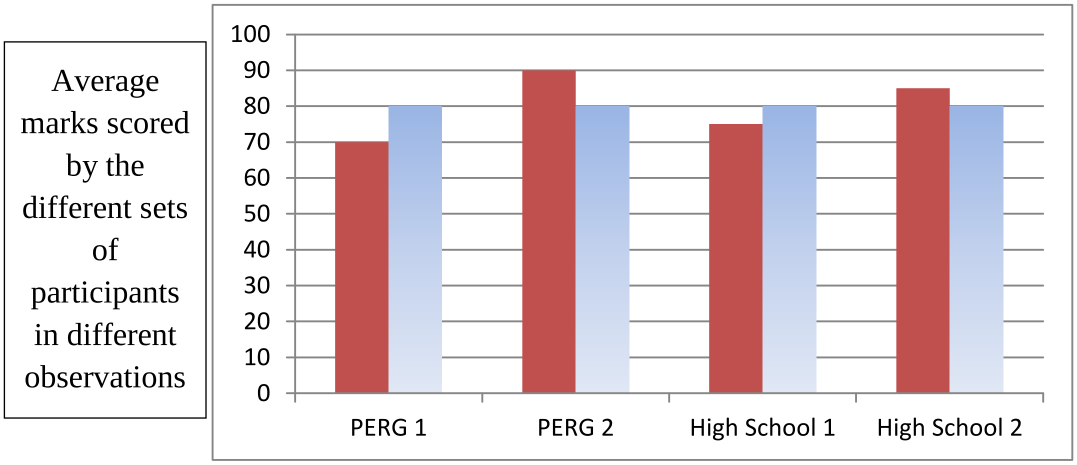
4.2 Questions framed by the participants:

1. How hard and where should you hit for the Gulli to pop up?
2. Why does the Gulli pop up when you hit it down? What force makes it go up?
3. How does wind influence the flight of the Gulli or what is the role of air resistance to minimize or maximize the range?
4. What would change if you modify the shape or weight of the Gulli or how does the different size of the Gulli affect the outcome of the game?
5. What about the energy, when it is at the ground and when it is in the air?
6. Which is the best place and best angle to hit the Gulli in order to reach the desired range?
7. What is the best timing to hit the Gulli when it is in the air in order to reach the maximum range?
8. Why is there horizontal translational motion sometimes?
9. How does the angle (or value) of applied force affect the trajectory followed by the Gulli?
10. Why does the Gulli rotate after being struck?
11. What is the relationship between force and location of impact?
12. How does the player judge when to hit it?
13. What will happen if you hit the Gulli at different locations?
14. How much force should be applied to the Gulli in order to get 3m? Is it possible to set the parameters like this?
15. Which degree of rotation is most useful when looking for the greatest distance?
16. Is there a certain angle measurement where the player hits the Gulli to make it spin or travel faster?
17. How is the orientation affected by air-resistance? Is this significant for the distance to be travelled by the Gulli?
18. How does the curvature of the ends affect the rotational and translational energy imparted?
19. How does the nature of the ground affect the initial upwards motion or how does the hardness of the ground affect the force needed to launch at a certain maximum height?
20. Why was there a small pit dug at the place of blowing the Gulli (in the video)?

It is very clear from the questions asked by various participants, each from a different place and a different setting, that there is a set range of questions for every activity or demonstration. Questions asked for a particular demonstration using a particular game revolved around a few concepts or queries. This indicates that props and models based on games can be developed and used in the curriculum which addresses specific concepts in science, particularly in physics.

4.3 Comparison of post – test results at different places:

An identical time based post – test was given to the participants after each demonstration at different places, (to physics education research groups (PER) and high school students) in the US. The average scores of all attempts and their comparison with the overall average is shown through the graphs as follows:



Comparison of average marks of individual observation with the overall average

The graph depicts that average scores achieved by the participants at physics education research group seminars (shown as PERG 1 and 2 in the graph) are 70% and 90% whereas the average marks scored by the participants at different high schools (shown as High school 1 and 2 in the graph) are 75% and 85%. Overall average score*of all these attempts is 80%.

(*Average score = Sum of the average scores of different attempts / Total number of attempts).

What emerged here is that strategies on 'using games as demonstration tools in the classrooms' worked well as participants were able to predict, infer, use previous knowledge, summarize and ask questions.

5. Discussion and outcomes:

This study discusses responses of learners towards an instructional method and its effectiveness over different class settings for detailed understanding of developing awareness of everyday thinking and learning. Everyday learning depends upon a number of factors like students' interest areas, surrounding environment, involvement of teacher and the taught, motivation to learn, challenges in learning and fear of failure. If the learning environment is made so comfortable for a learner such that learners get motivated as active participants in the learning process, I think half of the job is done. A similar attempt is made through study results reported in the presented article. Using Gulli – Danda as demonstration tool, to introduce physics ideas and to motivate learners to think independently, was acclaimed as a great idea at different places when it was presented. Such demonstrations were able to draw participants' attention and were able to engage them actively in hands on as well. Using games as low cost demonstration tools can be one of the methods to overcome traditional method of teaching.

Physics of Gulli – Danda is demonstrated comprehensively in section 3 which covers all key aspects of the motion involved with Gulli – Danda. Equation (2) gives the velocity achieved by the Danda which has been developed through the potential associated with it and then later converted into its kinetic energy. Equation (5) illustrates that the Gulli follows a parabolic trajectory and performs translational and rotational motion both depending on the nature of force and angle provided at the time of the hit. This equation is related to schematic diagrams 4 and 5. Position vector of center of mass of Gulli is given by eq. (6) and the power transformed by the Danda is shown in eq. (9). The radius of curvature, time of flight, horizontal range and maximum height attained along-with the coefficient of restitution are given in the successive equations and shown through the respective diagrams. How Gulli - Danda was used as demonstration tool with different sets of audiences at physics education research groups and schools in the United States has been explained in section 4. Evidence of providing opportunities for learners to think independently is also well discussed in this section. It is also established that this method nurtures an inquiry based scientific temperament in learners since framing questions after the demonstration is a key feature of the method.

Through this method, learners were able to learn to appreciate the culture and values associated with the game, which increases their awareness of a particular culture. A specific set of physics ideas were evolved during the demonstrations which are given in section 4.1. Questions framed by the participants are discussed in section 4.2 which clearly shows that learners were able to make connections with the seen and unseen world through demonstrations of Gulli – Danda which provides powerful evidence for a strong connection between the neuroscience of brain chemistry and play and joy. It advocates using traditional native games as props and 'play' as the basic component of enquiry and also shows that these games can be used to develop low cost pedagogical tools to address specific concepts which may possibly play a major role in designing an activity based physics curriculum. A comparison of post test results of different sets of audiences at different places is presented in section 4.3, which shows that no learner scored less than fifty percent marks in these tests. This is a clear indication of achieving average gain in conceptual understanding of learners towards physics concepts through using Gulli – Danda.

In totality, this paper addresses all the research questions focusing on pedagogical approaches used for developing activity based science curriculum. Furthermore, I plan to develop brain compatible animations, videos and video games explaining scientific principles using traditional games as low cost pedagogical tools (to connect classroom teaching to real life experiences) in science teaching which will disseminate use of these demonstration tools worldwide as reflective innovative practices resulting in sustainable changes for blended learning and flipping classrooms leveraging the power of technology.

Acknowledgements: I am grateful to Mr. Ashok K Pandey, Principal, Ahlcon International school for his guidance and support. My deep sense of gratitude goes to the referee of this paper whose valuable suggestions to clarify the concepts. I express my appreciation to all academics and students for this study at different places in the United States as well as in India. I would also like to thank Dr. Evelyn L. Turner from the United States and Shreya Garg from India for editing, G. S. Kasturi, Kritika Nangia and Pritish Chatterjee for their valuable inputs while developing the physics of Gulli – Danda and Sania Dua for diagrams 1 and 2. All time support of Ahlcon International School and financial assistance provided by the Department of State, U.S. during my Fulbright DAT grant in the United States is greatly acknowledged.

References:

1. National Curriculum framework – 2005: ISBN 81-7450-467-2 Published by NCERT, Sri AurobindoMarg, New Delhi 110 016, India.
2. http://www.math.tohoku.ac.jp/~kuroki/Sokal/misc/bethespoof.html
3. Khunyakari, R., et. al.; (2010) Bringing Design into the Indian School Curriculum. International Conference on "Designing for children- with focus on 'play' and 'learn', at Industrial Design Centre, IIT, Mumbai, Feb 2-6, 2010.
4. S C Joshi: Learning Physics through Games: A Game Based Teaching Model (GBTM) Builds an Inquiry Classroom Environment; Research Findings of Fulbright DAT Grant, Submitted to Institute of International Education, United States (December 06' 2012).
5. Guemez J, Fiolhais C, Fiolhais M, 2009, Toys in physics lectures and demonstrations – a brief review, Physics education No 44 vol 1 p.53 – 64.
6. Randel, J.M., Morris, B.A., Wetzel, C.D., &Whitehill, B.V. (1992). The effectiveness of games for educational purposes: A review of recent research. Simulation and gaming, 23(3), 261-276.
7. Ellington, H., Goedon, M., &Fowlie, J. (1998). Using Games and Simulations in the Classrooms. London: Kogan Page.
8. Perkins, K., Adams, W., Dubson, M., Finkelstein, N., Reid, S., Wieman, C., &LeMaster, R. (2006). PhET: Interactive simulations for teaching and learning physics. The Physics teacher, 44(1), 18-23.
9. Klietsch, R. G. (1969). An introduction to learning games and instructional simulations: A curriculum guide. Newport, MN: Instructional Simulations.
10. Hasse C, 2008, Learning and transition in a culture of playful physicists, European Journal of Psychology of education No 2 vol 23 p.149 – 164.
11. https://en.wikipedia.org/wiki/Gilli-danda.

Physics Education **Jan – Mar 2014**

Dye-doped gelatin films for phase-conjugation studies in undergraduate optics laboratory

T. Geethakrishnan[1*] and P.K. Palanisamy[2]

[1]Department of Physics
University College of Engineering
Villupuram605103, Tamilnadu, India.
[*]E-mail: tgeethakrishnan@hotmail.com

[2]Centre for Laser Technology
Department of Physics, Anna University
Chennai 600025, Tamilnadu, India.

(Submitted 02-01-2014)

Abstract

In the present paper, an overview of optical phase-conjugation and the experimental observation of optical phase-conjugation based on the degenerate four-wave mixing are presented. A continuous wave 633 nm He–Ne laser of total power 35 mW is used to generate phase-conjugate signal from the Acid blue 7 (Alphazurine) dye-doped gelatin film.A maximum phase-conjugate reflectivity of 0.22 % has been observed in these dye films. Degenerate four-wave mixing based optical phase-conjugation can easily be demonstrated for undergraduate students in any optics laboratory using low power lasers.

PACS: 42.65. –k
Key words: Holography, Optical Phase-conjugation, Dye-doped gelatin films.

1. Introduction

Optical phase-conjugation (OPC) is a technique that incorporates nonlinear optical effects to precisely reverse both the direction of propagation and the overall phase factor of a light wave. The process can be regarded as a unique kind of mirror with very unusual image-transformation properties. Optical phase-conjugation was first observed by Zel'dovich et al [1] in 1972, but its roots can be traced back further to earlier work in static and dynamic holography [2,3].

Optical phase-conjugation is an important technique with applications in many fields of science and engineering such as spectroscopy, adaptive optics, and real-time image processing orphase-conjugate mirrors. In general, OPC is a highly useful and very unique approach to explore the nonlinear optical properties of various materials and to investigate different physical processes occurring in those media under the action of strong coherent optical fields. Phase-conjugation (PC) by degenerate four-wave mixing (DFWM) has been demonstrated in numerous nonlinear media [4–8] and most of the experiments utilized high peak power pulsed lasers in order to obtain measurable signals. The majority of published works in the field of optical phase-conjugation and Four-wave mixing (FWM) had been based on studying the nonlinear optical properties of a great variety of materials which include dye solutions [9-12] or dye-doped matrixes [13-16], impurity-doped glasses [17-18], crystals [19,20] and fullerenes (e.g. C60) related materials [21,22].

This paper, presents the fundamentals of optical phase-conjugation and the experimental demonstration in Acid blue 7 dye-doped gelatin films through degenerate four-wave mixing geometry using low power continuous wave He–Ne laser.

2.Theory

Two optical beams are considered to be phase-conjugate to each other if they have same wavefronts but propagating in opposite directions. This means that the k-vectors of the two beams have the opposite sign and that the amplitude functions are complex conjugate of each other. When an optical beam is incident on a conventional mirror, only the component of the k-vector that is perpendicular to the plane of the mirror is reversed. If a divergent beam is incident on a conventional plane mirror its reflection will diverge in the same manner.

Hence for a beam reflecting from a flat mirror located in the x–y plane, reflection is given by

$$Aexp\ i\left(\left(k_x + k_y + k_z\right).r - \omega t\right) \Rightarrow A' \exp i\left(\left(k_x + k_y - k_z\right).r - \omega t\right) \tag{1}$$

However, the beam incident on a phase-conjugate mirror (PCM) reverses all three components of the k-vector and is given by

$$Aexp\ i\left(\left(k_x + k_y + k_z\right).r - \omega t\right) \Rightarrow A' \exp i\left(-\left(k_x + k_y - k_z\right).r - \omega t\right) \tag{2}$$

This causes the reflected wavefront to exactly retrace the path of the incident beam regardless of the initial spatial structure of the beam. There are many nonlinear optical processes that are capable of generating phase-conjugate (PC) waves. Holography can be considered as a static form of phase-conjugation [4,5]. The reconstructed field that forms a real image in conventional holography is actually a phase-conjugate replica of the original object field. Stimulated scattering processes are another important class of interactions that can be used for optical phase-conjugation. Degenerate four-wave mixing (DFWM) is one of the most convenient processes for the generation of high-quality PC waves.

3.Optical phase-conjugation through DFWM

There is a close analogy between the degenerate four-wave mixing (DFWM) process and the real time holography. As shown in the Fig.1(a), two counter-propagating pump waves (E_1 and E_2) pass through the nonlinear medium, and a probe wave (E_3) is incident on the nonlinear medium at an angle with respect to the pump wave E_1. The wave vectors of these three light beams are k_1, k_2 and k_3 respectively ($k = 2\pi/\lambda$). Under this experimental arrangement the backward propagating (phase-conjugate) wave can be generated through the reflection or transmission from two possible induced gratings. In the first case, the interference between the forward-pump beam E_1 and the signal beam E_3 that produces nearly parallel interference fringes (transmission grating) along the bisector direction of the crossing angle are considered. Since the refractive-index change in the medium is proportional to the local light intensity, one may realize that the interference fringes can produce an induced holographic grating within the nonlinear medium. In this case, the backward-pump beam acts as a reading beam during its passage through the induced grating. A diffracted (or reflected) wave E_4 is created and the wave vector corresponding to this newly generated wave is k_4.

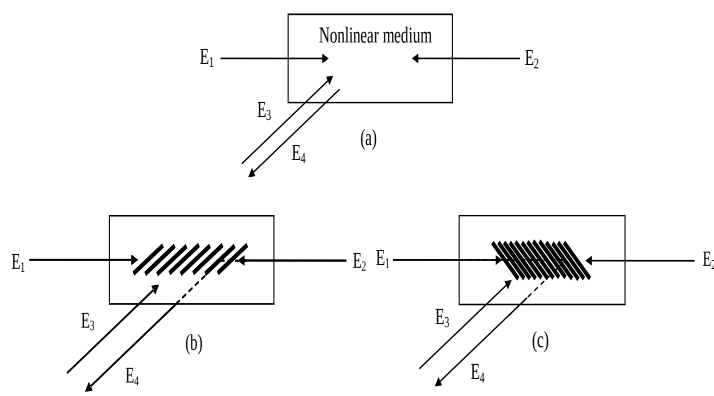


FIG.1: (a) A schematic of four-wave mixing, (b) a transmission grating formed by E_3 and E_1 (c) a reflection grating formed by E_3 and E_2

According to the holographic principle, it is known that this diffracted wave E_4 has the spatial information carried by the incident signal beam E_3. In other words, the waves E_3 and E_4 are phase-conjugated to each other. To further justify this conclusion one can treat the nonlinear medium as a holographic medium whose transmission function is determined by the interference-induced refractive index modulation and can phenomenologicallybe expressed as

$$T \propto (E_1 + E_3)(E_1 + E_3)^* = |E_1|^2 + |E_3|^2 + {E_1}^* E_3 + E_1 {E_3}^* (3)$$

where E_1 and E_3 denote the complex amplitudes of the forward-pump wave and the probe waves respectively, in the grating plane.As assumed the waves E_1 and E_2 are two counter-propagating plane waves and$E_2 = E_1^{\,*}$, so that the transmitted field of reading wave is given by

$$E_2^{'} \propto TE_2 = T{A_1}^* = [|E_1|^2 + |E_3|^2]E_2 + E_3\,({E_1}^*)^2 + E_1 E_2\,{E_3}^* \qquad (4)$$

Here on the right-hand side of the equation, the first term proportional to E_2 represents the zero-order diffracted wave that does not involve any spatial information and therefore of no interest to us. The contribution from the second term that actually involves a phase factor ofexp [-i (2k_1.r – k_3z)] can be neglected because its phase matching condition could not be fulfilled. The third term corresponds to the diffracted wave that involves the spatial information carried by the signal wave E_3 and can be written separately as

$$E_4 \propto E_1 E_2 {E_3}^* \quad (5)$$

As shown in Fig. 1(c), the backward wave E_4 can also be generated through the diffraction of the holographic grating (reflection grating) induced by the waves E_2 and E_3. In this case, the forward-pump wave E_1 plays the role of the reading plane wave, and the reflected wave E_4 is still phase-conjugated to the signal wave E_3. Since both these gratings contribute to the generation of the phase-conjugate wave E_4, the total conjugate field E_4 is due to the coherent superposition of both the processes. The period (Λ) of these two gratings are different and can be written as

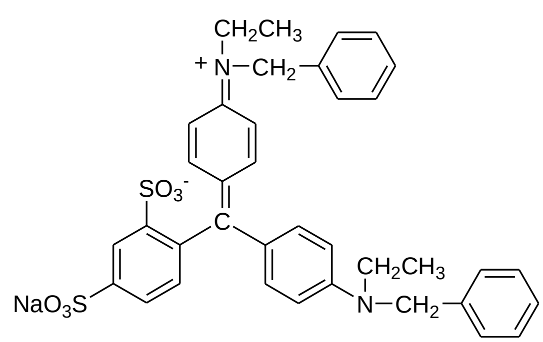
$$\Lambda_{13} = \frac{\lambda}{2\sin(\theta/2)} \qquad (6)$$

$$\Lambda_{23} = \frac{\lambda}{2\cos(\theta/2)} \qquad (7)$$

where λ is the wavelength of the waves in the medium. It is obvious that these periods of the induced gratings are determined by the corresponding spacing of the interference fringes formed by two appropriate waves.

4. Experimental studies

In this work, a commercially available synthetic dye, Acid blue 7 (Alphazurine A-C.I. 42080) was used as the nonlinear agent, which belongs to the triphenylmethane [23] groups. The general structure and formula of acid blue 7 dye are shown in Fig.2.



Molecular formula: $C_{37}H_{35}N_2NaO_6S_2$

FIG. 2: Chemical structure and formula of Acid blue 7 dye

The UV-visible absorption spectra of acid blue 7 dye were studied using UV-2401 PC spectrophotometer and it exhibits the peak absorbance at 637 nm as shown in the Fig.3.

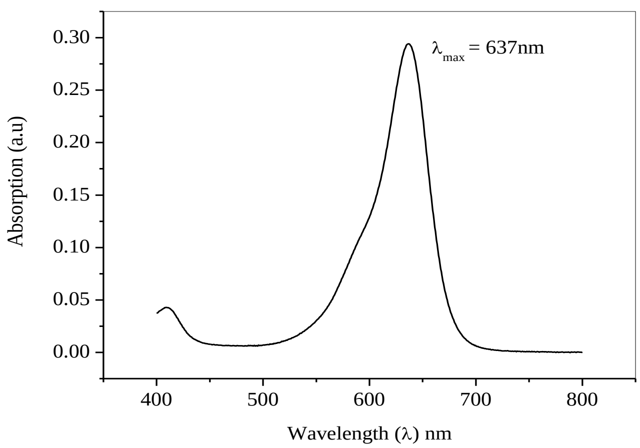


FIG. 3: UV-Vis absorption spectra of Acid blue 7 dye in water

Dye-doped gelatin films were prepared by removing silver halide from10E75 Agfa Gaevert holographic plates and soaking in aqueous dye solutions of appropriate dye concentrations for 2 minutes time duration and dried at room temperature. The thickness of the dye film used in this study was of the order of 10 microns. These films were used without any further process for this study.

Fig.4 and Fig.5 respectively show the schematic and the photograph of the experimental setup, which were used to realize the optical phase-conjugation. In this case, the standard degenerate four-wave mixing (DFWM) configuration was used. A He–Ne laser (Coherent, 31-2140-000) beam at 633 nm wavelength was used to generate the phase-conjugate wave from the Acid blue 7-doped gelatin films. The output beam from the laser was first split by a beam beam-splitter BS1 (~5:90). The beam reflected off from BS1 was used as the probe beam E_3 after reflected by the beam splitter BS_3; the transmitted beam from BS1 was further divided by another beam splitter BS2 (50:50) to provide the counter-propagating pump beams, called forward pump wave E_1 and backward pump wave E_2 respectively. Beam splitter BS3was used to direct the probe beam E_3 to the dye-doped sample and to transmit the phase-conjugated signal, which was opposite to the direction of the probe beam.

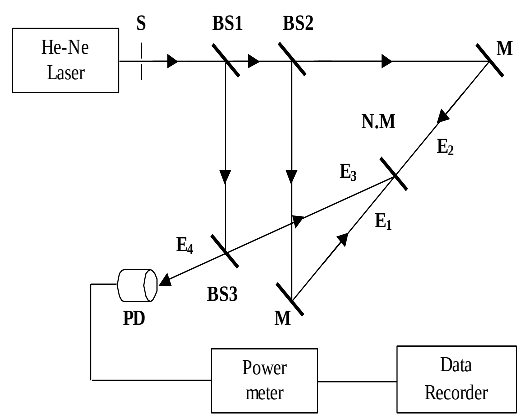


FIG. 4: Experimental set-up for the observation of PC wave, S-shutter, PD–photo detector, BS1, BS2, BS3–beam-splitters, M–mirror, N.M–nonlinear medium

The intensity of the phase-conjugated (PC) wave was measured by a photo detector fed to the digital power meter (Field Master ™ GS – Coherent). The optical path lengths of all the three beams were made equal, so that they were coherent at the sample in the DFWM geometry.The constant power ratio of the probe beam (E_3), forward-pump beam (E_1), and backward-pump beam (E_2) used in this study was ~1:10:10.

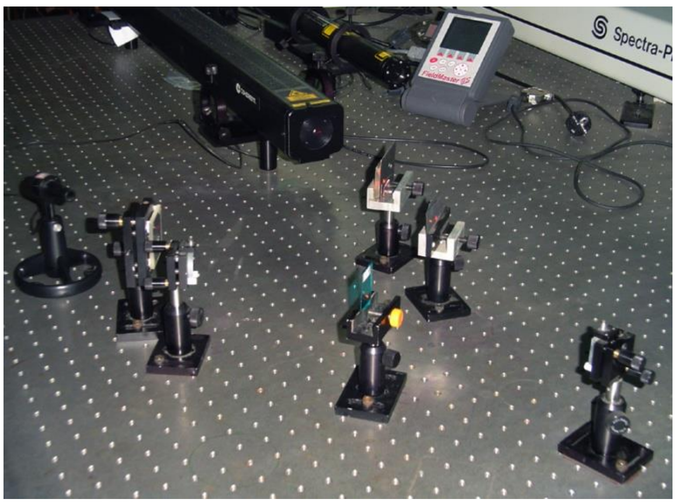


FIG.5: Photograph of the DFWM experimental set-up used for the generation of phase-conjugate waves from the Acid blue 7 dye-doped gelatin films

The spot size of each of the three (unfocussed) beams at the nonlinear medium (NM) was 1.25 mm in diameter. Interbeam angle between the probe beam and the forward-pump beam was varied between 5^0 and 10^0.

The grating spacing (Λ) of periodic lines can be determined according to the well-known formula (6) and (7). A shutter was introduced before the beam-splitter BS_1 to control the start of DFWM process. The phase-conjugate reflectivity is defined as the ratio of intensity of the phase-conjugate wave to the probe beam intensity.

5. Results and discussion

A systematic study has been made to investigate the influence of the dye concentration of the gelatin films on the intensity of the PC signal. Fig. 6 shows the PC reflectivity versus recording time for dye film with an optical density approximately equal to 0.7. For higher concentration of the films, the rate of formation is initially very slow, but the maximum value of PC signal obtained is high compared to the lower concentration films. These results confirm that the PC reflectivity increases with the increase in the concentration of the dye in the gelatin films. However, much higher concentrated plates need longer exposure time; this may lead mechanical instability and hence the reduction in PC reflectivity. A maximum phase conjugate reflectivity of 0.22% has been observed for probe beam intensity ≅0.1W/cm^2 and further increase of probe beam intensities leads to decrease of phase-conjugate reflectivity.

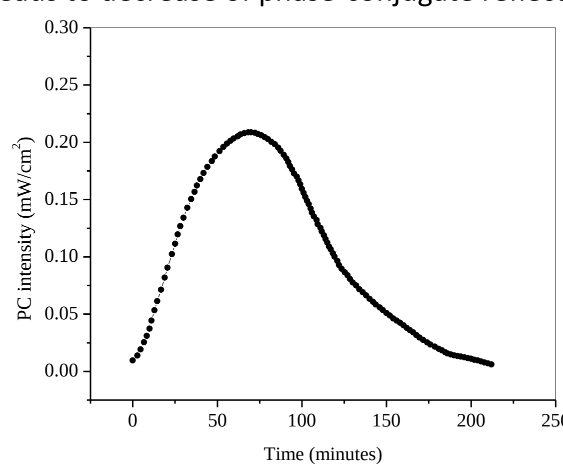


FIG. 6: Measured PC reflectivity in the Acid blue 7 dye-doped gelatin films as a function of recording time

As discussed earlier, two kinds of gratings are developed during the DFWM process towards phase-conjugate wave generation. To quantify their individual contributions, we measured the intensity of phase-conjugate signal by sequentially shutting and opening both the forward-pump (E_1) and backward-pump (E_2) beams incident onto the sample at various stages of the DFWM process. As shown in the Fig.7 when the reading beam E_1 isswitched off, the phase-conjugate signal obtained is only due to transmission grating. Similarly when the reading beam E_2 is switched off, the corresponding phase-conjugate signal obtained is only from reflection grating. We measured independently both transmitted and reflected components of the phase-conjugated signal strength at three different stages of DFWM process in progress and the results are seen in the Fig.7. From this plot itself it is very clear that the main contribution to the phase-conjugate signal comes from the E_2 diffraction from the transmission grating and the growth rate of transmission grating is comparatively very fast than the growth of reflection grating.

There are two main processes, which contribute effectively to the origin of phase-conjugation in acid blue 7 doped gelatin films. First one is formation of thermal grating and another is due to photo bleaching of dye molecules. We have experimentally verified these facts adopting the literatures [24] for the acid blue 7 dye-doped systems. Observation of decaying phase-conjugate signal gives information on whether or not there is a thermal grating contribution to the phase-conjugate signal.

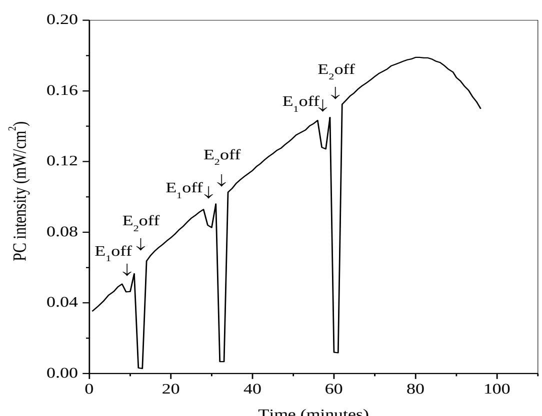


FIG. 7: Traces of the phase-conjugate signal as a function of time for Acid blue 7 dye films. A sequence of timed shutting and opening of the beams is shown in the Fig.

6. Conclusions

This paper reports the demonstration of optical phase-conjugation based on the degenerate four-wave mixing in Acid blue 7 dye-doped gelatin films by using a 633 nmHe–Ne laser of total power 35 mW. Since this work is focused on the undergraduate level experimentation, the fundamentals of optical phase-conjugation are discussed. The individual contributions from the induced holographic gratings to the phase-conjugate signal were quantified and discussed. The inexpensive dye-doped films can easily be prepared in the students laboratories for optical phase-conjugation and other nonlinear optics related studies using low power laser sources available in the market. The authors hope this article help the students who are interested in studies related to optical phase-conjugation, nonlinear optics and materials.

References :

[1]B. Ya. Zel'dovich, V. I. Popovichev, V. V. Raghul'skii, F. S. Faizullov, JETP Lett. 15 (1972) 109-113.

[2] H. Kogelnik, The Bell System Technical Journal. 44 (1965) 2451-2455.

[3] W. P. Cathey, Proc. IEEE. 56 (1968) 340-347.

[4] R. A. Fisher, *Optical Phase conjugation*, (Academic press, New York, 1983).

[5] R.W.Boyd, *Nonlinear Optics*, (Academic press, New York, 2003).

[6] E. I. Moses, F. Y. Wu, Opt. Lett. 5 (1980) 64-67.

[7] J. O. Tocho, W. Sibbett, D. J. Bradley, Opt. Commun. 34 (1980) 122-126.

[8] P. F. Liao, D. M. Bloom, Opt. Lett. 3 (1978) 3-4.

[9] M. T. Gruniesen, A. L. Gaeta, R. W. Boyd, J. Opt. Soc. Am. B. 2 (1985) 1117-1121.

[10] A. Costela, J. M. Figuera, F. Florido,Optics Commun. 100 (1993) 536-544.

[11] B. R. Reddy, P .Venkateswarlu, Journal of the Opt. Soc.Am. B. 10 (1993) 438-445.

[12] A. Costela, I. Garcia-Moreno, Chem. Phys. Lett. 249 (1996) 373-380.

[13] Kuebler S M & Denning R G, Chem. Phys.Lett. 250 (1996) 120-127.

[14] G. R. Kumar, B. P. Singh, K. K. Sharma, Opt. Commun. 73 (1989) 81-84.

[15] H. Nakatsuka, D. Masuoka, T. Yamamoto, Opt. Commun. 80 (1991) 215-218.

[16] E. Mohajerani, E .Whal, G. R. Mitchell, Opt. Commun. 92 (1992) 403-410.

[17] S. Miyanaga, H. Ohtateme, K. Kawano, H .Fujiwara , Journal of the Opt. Soc. Am.B. 10 (1993) 1049.

[18] R. K. Jain, R. C. Lind, Journal of the Opt. Soc. Am. 73 (1983) 647-653.

[19] P. Roussignol, D. Ricard, K.C.Rustagi, C. Flytzanis , Opt. Commun. 55 (1985) 143-148.

[20] D.G.Steel, S. C. Rand, J. Liu, Journal of the Opt. Soc. Am. B. 4 (1987) 1794-1800.

[21] T.Shimura, S.A.Boothroyd, J. Chrostowski, P. Myslinski, Opt. Commun. 101 (1993) 124-132.

[22] R .Vijaya, Y. V. G. S. Murti, G. Sundararajan, C .K. Mathews, P. R. Vasudeva Rao,Opt.Commun. 94 (1993) 353.

[23] K.Venkataraman, '*The Chemistry of Synthetic Dyes*', (Academic press, New York, 1971).

[24] A. Costela, I. Carcia-Mareno, J. L. Saiz, Journal of the Opt. Soc.Am. B. 14 (1997) 615-619.

Development and Validation of a Scale to Measure Physics Laboratory Attitude Level of University Students

Namudar İzzet Kurbanoğlu[1] ve Ahmet Akın[2]

[1]Sakarya University Faculty of Education,
Department of Science Education, SAKARYA-TURKEY

[2]Sakarya University Faculty of Education Department of Psychological Counseling and Guidance, SAKARYA-TURKEY

(*Submitted:* 21-01-2014)

Abstract

The aim of this research is to develop a scale for measuring physics laboratory attitudes. Factor analytic evidence from a sample (n=503) of university physics students shows the Physics Laboratory Attitude Scale is a uni-dimensional scale. The amount of total variance explained by one factor was nearly 59%. Factor loadings of the items ranged from .70 to .91. For convergent validity the relationships between the physics laboratory attitudes and self-efficacy related to learning and performance was found as .21. The internal consistency reliability coefficient of the scale, using Cronbach alpha was .94. Overall findings demonstrated that this scale had high validity and reliability scores and that it may be used as an efficient instrument in order to assess attitude level of physics laboratory students.

1. Introduction

Introductory physics is a required course for many sciences such as engineering, chemistry, mathematics, and of course, physics. Furthermore physics, a typical introductory course for most engineering, science, and mathematics students, provides a context for which students can study change in a concrete setting. Physics education starts in the fourth grade as science courses, and it continues all through high school and university. Also, physics is taught in many academic programs such as chemistry, biology, medicine, dentistry, pharmacology, environmental sciences, engineering, and architecture as a compulsory course at university level. However, students traditionally have difficulty with physics courses [1]. As a result, many students change their major after failing physics several times [2].

Introductory physics requires a laboratory to accompany the lecture sequence. There are a variety of improvements [3, 4] designed for introductory physics laboratories that show promise for improving student learning. Laboratory experiences have always been important components for the reinforcement and understanding of physics concepts. Therefore, laboratory application should be considered more seriously to make learning in physics lessons reach higher degrees than just knowledge and comprehension level.

Laboratory activities [5, 6, 7, 8], have a crucial role in the science curriculum, and science educators have proposed that many benefits accrue from engaging students in science laboratory activities. In addition, it was suggested that inquiry-centred laboratories have the potential to enhance students' meaningful learning, conceptual understanding, and their understanding of the nature of science [9]. According to Hershey [10], laboratory experiences provide students with the important experience of meeting "nature as it is, rather than in idealized form" and [11, 12] and with the opportunity to develop skills in scientific investigation and inquiry. Moreover, these experiences provide support for high-order learning skills that include observing, planning an experiment, asking relevant questions, hypothesizing, and analyzing experimental results [13, 14].

The learning in the introductory physics laboratory is related to several variables and attitude is one of these variables. Attitudes, like academic achievement, are important outcomes of science education in secondary school and university [15]. Researchers [16, 17, 18, 19] have confirmed that attitudes are linked with academic achievement. An attitude may be defined as a predisposition to respond in a favorable or unfavorable manner with respect to a given attitude object. The development of students' positive attitudes regarding science as a school subject is one of the major responsibilities of every science teacher [20].

Most of the researches on the attitude have concluded that student's attitude is an integral part of learning and that it should, therefore, become an essential component of physics concept learning pedagogy. According to Schibeci [21], attitudes toward science involve an attitude object such as "science" or "science lessons," "laboratory work" [19]. Efficiency of learning physics concept depends on students' attitudes towards physics laboratory. Ensuring that students develop positive attitudes towards physics laboratory will enhance students' abilities to learn physics topics. When tools are developed for measuring the dimensions of the factors affecting the learning of concepts in introductory physics such as attitude, physics teaching will reach the intended destination. Thus, the purpose of this study is to develop an assessment tool in order to be used to measure the attitudes of college students towards physics laboratory.

2. Methodology of Research

Participants: In an attempt to obtain a wide variation in responses, efforts were made to obtain respondents from six different universities, Turkey. Participants were 503 university students (273 male, 230 female) from department of physics who enrolled physics laboratory course.
Scale Development: The Physics Laboratory Attitude Scale was designed to measure the attitudes of students who take physics laboratory course. A total thirteen items were written. Respondents were asked to respond to each item using a 5-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree). The scale items are included in Table 2. Demographic information also was requested, including gender and department and university information.

3. Measures

Self-efficacy Related to Learning and Performance: Self-efficacy was measured by using the Turkish version of the Self-efficacy related to learning and performance subscale of the Motivated Strategies for Learning Questionnaire (MSLQ) [22]. Turkish adaptation of this scale had been done by

Büyüköztürk, Akgün, Özkahveci, and Demirel [23]. The Self-efficacy subscale consists of eight items and each item was rated on a 7-point scale (1= not at all true for me to 7= very true for me). As a result of factor analysis in construct validity, it was found that factor loadings of items were between .52 to .65. In the reliability study, the internal consistency alpha coefficient was calculated .86.

Procedures: Physics laboratory classes (ranging in size from 20 to 30 students) were selected randomly by the on-site data collector at the six universities. The scale was administered in a group format to each physics laboratory courses the first and second semesters during the 2011-2012 academic years. Prior to administration of measures, all participants were told about purposes of the study and administration typically required 10 to 15 minutes. The person administering the scale collected and returned the scales to the researcher. Completion of the questionnaires was anonymous and there was a guarantee of confidentiality. The instruments were administered to the students in groups in the classrooms. Analysis of the data took place in two ways: (a) calculating item total correlation estimates for item analysis to identify any faulty items, obtaining internal consistency reliability estimates of the scale scores and b) testing the construct validity by exploratory factor analysis and the convergent validity by estimating the relationship between physics laboratory attitude and self-efficacy related to learning and performance subscale of the motivated strategies for learning questionnaire.

4. Results of Research

Item Analysis and Reliability: The corrected item-total correlations of the 13 items ranged from .64 to .76. Estimated Cronbach's reliabilities were .94. Table 1 shows means, standard deviations, and the item total correlations of the 13 items.

Table 1: Means, standard deviations, and item–total correlations of the draft Physics Laboratory Attitude Scale

Items	$\overline{X}$	**SD**	***rjx***	Items	$\overline{X}$	**SD**	***rjx***
Q1	4,50	0,80	.64	Q8	4,53	0,79	.76
Q2	4,45	0,82	.72	Q9	4,41	0,83	.75
Q3	4,53	0,79	.72	Q10	4,30	0,92	.76
Q4	3,98	1,00	.65	Q11	4,47	0,85	.75
Q5	4,28	0,86	.75	Q12	4,30	0,91	.69
Q6	4,34	0,89	.71	Q13	4,42	0,89	.73
Q7	4,31	0,90	.75				

	Items	Factor loadings
Q1	Making experiments in physics class increases my interest to the subject.	0.70
Q2	Learning new information while having physics experiments.	0.77
Q3	Without doing physics experiments the information will not be permanent.	0.77
Q4*	Making experiment in physics lesson does not affect my performance in a positive way.	0.70
Q5	Laboratory experiment teaches how to work with discipline.	0.79

Table 2: Items and principal components un-rotated factor/structure component matrix solution of the Physics Laboratory Attitude Scale

Q6	Having more physics laboratory will increase my interest.	0.76
Q7	Laboratory works makes me more practical in my daily life.	0.80
Q8*	Physics experiments do not affect the learning of the physical terms.	0.81
Q9	It is hard to learn physical terms without doing experiment.	0.79
Q10	Physics lessons should have more laboratory work.	0.80
Q11	Having laboratory experiment will contribute to the development of mental and manual ability.	0.79
Q12	It is more interesting to learn physical terms by doing experiments.	0.74
Q13*	Doing experiments in physics courses reduces my interest in the subject.	0.78

*Negatively keyed items.

Construct Validity: For construct validity, exploratory factor analysis was conducted to validate the underlying structure of the model. Prior to the conduct of exploratory factor analysis, Kaiser-Meyer-Olkin (KMO) statistic and Bartlett's Test of Sphericity was calculated. The KMO value (KMO=.952) indicated that the degree of common variance among the variables was marvelous.

The Bartlett's test of sphericity indicated a Chi square 4789.335 with an observed significance level of $p<.001$. Based on the results, it was inferred that the relationship between the variables was strong and appropriate for factor analysis. One factor explained 59% of variance. All of the items had factor structure coefficients exceeding an absolute value of 0.30.

Convergent Validity: Convergent validity of the Physics Laboratory Attitude Scale scores was assessed via correlation with the self-efficacy related to learning and performance scale. The Physics Laboratory Attitude Scale correlated positively with the self-efficacy related to learning and performance subscale of the motivated strategies for learning questionnaire (r =.21).

5. Discussion

Physics laboratory is one of the most important courses for undergraduate students majoring in applied chemistry, polymer chemistry, material chemistry, chemical engineering, life science, and environmental engineering and science. Therefore, in this study it was developed an assessment tool in order to be used to measure the attitude level of college students' towards physics laboratory.

Based on the principle of measuring physics laboratory attitude by a global scale, we proposed a thirteen-item global measure of physics laboratory attitude. The proposed physics laboratory attitude scale was examined with a sample of 503 university students. In general, the physics laboratory attitude scores demonstrated good internal reliability, construct validity, and convergent validity. Specifically, the physics laboratory attitude correlated moderately with self-efficacy related to learning and performance. As a result, it can be said that this scale had high validity and reliability scores. Therefore, the physics laboratory attitude scale could serve as a useful tool for engineering, chemistry, mathematics, and of course, physics fields to collect information about the physics laboratory attitude levels of students. However, further studies that will use the physics laboratory attitude scale are important for its measurement force.

References:

[1] Byun, T., Ha, S. and Lee, G. (2008). Identifying student difficulty in problem solving process via theframework of the house model, *Proceedings of the Physics Education Research Conference* (Vol.1064, pp. 87-90). Edmonton, Alberta: AIP.

[2] Tuminaro, J. and Redish, E. (2004). *Understanding students' poor performance on mathematical problem solving in physics,* Paper presented at the Physics Education Research Conference.

[3] Hake, R. R. (1998). Socratic pedagogy in the introductory physics laboratory, *Am. J. Phys.* **66**: 64-74.

[4] Laws, P. W, Sokoloff, D. R. and Thornton, R. K. (1995). *Real Time Physics,* (OR: Vernier Software).

[5] Hofstein, A., and Lunetta, V. N. (1982). The role of the laboratory in science teaching: neglected aspects of research, Review of Educational Research, **52**: 201–217.

[6] Hofstein, A. and Lunetta, V. N. (2004). The laboratory in science education: foundations for the twenty-first century, Science Education, **88**: 28–54.

[7] Lunetta, V. N. (1998). The school science laboratory: Historical perspectives and context forn contemporary teaching. In B. Fraser & K. G. Tobin. (Eds.), *International handbook of science education* (pp.249-262). Dodrecht, The Netherlands: Kluwer.

[8] Tobin, K. G. (1990). Research on science laboratory activities. In pursuit of better questions and answers to improve learning, School Science and Mathematics, **90**: 403–418.

[9] Taitelbaum, D., Mamlok-Naaman, R., Carmeli, M. and Hofstein, A. (2008). Evidence for teachers' change while Participating in a continuous professional development programme and implementing the inquiry approach in the chemistry laboratory. International Journal of Science Education, **30**(5): 593–617.

[10] Hersey, T. (1990). *Teacher's guide to advanced placement courses in physics: Physics B and Physics C.* New York: Advanced Placement Program, The College Board.

[11] Hoffer, T., Radke, J. and Lord, R. (1992). Qualitative/quantitative study of the effectiveness of computer-assisted interactive video instruction: The hyperiodic table of elements. Journal of Computers in Mathematics and Science Teaching, **11**: 3–12.

[12] Shymansky, J., Kyle, W. and Alport, J. (1983). The effects of new science curricula on student performance, Journal of Research in Science Teaching, **20**: 387–404.

[13] Bybee, R. (2000). Teaching science as inquiry. In J. Minstrel and E. H. Van Zee (Eds.), Inquiring into inquiry learning and teaching in science. Washington: AAAS.

[14] Hofstein, A., Shore, R. and Kipnis, M. (2004). Providing high school chemistry students with opportunities to develop learning skills in an inquiry-type laboratory: a case study. International Journal of Science Education, **26**: 47–62.

[15] Cheung, D. (2009). Students' Attitudes toward Chemistry Lessons: The Interaction Effect between Grade Level and Gender, Research in Science Education, **39**: 75–91.

[16] Weinburgh, M. (1995). Gender differences in student attitudes toward science: A meta-analysis of the literature from 1970 to 1991, Journal of Research in Science Teaching, **32**: 387–398.

[17] Freedman, M. P. (1997). Relationship among laboratory instruction, attitude toward science, and achievement in science knowledge, Journal of Research in Science Teaching, **34**(4): 343–357.

[18] Bennett, J., Rollnick, M., Green, G. and White, M. (2001). The development and use of an instrument to assess students' attitude to the study of chemistry, International Journal of Science Education, **23**(8): 833–845.

[19] Salta, K. and Tzougraki, C. (2004). Attitudes toward Chemistry among Eleventh Grade Students in High Schools in Greece, Science Education, **88**: 535.

[20] Oskamp, S. and Schultz, P. W. (2005). Attitudes and opinions (3rd ed.). Mahwah, NJ: Lawrence Erlbaum Associates.

[21] Schibeci, R. A. (1983). Selecting appropriate attitudinal objectives for school science, Science Education, **67**: 595-603.

[22] Pintrich, P. R., Smith, D. A. F., Garcia, T. and McKeachie, W. J. (1991). *A manual for the use of the motivated strategies for learning.* Michigan: School of Education Building, The University of Michigan. (ERIC Document Reproduction Service No. ED338122).

[23] Büyüköztürk, Ş., Akgün, Ö., Özkahveci, Ö. and Demirel, F. (2004). The validity and reliability study of the Turkish version of the Motivated Strategies for Learning Questionnaire, Educational Science: Theory and Practice, **4**(2): 207–239.