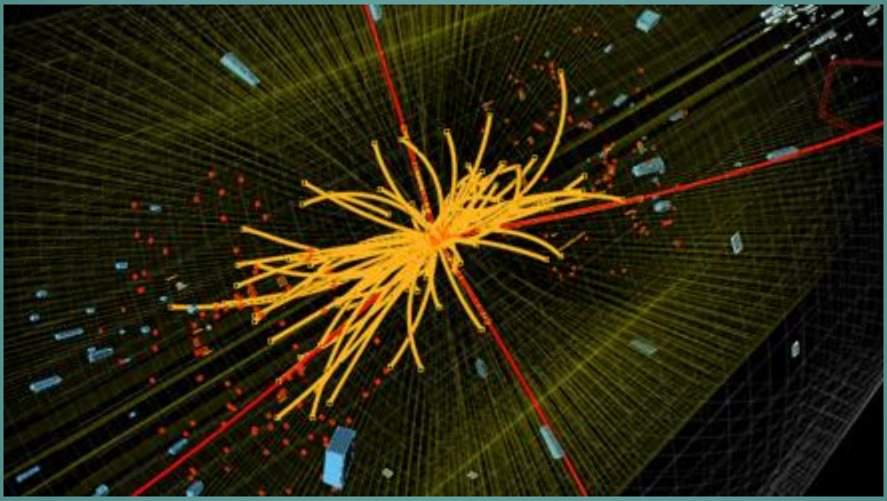
Vol 28 No. 2 Apr - Jun 2012 ISSN 0970-5953

INDIAN ASSOCIATION OF PHYSICS TEACHERS
IAPT

PHYSICS EDUCATION



Higgs Candidate

Proton-proton collision in the CMS experiment producing four high-energy muons (red lines). The event shows characteristics expected from the decay of a Higgs boson but it is also consistent with background Standard Model physics processes (Image: CMS)

www.physedu.in

Physics Education **Apr - Jun 2012**

Volume 28, Number 2

In this Issue

EDITORIAL

(*Submitted* 30-06-2012)

IIT JEE and College Entrance Examinations

IIT-JEE is indeed a most challenging college entrance test in the country and is known for its largely unblemished record for about half a century. It is considered a challenge to crack by many a bright student of the final year in School, a cherished dream for most and is regarded as one exam that will ensure a bright career - an obsession, particularly among the middle class. Like all entrance examinations it is simply a tool to select a few for admission when the number aspirants exceed the capacity. It does not really test the aptitude of the candidates; neither does it try to know their motivations for the programs. Just a simple test, perhaps to some extent evaluates the extent of the preparation of the candidates for the curriculum ahead. On the basis of the rank (s)he gets, the successful students make a more or less a firm decision on the branch of Engineering (or Science), they will get to study a few semesters later after the core initial years and eventually graduate. If you have top ranks you will get to choose (and most do) the most prestigious Computer Science label, whether or not you have natural inclinations for it. Whatever is your rank, if it is good enough to earn you an admission in one of the IITs, you are sure that a lucrative career is waiting for you at the end of the four year term. It may not even have to be technology related occupation for which you are given the rigorous training. Better ones apparently choose Management, Finance and even Civil Services. Many doors open up for you and it is important that you acquire the IIT label. Preparing you to tackle this exam is therefore has become a big business. When coaching schools advertise that they have programs that take in 9th class students for their “early start” and find many wish to avail such extended tuitions, one can understand the level of seriousness attached to this simple test. Coaching Institutes tell the students how to optimize their scores, tell them a few tricks and strategies and do better in the test than what their natural talent would warrant. IITs, in turn, are trying their level best to so constantly modify the test that such tricks do not give an unfair advantage to the specially coached student. This is a never ending duel.

As IITs keep trying steps to minimize the coaching school distortion in their admission, the nation is rightly concerned with the enormous pressure the school children face in writing many different such entrance tests just to ensure that they will get admission somewhere. Proliferation of tests and huge uncertainty is indeed a cause for worry. Minister for Human Resource Development Kapil Sibal thought a simple prescription – one National Joint Admission Test and accord some weightage to the performance of the school final examination (suitably normalized for each board). He thought this way students will pay attention to their respective School Board examinations. IIT faculty reacted at attempts to ‘dilute’ their pre-eminent JEE and felt abhorrent at the idea of ministry interfering in the admission process, which is the natural domain of their Academic Senates. They felt that tampering with one well run program is no way to remedy systemic faults elsewhere. Finally a compromise has been reached by which a Common Entrance test will indeed be held, but the IIT’s will use both common entrance test and the School Board marks only as a screening process. They will conduct an advanced more incisive test for this manageable set of screened aspirants. Hopefully this will also let them attempt to further minimize the distortions introduced in the process by the Coaching School. Whether this will indeed be pressure busting exercise for the aspiring students remains to be seen.

As teachers we must be concerned with another aspect. The objective of any examination/test is diagnostic. It is to let the student know whether (s)he has grasped to the intended level the instructions received and for the teacher to know if (s)he has succeeded in the task. Are we distorting the various instructional programs, particularly in the Schools, to simply become an objective of students getting through well in the end of term examinations? Is the object of pedagogy simply to prepare the students for the examinations that leads to the certificates and diplomas they will earn? Where is the joy of learning? Where is the curiosity to know? Is it just a game we are playing with no consequence to the needs of the Society? Are our Schools and Colleges merely treated as some sort of a filter to let a few people earn more than others in the job market. This indeed is unsatisfactory. Quite clearly we know that the Teacher is a better judge than any single or combination of examinations, however objective and impartial the test may be. The society must place greater importance to the Continuous Comprehensive Evaluation (CCE), which can be performed by the teacher only. It is time we give more importance to the Teachers' recommendation and their opinions on their ward. This is perhaps what was meant in the concept of the veneration of *Guru*, emphasized in our culture. In our quest for objective evaluation, we seem to have slowly degraded the teacher and forgotten the vital role (s)he is called upon to play in the learning program. Joint admission Board of IIT has no way of knowing what the school teacher thinks about the aspirant. In contrast, Admission Committees abroad pay a lot more attention to the Statement of Purpose (SOP) that the student applicant turns in with the application and looks closely the Recommendation Letters of the Teachers, who knows the students well in view of their long association. I would like to see a day when it will be natural for Admission Boards of not only IITs, but many organizations to add Teacher input as a vital component in admissions or recruitments.

I will end this piece with an optimistic note. With the enactment of Right to Education (RTE) bill there is a clear signal that it is the right of every child to be in the school between 6 and 14 years and the School should be, the same Act ordains, *child centric.* In order to achieve this you have to necessarily empower the Teacher, since (s)he is the one to2 be aware of what it is to be *child centric.* Let us once again look forward for the society to place the Teacher (*guru)* at the exalted level. May we, as teachers equip ourselves with the skills and attitudes to be worthy of that respect.

R Ramachandran
rr_1940@yahoo.co.in
Consulting Editor
Pune 411008

Evaluation of density of electron states – A simple approach

N.V. Karthikeyan and K. Navaneethakrishnan
Department of Physics, K.L.N. College of Engineering,
Pottapalayam-630611, India.

(*Submitted* Aug 2011)

Abstract

The density of electron states in one, two and three dimension is evaluated by following a simple procedure which makes the significance of Fourier space apparent. Using this approach, the relativistic electron gas is also discussed. The approach is easily adaptable to an undergraduate physics course.

PACS numbers : 71.10.ca.,81.05.BX,
Key words : Electron gas; Density of states; Relativistic electron gas.

1. Introduction

Density of states, D(E) = dn/dE, which is the number of energy states per unit energy at E, is an important quantity in many – particle systems , such as a gas or a solid. In terms of D(E),many physical quantities such as the number of particles or the total energy or any other quantity such as the specific heat may be obtained.

$$N = \int dE\, D(E) f(E) \qquad (1)$$

$$E = \int dE\, E\, D(E) f(E) \qquad (2)$$

where *f(E)* is the suitable distribution function for the many – particle system. For example, it is the Fermi –Dirac distribution function for a many electron system. Several text books on Solid State Physics [1] and Statistical Mechanics [2] contain detailed discussions on *D(E).*

At present there is considerable interest on the physics of low dimensional systems such as quantum wells (QW),quantum well wires (QWW), quantum dots (QD) and super lattices (SL).[3]. Fabrication, observation of novel properties and applications of these systems have revolutionized the semiconductor field. Several interesting properties exhibited by these systems arise due to the peculiar nature of *D(E)* in different dimensions. Properties in between these dimensions (quasi two dimension, for example) may also be obtained. In general, the derivation of *D(E)* in different dimensions is different [3]. In the present work we give a unique, yet simple method of evaluation of the density of states in different dimensions.

2. Density of electron states.

A. Non relativistic case

We consider a non interacting electron system (a gas) to which Fermi – Dirac statistics is applicable. The conventional derivation for *D(E)* is as follows, for a 3D gas. At 0 K, we have

$$2 \times \frac{4}{3}\pi k_F^{\,3} \times \left(\frac{L}{2\pi}\right)^3 = N \qquad (3)$$

where $\frac{4}{3}\pi k_F^{\,3}$ is the volume of the Fermi sphere, $\left(\frac{L}{2\pi}\right)^3$ is the density of electron states obtained from periodic boundary conditions, N is the total number of electrons and the factor 2 arises due to spin degeneracy. The above result not only gives, $k_F = (3\pi^2 N/V)^{1/3}$, $E_F = (\hbar^2 k_F^2)/2m$ and also shows that $D(E) \propto E^{1/2}$. The density of states varies as the square root of energy at the Fermi energy. Usually, this is taken to be valid for all $E \le E_F$.

The derivations is one and two dimensions are similar to the above. The results are

$$2 \cdot k_F \cdot \left(\frac{L}{2\pi}\right) = N \qquad \text{(1D)} \qquad (4)$$

and

$$2 \cdot \pi k_F^2 \cdot \left(\frac{L}{2\pi}\right)^2 = N \qquad \text{(2D)} \qquad (5)$$

Thus $D(E)$ is proportional to $E^{1/2}$ in 3D, constant (independent of energy) in 2D and $E^{-1/2}$ in 1D.Thus the evaluation of D(E) requires the information available in momentum space. In the discussion presented above, the use of this information is indirect. To bring out the information in a more apparent manner we suggest writing $D(E) = \frac{dN}{dE} = \frac{dN}{dk}\frac{dk}{dN}$ and use $E = \frac{\hbar^2 k^2}{2m}$ for a free particle.The periodic boundary conditions on free particle energy eigen functions lead to $e^{ikx} = e^{ik(x+L)}$. This gives $e^{ikL} = e^{2\pi ni}$ where n is an integer. Quantization follows and we get $k_n = \frac{2\pi n}{L}$, where L is the size of a box; many such huge boxes are assumed to fill the space.

In 1D, $\frac{dn}{dk} = \frac{L}{\pi}$ from the above expression for quantization of wave vectors, We multiply the above by 2 to take into account spin degeneracy. Hence

$$D(E) = \frac{L}{\pi}\frac{m}{\hbar^2 k} = \frac{L}{\pi}\frac{m}{\hbar^2}\frac{\hbar}{(2mE)^{\frac{1}{2}}} = \frac{L}{\pi}\left(\frac{m}{2\hbar^2}\right)^{\frac{1}{2}} E^{-\frac{1}{2}}.$$

Hence the density of states per unit length g(E) is $\left(\frac{m}{2\pi^2\hbar^2}\right)^{\frac{1}{2}} E^{-\frac{1}{2}}$.

In 2D, we write the quantization for k_x and k_y as

$$k_x = \frac{2\pi}{L} n_x \text{ and } k_y = \frac{2\pi}{L} n_y .$$

Therefore, $dk_x\, dk_y = \left(\frac{2\pi}{L}\right)^2 dn_x\, dn_y$.

In polar coordinates this becomes $2\pi k\, dk = \left(\frac{2\pi}{L}\right)^2 dn$.

Hence $\frac{dn}{dk} = 2\pi k\left(\frac{L}{2\pi}\right)^2 = \frac{4\pi k A}{4\pi^2} = \frac{k}{\pi}A$. The factor 2 is used for spin degeneracy. A is the area L^2. Hence $D(E) = (Am/\pi\hbar^2)$. So, density of states per unit area of the sample is $m/\pi\hbar^2$.

A similar procedure in 3D gives for the density of states per unit volume, $1/\pi^2\hbar^3(2m^3)^{1/2}E^{1/2}$.

These results are the same as those obtained earlier.

B. Relativistic Electron gas.

In the relativistic case we use $E^2 = c^2p^2 + m_0^2c^4$, where m_o is the rest mass of the electron. In the extreme relativistic case $cp \gg m_0c^2$, so $E = cp = c\hbar k$. This immediately leads to the following expressions for the density of states (per unit length in 1D, area in 2D and volume in 3D):

$$g(E) = \begin{cases} \dfrac{2}{c\hbar} & \text{in 1D} \\[2ex] \dfrac{1}{2\pi(c\hbar)^2} E & \text{in 2D} \\[2ex] \dfrac{1}{2\pi^2(c\hbar)^3} E^2 & \text{in 3D} \end{cases}$$

A comparison of the non relativistic and relativistic cases is provided in Table 1

3. Applications of relativistic case

It is interesting to note that the average energy of a 3D relativistic electron gas at 0 K is given by

$$\langle E\rangle_{3D,R} = \int_0^{EF} dE\, E\, D(E) f(E) \Big/ \int_0^{EF} dE\, D(E) f(E)$$

where $f(E)$ is the Fermi –Dirac distribution function whose value is 1 at 0 K for $E \le E_F$

We obtain $\langle E \rangle_{3D,R} = (3/4)E_F$ The result for the non relativistic case is $(3/5)E_F$.

As another application of the relativistic case, we obtain the Fermi energies in 1, 2 and 3 dimensions.

We obtain

$$n=\int_0^{EF} g(E)dE = \begin{cases} \frac{2}{c\ \hbar} E_F & \text{in 1D} \\ \frac{1}{2\pi c^2 \hbar^2} {E_F}^2 & \text{in 2D} \\ \frac{1}{3\pi^2 c^3 \hbar^3} {E_F}^3 & \text{in 3D} \end{cases}$$

where n is the electron concentration.

Hence E_F is proportional to n (no. of electrons per unit length, in 1D), $n^{1/2}$ (electrons per area, in 2D) and $n^{1/3}$(electron concentration per volume, in 3D)

One can evaluate other properties like specific heat etc.

Conclusions:

We have provided a simple procedure to evaluate the density of electron states in different dimensions which brings out the significance of momentum space. Application to relativistic electron gas gives interesting results for Fermi energy and average energy. These results differ substantially from the corresponding expressions of the non relativistic case.

Table 1. Variation of physical quantities in 1D,2D and 3D : Relativistic & Non-Relativistic cases

Quantity	NR			R		
	1D	2D	3D	1D	2D	3D
g(E)	$E^{1/2}$	Constant	$E^{-1/2}$	Constant	E	E^2
E_F	n^2	n	$n^{2/3}$	n	$n^{1/2}$	$n^{1/3}$

References

1. C.Kittel, Introduction to solid state physics, 7th edition (John wiley,2003)

2. R.K.Pathria, Statistical Mechanics, (Pergamon Press, 1972)

3. Paul.Harrison, Quantum wells, wires and Dots: Theoretical and computational Physics of Semiconductor Nanostructures,3rd Edition,(John wiley,2010)

Activity-based Interactive Engagement in Science (Physics) Laboratory

An Approach to Stimulate Pre-Service Teachers' Conceptual Understanding

Funda Ornek[1] and Metin Orbay[2]

[1] Math & Science , Bahrain Teachers College,
University of Bahrain,
Kingdom of Bahrain
(fundaornek@gmail.com)

[2] School of Education,
University of Amasya, Turkey.

(*Submitted 5/7/2010*)

Abstract

The purpose of this study was to investigate pre-service teachers' views with regard to the teaching approach used in the course of Science Laboratory and Its Applications based on activities and interactive engagement (Örnek, 2008; Orlik, 2005; Ornek, 2006) to stimulate their conceptual understanding of science concepts. Its aim was also to consider how physics may aid the teachers/instructors in bringing physics successfully to all levels. An open-ended exit survey was administered to 216 pre-service teachers, who were at Balikesir University in Turkey at the end of the fall 2007 semester and 210 pre-service teachers at the end of the spring 2008 semester and 140 pre-service teachers at Amasya University in Turkey at the end of the Fall 2008 semester to collect data in terms of students' conceptions about the course and the approach used. In addition, semi-structured interviews with 62 volunteer pre-service teachers from Balikesir University and 20 volunteer pre-service teachers from Amasya University were conducted. The findings of this study have potential in translating research insights into practical recommendations for teachers regarding with making science labs more effective and efficient and provide guidelines for teachers (Chin, 2007) to increase students' construction of knowledge in science and making connections to the real life and other fields such as health science.

Key words: Activity-based learning, interactive, physics lab, science lab

1. Introduction

Students' learning experiences, their experiences with the teacher, teaching approaches of the teacher, and the subject matter itself play a crucial role in formation of students' conceptions concerning the course. Students' conceptions affect their behavior, influencing what the learner selects from the class environment, how they will react to the teachers, the materials being used and their classmates (Shah, Riffat, & Reid, 2007). Science laboratories that are one of the class environments are broadly considered as a key component of science instruction since most science fields such as physics, chemistry, or biology are founded on activity-based investigations in the world even though observations, inferences, imagination, and creativity are parts of nature of science. A constructivist-inquiry approach can be used for effective teaching and learning science as a contemporary teaching approach that unfortunately teachers/instructors sometimes may not be able to manage properly to arrive at competent level of students' knowledge and skills (Orlik, 2005).

The usage of this approach is based on students' actively participation and emphasizes the cooperative and constructive nature of scientific activity. Therefore, students are expected to engage in explaining concepts with their peers and teacher/instructor (Akkus, Gunel, & Hand, 2007). Likewise, students are encouraged to make explicit and sound connections among questions, observations, and evidence (Akkus et al., 2007). This approach requires a high level of interaction among the students, the teacher/instructor, the area of study, and available resources (Aladejana & Aderibigbe, 2007). Traditional laboratory activities are already designed for students who follow instructions in their lab manual. Science teachers and science laboratory manuals in general emphasize procedures (Perkins-Gough, 2007). Teachers/instructors often prepare the questions and the tasks to be followed by students before starting the experiments. In other words, problem/questions and procedure/method are always provided to students. Solution can be either provided to students or constructed by students (Fay & Bretz, 2008). In general, the role of the teacher in the traditional lab classes is to transmit the knowledge to students. Students are also expected to receive or memorize the given information. This kind of classes likens teacher-centered instruction even though experiments are involved. Thus traditional teacher-centered instruction can have an absence of student-centered learning activities (Akkus et al., 2007).

According to the most research, it has been found that student achievement and skills improved when an activity-based interactive engagement curriculum was used to teach science (Aladejana & Aderibigbe, 2007; Turpin & Cage, 2004; Welch & Walberg, 1972; Fraser, 1986; Bredderman, 1983; Wong & Fraser, 1996). Students can recognize, design, and apply fundamental science concepts into practice (Akkus et al., 2007). Since students do have some difficulties in understanding the underlying scientific concepts, this activity-based teaching approach can have the potential to excite and enlighten students about the importance of science in their daily lives. It exposes teachers/instructors to classroom activities that combine excitement, cooperative learning, and participatory activities with real-world relevance" (Conlon, 2004). These approaches will motivate students to pursue careers in science, engineering, and technology if it is started to implement at the early level of schools such as primary and secondary schools.

2. Theoretical Underpinnings

The principal theoretical framework underlying this study is social constructivism that focuses on the environment in which the knowledge is formed and how this environment may influence the individual (Bodner, Klobuchar, & Geelen et al., 2001). Social constructivism would occur when a group of people collaborate to solve a problem or create and prepare an activity. Each person brings a little bit to the interaction, and together they can build knowledge that leads to a solution which each would have been unable to do alone. In this case teacher assists student performance by guiding the discourse among students to support student learning (Chin, 2007).

Meaningful learning in constructivist approach is a cognitive process that students make sense of the world with regard to the knowledge that they have already constructed (Wilson, 1996; Fosnot, 1996; Steffe & Gale, 1995). Some features of the constructivist classroom settings include carrying out some experiments or activities, engaging in meaningful problem-based work and working collaboratively with each other. In other words, students can construct knowledge and skills through their own experience (Windschitl, 2002) and they can construct an accurate representation of the real world (Doolittle & Camp, 1999). The science laboratory environment or setting is a major path for the students to be involved actively and to perform activities, construct new knowledge-sometimes modify the previous knowledge- onto their existing mental framework for meaningful learning to take place (Huitt, 2003; Sherman, 1995).

In addition, constructivism is used to describe a large number of different theories which fall under the general thought that knowledge is constructed (Philips, 1995). Rather than receiving knowledge as a transmission of knowledge that is already complete and ready to use, students build their knowledge on the foundation of what they have previously learned. Students approach a situation with prior knowledge influencing them (Hoover, 1996). For example, students in a physics class will apply what they already know about how objects react when they are sitting in a car going around a sharp turn (Churukian, 2002). The different theories of constructivism are often delineated by adjectives which describe their primary focus. There are different constructivism thoughts of which one is social constructivism is central for this study.

A Purpose of the Study

The purpose of this study was to investigate pre-service teachers' views with regard to the course of science laboratory and its applications. Its aim is also to consider how physics aid the teachers/instructors in bringing physics to all levels.

The focus of the study was: What are pre-service teachers' views and expectations of the role of activity-based approach in enhancing conceptual understanding of physics and constructing knowledge of physics by means of activity-based and interactive engagement?

B Structure of the Course and the Teaching Method

A course named "Science Laboratory and its Applications" for pre-service teachers was taught by using an activity-based interactive engagement approach in Turkey. This course was a two-semester course that the fall semester covers fluids, optics (light), energy, energy transfer and transformation, and sound and the spring semester covers electricity and magnetism. The teaching approach was the first time applied in the universities listed before.

In Science labs, the teacher should assist students in making sense of scientific ideas and support them in applying the ideas (Chin, 2007) and making connections to real life and other disciplines while teaching science. Whereas in traditional science labs, firstly content knowledge or theoretical information is provided by the teachers then students are asked to conduct experiments based on the lab manual which is a kind of cook book. For activity-based interactive engagement approach, students are asked to do activities and the teacher asks questions for brainstorming to explore students' views and gives importance on their views even though their ideas can be different from the scientific views. During continuing discussions, the teacher always asks conceptual questions to elicit students' ideas and facilitate productive thinking, gives constructive and encouraging feedback to students encourages multiple responses (Chin, 2007). These all help students construct knowledge of science by the constructivism- inquiry approach (Roth, 1996; Van Zee & Minstrell, 1997a).

The other advantageous of using this approach is that students are asked to make connections to the real life and other disciplines such as chemistry, biology, or health science. Before this contemporary teaching approach, the traditional teaching method as known, there are experiments that students are supposed to do and write reports after that, was employed to teach this course. Also, students were not able to do same experiments at the same course period due to the lack of materials so students had to do different experiments in each course period. In that approach, students memorized some information or were just taking measurements instead of construction knowledge.

The course had four sections and each section had about 40-55 students and was divided into 8 small groups. Each group had six or seven members and each one had to prepare and present one activity and manage the classroom in each class period as if he/she was a teacher. The students in each group presented in every week alternately. The purpose was to have students be actively involved and discuss the results. Each group presented the same topic in a class period such as density.

An example from the activities that was conducted is shown in Appendix A: Students do not have to do the same activity exactly shown in Appendix A. They need to do an activity about density though. The noteworthy thing about these activities students experienced are very easily duplicated with common, ordinary household items that can be probably found around your school or homes.

As seen in the activity in Appendix A, pre-service teachers should start with an interesting question to draw attention and make students engage in the activity. In other words, brainstorming should take place and is important before starting the activity which is called as pre-activity. During brainstorming a teacher or instructor can ask several questions and make students be curious about the phenomena. After that while the teacher is doing the activity, she/he asks students do it at the same time. In the time of performing activities which is called as during activity as well, you can ask some questions such as “what will happen now?” or “what did you observe?” So the teacher can involve students in the course actively.

In the subsequent section (What is happening?) called as post-activity as well, the teacher can ask students some questions such as what is happening and why this happened in science lab discourse to scaffold student’s thinking and assist students construction of scientific knowledge (Chin, 2006) to encourage them to explain the reasons of the phenomena. If their explanations and predictions are different from scientific knowledge, then ask more questions to make them to elaborate on their previous answers and ideas about the phenomena and assist them construct conceptual knowledge. Students may have some misconceptions or some knowledge that they bring from their previous learning or experience.

Thus, this teaching approach can provide a resource for students to clarify misconceptions regarding material covered in the other aspects of the course. For instance, some students may misuse the concepts of density and mass such as if one thing has more weight then it sinks first. Whereas it should be examined with respect to its density.

The other most important issue is that students do not have enough knowledge about the nature of science (NOS) because their teachers or instructors unfortunately are not in general aware of the importance of NOS in teaching science courses, so do students. One of the most important aspects of NOS is *tentativeness of scientific knowledge* that means all scientific knowledge is subject to change (Lederman et al, 2002; Akerson & Hanuscin, 2008; Abd-El-Khalick & Akerson, 2004; Akerson, Abd-El-Khalick, & Lederman, 2000). When new knowledge is found, the old one should be modified or omitted. This aspect of NOS is really crucial because if teachers, instructors, or prospective teachers do not know about that they can teach that scientific knowledge is durable and not easily changed. So, they learn and trust in old knowledge and they will not update their knowledge. For instance, when we did an activity concerning the concept of “static electric”, I asked students a question with regard to the real life. The question was “how can electric sparks occur in air?” An electric spark may be occasionally seen when you flip a switch, unplug a power cord, shake one’s hand, or open a door. All my students, approximately 200, answered this question incorrectly. Their responses were that during a spark, charge is transferred from one object to another object. In other words, during a spark electrons could simply jump from the negative object to the positive object. Whereas this knowledge is old knowledge and this is not a possible mechanism because a free electron can travel only about $5x10^{-7}$ meters before colliding with a gas molecule (air molecule or ion) and losing much of its energy (Chabay & Sherwood, 2002). The correct answer to the question is related to positive ions and electrons moving in ionized air. This ionized air model is significantly better explanation that jumping electrons model because no particle in the ionized air model travels farther than approximately $5x10^{-7}$ meters.

Therefore some activities can positively influence students' views of NOS. Smith, Maclin, Houghton, & Hennessey (2000) also found that appropriate science practices could positively influence students' views of NOS. The results of the study may provide evidence that activity-based instruction can be effective for developing students' NOS views.

Furthermore, in the activity, the section of "what to do" is the step three. That step has also other aspect of NOS that observations can be affected by the observers' anticipation, pre-knowledge, experience, background, and preconceptions (Chen, 2006). After students make careful observations, they will make some inferences based on their pre-knowledge and anticipation. So they will learn how to use available data to draw results.

3. Methodology

Settings and Participants

216 pre-service teachers in the fall 2007 and 210 pre-service teachers in the spring 2008 from Balikesir University and 140 pre-service teachers in the fall 2008 from Amasya University in Turkey were participated in this study. 62 pre-service teachers from Balikesir University and 20 from Amasya University also volunteered for interviewing in this study.

Data Collection and Analysis

An open-ended exit survey, which was adapted from Ornek's study (2006) and modified, to explore pre-service teachers' thoughts about the course and the manner it was taught in the course. This survey includes 10 open-ended questions and establishes students' views about the course. The questions are on what students liked and disliked about the course as well as what they would change in the course and how their activities provide them to understand physical phenomenon. The semi-structured interviews were conducted after the final exams since we were the instructor of the courses students may think their responses can affect their final grades. We used a qualitative approach to address our research question and analyze the data. The qualitative details from the semi-structured interviews were used not only to validate the survey results, but also to provide some plausible interpretations for the findings drawn from this study.

Each volunteer was interviewed in the offices or the science lab in personal. The interviewing time varied depending on students. Once we transcribed all the interviews, we created initial codes or concepts through the process of open coding by using transcripts from the interviews and the exit survey results. We coded them and analyzed them using inductive analysis with the help of a data-management software program called ATLAS.TI. Inductive analysis begins with specific observations and builds toward general patterns. Categories of analysis emerge from interviews and survey results as the researcher comes to understand patterns which are in the phenomenon being studied (Patton, 2002). This procedure involved grouping and regrouping the quotes until we developed the categories of descriptions. We interpreted and compared the meanings of the categories.

Theoretical Framework for Qualitative Study of Phenomenography

Since this study is concerned with student experience within a science laboratory course, the design of this qualitative study is best viewed within a phenomenographic framework. Phenomenography (Marton, 1986) is the study of the different ways in which people notice the world. In phenomenographic research, the researcher chooses to study how people experience a given phenomenon, not to study the phenomenon. Here we apply this idea to ascertain the ways of experiencing of a science laboratory and applications courses by students.

4. Results and Conclusions

The transcripts from the interviews and the informal conversations and quotations from the open-ended exit survey contain the following shorthand notation: [ ] represents comments about the interviews and the informal conservations with students and the survey results added after the fact, {…} indicates that unimportant words were omitted from the transcript, and unnecessary words or sentences that are not included.

Examining the data from the survey and the semi-structured interviews have given a deeper understanding of the students' conceptions about the course. First, all survey results and the interviews were examined and the results were supported by the quotations from the survey results. Second a researcher, who was not involved in this study, analyzed the same data independently and compared the findings. After that, we found that our results were compatible. Because of space constraints, a few quotes are chosen to support assertions. Quotes chosen from the survey results and interviews are representative of other students.

The data were analyzed into the following categories that emerged while reading and re-reading the transcripts and the survey results: *expectations; instructions; difficulties; understanding and learning; traditional science lab vs. activity-based lab; student-instructor interaction; assignments; interactions with students' group mates.*

Students enter a course with expectations of how it will be conducted, what will happen, and how they will interact in the class. In the case of these students, this activity-based teaching was a totally new environment for them. Also, the expectations concerning involvement of the instructor acting sometimes like a student and their interactions with the instructor were realized. The following quotes describe that students did not expect this kind of group work to be a part of the class and were not expecting involvement of the instructor that much.

> *S1: Um, I'm not sure. I know there's a lab but I don't know what we're doing in the lab. Actually I heard from students who already took this course before that it was going to be traditional science lab.*
>
> *S2: ... Um, I didn't expect you [the instructor] to be very involved. Like acting a student and being interested in students' any kind of problems.*
>
> *S3: ...And this course isn't like all the other class- or classes that I have. Like I have math and chemistry, and they're completely different from them.*
>
> *S4: The interesting thing for me is that I was very surprised at the beginning and did not like the way in which the course was taught because each group had to prepare and present an activity about the same topic. For example, each group did an activity about density in a class period and some of them did the same activities. It was not good for me because I felt that we were going to just repeat the same things; however what I saw was that each group presented in a different way and used different materials and made different connections to the real life and other majors. At the end of the class, I just said "wavvv" as being amazed [laugh].*
>
> *S : Honestly, this is the course I enjoy most in this semester and every week I was*

> *looking forward to doing my activity and seeing others' activities. To learn the reason of phenomenon that we come across in our life is very good. But at the beginning, I was very nervous while I was presenting my activity and acting like a teacher in the class. It was like the act of rehearsing before becoming a teacher [laugh]. On the other hand again frankly it was difficult to prepare the assignments because every week we have to prepare a new and different assignment. So I am not used to this kind of things.*

The students revealed strategies to help them learn and understand concepts being taught in the classes. In their comments, they stated about their own learning and understanding styles and whether those styles were or were not addressed. Students' natures vary and cannot be the same. They can learn through different styles. However, they learn by asking questions and discussing things with the instructor or some learn from people who are closer to their ages in general. In other words, some of students are comfortable to discuss things with their classmates. Students pointed out that they liked the fact that they could work, discuss, and share with their peers and learn from their peers. They had an opportunity to investigate and discuss the concepts with peers and the instructor as well. That's; this method can have the potential that students construct the knowledge and understand the science concepts by discussing concepts and questions, sharing idea, and getting help from the instructor and their peers. Here are several quotations that explore these ideas.

> *S4: First, I would like to thank you [the instructor] for employing this method because our teachers had tried to teach courses including science, mathematics, and others by using traditional methods that we had to memorize scientific truths and learn some concepts which are very abstract for us. As for science laboratory, the only purpose of school laboratory experiments was to help us memorize the scientific truth and take some measurements. I do not mean we did not learn, actually we learn for that moment but later we forget almost all knowledge that we learned because we did not make any connections to real life or other disciplines. Moreover, we forgot most of that knowledge. If we were taught our courses in that way, probably we would not forget almost all information and remember them. Also, I'd like to constitute activities with our group mates. That means we are not restricted to do same activity or experiment in the class. Also, we search, find, create, construct, and conduct the activities and the knowledge. So, it will be not easy to forget concepts. As a prospective teacher, when I become a teacher in the future I will definitely use the method, which you applied in the class, even though I will not teach science. So, I can adapt this method to mathematics.*

> *S5: It [the small group]'s helpful, because sometimes, well even more than sometimes, I'm wrong and it's good for other people to show me how I'm wrong. It's also good for me to explain to together people how they're wrong. If you work well together you can usually get most of the good activities and presentations. But if you don't work well together things don't always get done very well.*

> *S6: Small group work. Ok. It gives us a chance to like openly discuss, you know, the problem at hand and throwing out different ideas and seeing which ones work. Agreeing and disagreeing. Just working out the whole activity. Just, you know, that really helps develop the ability to prepare an activity.*

> *S7: Um, working in small groups for me has always been beneficial because I can talk to somebody, who knows what their*

> *talking about and learn from them. And if I know what I'm talking about or have a general idea and another person does not then I can teach what I know to them. This greatly helps me because I am rethinking what know and being `is this really right? Is- do I know this that well?' And I can find then that as I discuss these things and try to teach the concepts to somebody else it reinforces my own idea of the concepts. And also if, uh, nobody knows the answer, uh, then it does help to have many people, you know, looking in the book and searching their own notes for the answer. So, overall I think small groups are, uh, are very helpful. Um, in general the small group that I'm in right now we don't- nobody is really- had that firm a grasp, um, and so it's not as helpful as I would like it to be and that I can get more information from it- form my peers…(Fall 2004).*

Some students complained that the method employed in the course, especially writing assignments, was difficult and made the course harder, but also they were aware that it provided a better understanding of physical phenomenon. They indicated this because they were not taught by using this teaching method so far. Also investigating and preparing the activities and writing assignments take a log time and the other problems are that we do not always have computer labs and internet access to be used by students for their search and libraries to search. Here are some comments from the study participants concerning their worries at the beginning and how their views are changed throughout the course.

> *S7: To be honest, at the beginning of the class, I was not that much happy because it [the teaching method] was new for me and I did not know how to do. So, I had some troubles in the beginning then I liked it. It is like exploring and discovering something. The most difficult part for me was to write and prepare assignments because the format of it is different from that of we had used so far. That's; we first start with activities then write or give theoretical information. Also, the best part for me is to make connections to everyday life. You know that it is good to know what is happening around us and how things work. Also, we made connections to the other disciplines. Moreover, we are asked to construct another activity that related to the topic. For example, if we prepare an activity on light, at the end of the assignment we need to write another activity that related to the light. Therefore, this makes us reinforce to comprehend the concepts. Writing the assignment in a different way provided our horizons expand. I really would like to thank you with regard to using this method and having us explore and investigate science concepts. Believe me that I will use your teaching style in the future.*

Students think that this method is conceptually-oriented and made students understand physics concepts since some steps in activities are related to real life and other fields such as biology, chemistry etc. It assisted students figure out how things around the world work. Also, they stated how different this method is from the traditional science lab. Some quotes that illustrate these points from students follow:

> *S8: The teaching method is inquiry-based and student-centered so that it provided us to investigate the given topic and construct an activity that can be already created or used before. However, it requires us to digest all information and then to form the activity. Moreover, the best part of it is that we learned how to connect all things to our everyday life. To be honest, I hate physics and science before this course because everything was limited with the equations and some facts. I was always thinking that why we were learning this mass. I got to know that physics is not that much*

difficult! How and where I am going to use all these junk information. In this course, I learned that the reasons of the phenomenon occurred around me or us. For instance, when I take a shower, now I know why a shower curtain gets sucked inwards when the water is turned on. The reason for this is Bernoulli principle. [The students explained the Bernoulli principle during the interview because I asked to make sure if they really learned or not] near the water stream flows into the lower pressure stream and is swept downward. Air pressure inside the curtain is reduced and the curtain is pushed inward. I can say that honestly I have not been aware of all physical principles that explain many phenomenon around us.

S9: Frankly, there are too many things to do in this class. I do not like this. Maybe I am not used to doing this kind of things in my education life. We have not done too much so far. So, this much load makes me crazy. On the other hand, when I think what I have learned so far in this class is that I know how things work and I can make connections with real life and other majors such chemistry. Throughout the course, the class seems to be fun and enjoyable. Also, I enjoyed doing a project about how a magnetic field affects plants. It is wonderful to do this project because I can combine physics with biology. So I saw how physics are related to biology. But again there is too much to do. This is my problem maybe I am a lazy person [laugh].

S10: *...Of course, laboratory work makes me clear about the concepts presented in the teacher's lectures and textbooks.*

S11: The activities we did and presented in the class gave us some concrete experiences that help us to understand the scientific concepts.

S12: The activities are related to everyday life. We did some activities that explore some interesting life-related questions, and this is wonderful...For instance, in density activity, we answered a question about why fat people swim more easily in the sea than slim people. In other words, a fat person floats, as you've probably heard, while your bones and muscles, denser than fat, are not as willing to float.

S12: ... I really like the way in which the course has been taught because that is learning as doing, practicing, and living. For me it is good teaching. In other words, students investigate, prepare, and present the activities in the class and the most beautiful thing about that you do not need to buy expensive materials; you can probably have them in your home or around you. Or even if you need to buy some of them, they are very cheap materials. After students present their activities of course all students participate actively during that process, the instructor if she feels and sees the concepts are not fully understood or not complete, she makes explanations and clarifications. What I mean we do everything we act like a teacher, she only acts like a helper. That is very nice. If we had been taught in a traditional method, I am sure that we would not acquire of permanent knowledge or learn anything- just memorizing scientific facts and take measures and make calculations and write a report. The other thing that I think is very important is that if we did not learn this activity-based teaching method, we would not use in our classes in the future when we become teachers.

S13: As I know so far, the purpose of school laboratory exercises is to help me memorize the scientific truths, however ,in this course we constructed our knowledge

> *and learned the idea behind physical phenomena.*
>
> *S14: I liked the fact that it was very concept-oriented and very helpful in understanding Physics.*
>
> *S15: I liked that we were taught and expected to know the underlying concepts of the*
>
> *equations and principles. Like-learned how things work. The best thing I love about this course is that we learn different things that I do not know before. Also, even if several years pass, I believe that I remember that the activities and what I learned in the class.*
>
> *S16: …to be honest, there was really too much to do in this course. It was tiring. But, we accomplished new things even if everything is already done before. On the other hand, we had to find the information, explore, digest them then we started to write our assignments. So, we learned and understand many topics for example now I know why I have a headache while talking on the cell phone even if we use headphones. The reason is that the headphones have magnets and they cause a magnetic field that affects our body.*

Students declared that the instructor was really helpful because she knows the subject matter well and asks questions to make them think about the answer instead of showing the way to answer the questions or problems directly. Also, she/he [the instructor] helps in class, out of class, and in her/his office. She/he makes students be very comfortable during class period as well.

> *S16: During the courses of science laboratory and applications I and II, we had very enjoyable learning time. Being active [student-centered] in the class and making connections with everyday life makes our interest of science increase. In addition, regarding classroom environments, it is very flexible and enjoyable because when we present our activity like a teacher, the instructor acts like a student and ask questions to us-showing how a student should be also- so it makes us very comfortable. As a result, the instructor makes us be involved actively in the class, she acted like a facilitator. As for science concepts I believe that we learn some science concepts better…I want to add one more thing that I wish we will have the same format in the second semester [spring 2008]. The instructor is very helpful and has a good manner.*
>
> *S17 : I am happy with my science lab, the method, and the instructor…the instructor changed the teaching method for traditional science lab to have gone on in certain way for a long time and she aimed students to construct and reach the knowledge by themselves instead of giving the knowledge. The other good thing is that I really love is that the instructor acts like students during class session and asks questions such as "why this happened?" This is really cool and makes us very comfortable during presentations.*

Students stated how the course helped them have teaching experiences and social interactions with other students and other people.

> *S18: …by means of this course we learned how to teach and how to behave in the class and class management. Before that class, we feel embarrassed and our communication skills were not good. But now we are more comfortable during class presentations. By means of this class we can be more successful as a teacher in the future.*
>
> *S19: We had experiences and we had social interactions with our group mates. It*

> *is really good. The other thing we are enjoying during the class.*
>
> *S20: The course is very constructive. I plan to share what I learned from this course with my students in the future when I become a teacher.*

The exit survey results and the interview results are summarized. Students' interview results were basically consistent with their responses on the exit survey. For the most part, the students were positive about the course based on activities and interactive engagement. On this exit survey, the responses the students made to the "what did you like and dislike about the course and the instructor" questions tended to be about the structure and format of the course. Students liked the ability to get individual attention in a classroom setting, interactions and discussions they had with their peers in the form of small group, interactions and discussions with the instructor and their peers. Students liked to have activities that can be related to the real-life problems and understand the science concepts behind the complicated, confusing equations and concepts in science. The responses for what they disliked about the course or what they would change about the course focused on the operations of the course or on small details. For example, at the beginning and throughout the course, some students thought that assignments were hard to prepare because there were too much to do and the assignment structure was different from the one we used before. Here we had to construct everything while preparing our assignments and activities. On the other hand, when we deduced new things or investigated, combined and compiled all information related with the given topic we felt excellent because we achieved a success. Most of the students, however, indicated that there was nothing they would change about the course. The responses the students gave in the interviews corresponded closely with the exit survey results.

Consequently, students revealed the following thoughts about the course such as the way in which was taught and the instructor of the course which are retrieved from students' interviews and the exit survey results.

- The course based on activity and interactive engagement provided an application of the knowledge to real world situations-made to see how things work in the real world instead of just looking at equations or some scientific facts. In other words, the course helped students learn and understand physical phenomenon and their connections to the real life and other disciplines.
- The teaching method provided some opportunities to construct student activities and knowledge as they investigate, explore, and digest.
- It provided assistance in learning how to analyze the systems, and make connections to the real world and everyday life which make for students physical meaning and interpretation.
- The method provided an opportunity for self learning especially during small group work and investigating the topics or concepts. Thus the knowledge that they explored will not be temporary.

Discussions and Implications

The aim of this study was to investigate students' views about the course and the way in which it was taught. In addition, we were interested in applying a novel teaching approach in Turkey that can guide teachers/instructors to use in their classes instead of the traditional science laboratory course because the science laboratory course is an important defining aspect of the academic performance of the students in science.

The usage of this activity-based method is predicated on student participation and lets students pose appropriate questions, perform helpful activities (Aladejana & Aderibigbe, 2007; Aladejana, 2006; Adelson, 2004; Mayer, 2003). This method requires certain amount of interaction between the students and between the students and the teacher.

Major benefits of this activity-based learning are that it makes the subject matter more comprehensible because students discover and present after making searches about the subject matter. Also, all students in the class present same subject in each week, so they can see what they have not yet comprehended well and learn different perspectives of the same concept. The other important benefit, which is indeed crucial, is that it makes to minimize memorizing since students can acquire ability to transfer of knowledge to the real life and other disciplines. They can make connections with the real life and other disciplines too. Moreover, students do not have to make a given activity by a teacher or instructor. They have a chance to choose their activities on certain topics. Therefore, they can be creative. In other words, this method promotes student curiosity, rewards creativity, encourages reasonable questioning, avoids dogmatism, and promotes meaningful understanding. Thus, it strives to enable all motivated students to be successful (Aladejana & Aderibigbe, 2007).

It is found that students' achievement and skills can be improved when students are taught science in an activity-based curriculum (Turpin & Cage, 2004; Fraser, 1986; Bredderman, 1983) which provides constructivist students' learning is oriented to real understanding not just memorizing.

The science laboratory course is a major part of the setting for learning and most science activities designed for learning and understanding science. In addition, it encompasses different kinds of tools and information resources, the interactions, the relationships between and among students and teachers, as well as the expectations and norms for learning and behavior (Aladejana & Aderibigbe, 2007).

Furthermore, it can be surveyed students' current knowledge while using these activities and update their knowledge and inform them in terms of the nature of science and how new knowledge arises by refining old knowledge or remove it. Also they can learn how to draw conclusions when they make observations and inferences based on their observations.

Based on these findings, it can be recommended that the approach can improve science instruction, construction of scientific knowledge, and understanding of scientific concepts, understanding the nature of science (NOS), developing practical skills, developing teamwork abilities, and developing scientific reasoning. Thus, students' academic performance can be increased and the method can have great potential in school and university settings so that remarkable increases in interest of learning and understanding in science can be quite possible.

Acknowlegements

We would like to thank to Dr. Saime K. Solmaz and Dr. Hüseyin Küçüközer for their support to utilize this activity-based teaching method at Balıkesir University. Likewise, we are grateful for our students for their enthusiasms and help in the department of primary mathematics teacher education at Balikesir University and Amasya University in Turkey.

References

Abd-El-Khalick, F.S., & Akerson, V.L. (2004). Learning about Nature of Science as conceptual change: Factors that mediate the development of preservice elementary teachers' views of nature of science. *Science Education*, *88*, 785–810.

Adelson, R. (2004) Instruction versus exploration in science learning. PsychNet, *American Psychological Association*, *35*, 34–47.

Akerson, V. L. & Hanuscin, D. L. (2008). Teaching Nature of Science through Inquiry: Results of a 3-Year Professional Development

Program. *Journal of Research in Science Teaching*, *44*, 653-680.

Akerson, V.L., Abd-El-Khalick, F.S., & Lederman, N.G. (2000). Influence of a reflective explicit activity-based approach on elementary teachers' conceptions of nature of science. *Journal of Research in Science Teaching*, *37*, 295–317.

Akkus, R., Gunel, M., & Hand, B. (2007). Comparing an Inquiry-based Approach known as the Science Writing Heuristic to Traditional Science Teaching Practices: Are there differences? *International Journal of Science Education*, *29*, 1745-1765.

Aladejana, F. & Aderibigbe, O. (2007). Science Laboratory Environment and Academic Performance. *Journal of Science Education and Technology*, *16*, 500–506.

Aladejana, F. O. (2006). Concept of teaching. In Ehindero O. J., & Aladejana F. O. (eds) Introduction to the teaching profession. Literamed Publications Ltd, Lagos, pp 12–19

Bodner, G., Klobuchar, M., & Geelen, D. (2001). The many forms constructivism [Electronic version]. *Journal of Chemical Education*, 78, 1107.

Bredderman, T. (1983). The effects of activity-based elementary science programme on students' outcomes and classroom practices. *Review of Educational Research*, *53*, 499–518.

Chabay, R. W. & Sherwood, B. A. (2002). "Vol I: Matter and Interactions: Modern Mechanics", New York: John Wiley & Sons, Inc.

Chen, S. (2006). Development of an Instrument to Assess Views on Nature of Science and Attitudes toward teaching Science. *Science Education*, *90*, 803-819.

Chin, C. (2007). Teacher questioning in science classrooms: Approaches that stimulate productive thinking. *Journal of Research in Science Teaching*, *44*, 815-843.

Chin, C. (2006). Classroom interaction in science: teacher questioning and feedback to students' responses. *International Journal of Science Education*, *28*, 1315-1346.

Churukian, A. D. (2002). Interactive engagement in an introductory university physics course: Learning gains and perceptions (Doctoral dissertation, Kansas State University, 2002). *Dissertation Abstracts International*, *62*, 1685.

Conlon, J. (2004). Physics outreach for WYP. *The Physics Teacher*, *42*, 468-469.

Doolittle, P. E. & Camp, W. G. (1999). Constructivism: The career and technical education perspective. *Journal of Vocational and Technical Education*, 16.

Fay, M. E. & Bretz, S. L. (2008). Structuring the level of inquiry in: A rubric helps teachers compare experiments and plan inquiry. *The Science Teacher*, 38-42.

Fosnot, C. T. (1996). *Constructivism: Theory, perspective, and practice*. New York: Teachers College Press.

Fraser, B. J. (1986). *Classroom Environment*. Croom–Helm, London

Holbrook, J. & Rannikmae, M. (2006). The Nature of Science Education for Enhancing Scientific Literacy. *Science Education*, *90*, 803-819.

Hoover, W. A. (1996). The practice implications of constructivism. SEDLetter. Adapted from htpp://www.sedl.org/pubs/sedletter/v09n03/welcome.html.

Hsiao, W. D. L. (nd). CSCL theories. Retrieved May 8, 2008, from http://www.edb.utexas.edu/csclstudent/Dhsiao/theories.html

Huitt, W. (2003). Constructivism. Educational psychology interactive. Retrieved May 4, 2008, from Valdosta State University, Valdosta. http://chiron.valdosta.edu/whuitt/col/cogsys/construct.html

Lederman, N. G., Abd-El-Khalick, F., Bell, B. L., & Schwartz, R. S. (2002). Views of Nature of Science Questionnaire: Toward valid and meaningful assessment of learners' conceptions of nature of science. *Journal of Research in Science Teaching, 39*, 497 – 521.

Martin et al. (1992). Displacement, velocity, and frames of reference: Phenomenographic studies of students' understanding and some implications for teaching and assessment [Electronic version]. *American Journal of Physics, 60*, 262-269.

Marton, F. (1986). Phenomenography-A research approach investigating different understandings of reality. *Journal of Thought, 21,* 28-49.

Mayer, R. E. (2003). Learning and instruction. Pearson Education, Inc, Upper Saddle River, pp 287–288

Orlik, Y. (2005). Using of active educational strategies and methods - the way to improve the quality of science education. *Journal of Science Education, 6*, 44-46.

Örnek, F. (2008). *Örnek Aktivitelerle Fizik ve Günlük Yaşam*. Ankara: Pegema yayıncılık.

Ornek, F. (2006). Modeling-based interactive engagement in an introductory physics course: Students' conceptions and problem solving ability, *Dissertation Abstract International* DAI-A 68/02.

Perkins-Gough, D. (2007). The status of the science lab. *Educational Leadership*. *64,* 93-94.

Philipps, D. C. (1995). The good, the bad, and the ugly: The many faces of

constructivism. *Educational Researcher*. 24, 5-12.

Roth, W. M. (1996). Knowledge diffusion in a Grade 4–5 classroom during a unit of civil engineering: An analysis of a classroom community in terms of its changing resources and practices. *Cognition and Instruction, 14,* 170–220.

Shah, I., Rıffat, Q., & Reid, N. (2007). Students perceptions of laboratory work in chemistry at school and college in Pakistan. *Journal of Science Education, 2*, 75-79.

Sherman, L.W. (1995). A postmodern, constructivist and cooperative pedagogy for teaching educational psychology, assisted by computer mediated communications. The first international conference on Computer support for collaborative learning, USA, 308-311.

Smith, C., Maclin, D. , Houghton, C., & Hennessey, G. M., (2000). Sixth-grade students' epistemologies of science: The impact of school science experiences on epistemological development. *Cognition and Instruction* **18** (2000), pp. 349–422.

Steffe, L. P., & Gale, J. (Eds.) (1995). *Constructivism in education*. Hillsdale, NJ: Erlbaum.

Turpin T. & Cage B. N. (2004). The effects of an integrated activity-based science curriculum on student achievement, science process skills and science attitudes. *Electronic Journal of Literature Science, 3,* 1–17.

Van Zee, E. H. & Minstrell, J. (1997a). Reflective discourse: Developing shared understandings in a high school physics classroom. International Journal of Science Education, 19, 209-228.

Wilson, B. G. (1996). Introduction: what is a constructivist learning environment? In: Wilson BG (ed). Constructivist learning environments. Case studies in instructional design. Educational Technology Publications, Englewood Cliffs, pp 3–8

Windschitl, M. (2002). Framing constructivism in practice as the negotiation of dilemmas: an analysis of the conceptual, pedagogical, cultural and political challenges facing teachers. *Review of Educational Research, 72*, 131–175.

Appendix A

Case of the Sunken Ice Cube

Why does ice float in one liquid but not the other?

What is Needed?

- Two tall clear beakers or plastic containers, one filled with tap water, the second filled with isopropyl alcohol or ethanol
- Two ice cubes (you should have extras)

What to Do (procedure)

Step One: Begin by telling students: The most important tool a scientist has is his or her powers of observation. So today, we are going to stretch and build those powers. Reveal to the students two identical beakers, one containing about 200 ml of alcohol, the other containing the same amount of clear water.

Step Two: Show them the two ice cubes. Pose the question; "What will happen if the ice is placed in the beakers?" Ask for predictions and hypotheses. Point out that our next step is a test. The ice is placed in each liquid. It sinks in alcohol, and floats in water.

Step Three: What did you "observe"? Review the initial observations the students made. Point out the distinction between true observations, such as "There is clear liquid in each beaker" and statements such as "there is water in each beaker." The second statement is an inference, which means it is a judgment we have made, drawn from our observations. One of the things scientists do is to make inferences, but we need to check those inferences, which is one reason we do experiments.

(Note: Astute observers may pick up on some other details that are quite informative. For example, the ice in alcohol will, after a few minutes, begin to melt, and actually float up off the bottom on a layer of water. This layer will be colder than the alcohol above, so students may notice condensation on the bottom part of the beaker only. These observations may help the students later.)

What is Happening?

Density is the quantity of matter in a given unit of volume, stated as density = mass/volume, generally given in SI units of g/cm^3. Density is an *internal physical property* and thus is often used to identify a substance. Water has a density of 1.00 g/cm^3 at room temperature, 25 degrees Celsius, meaning that a mass of 1.00 g of water occupies a space of 1.00 cubic centimeters. Materials that are less dense than water (have a density less than 1.00 g/cm^3) will float in water, while substances more dense than water (have a density greater than 1.00 g/cm^3) will sink. The same is true of any liquid, such as ethyl alcohol with a density of 0.79 g/cm^3. Materials less dense than ethyl alcohol will float in it, while materials more dense will sink. Therefore, as observed in this demonstration, ice (with a density of 0.92 g/cm^3) will float in water but sink in ethyl alcohol.

When water solidifies to form ice, the solid ice phase is less dense (as observed) than liquid water. This can be attributed to the hydrogen bonding that occurs in water in both the liquid and the solid states. ***Hydrogen bonding*** is the strong intermolecular force between the hydrogen and oxygen atoms of neighboring water molecules (this explanation is purposefully simplified!!) When water freezes, hydrogen bonding holds the molecules rigidly in a three-dimensional crystal. There are holes, or empty spaces, within the ice crystal. As water freezes to form ice, it must expand (rather than contract) to form this open crystal. As a result, the density of ice is less than the density of liquid water, which explains why ice floats in water. This is a very unusual phenomenon. Most substances are denser in the solid state than in the liquid state because particles are usually closer together in the solid state. Water is the rare and unusual exception.

Relationships with Real Life

Winter pond temperatures can remain very cold for many months in northern climates. There’s only so much water in a pond. There can only be so much oxygen in the water. If the ice caps over the pond, the fish, other animals, and any decaying organic material may eventually consume all the oxygen, the fish will suffocate.

This is referred to as “winter kill” and occurs commonly in natural ponds and our over-stocked backyard ponds are much more susceptible. It’s easy to avoid this by simply keeping a small area of the pond ice-free for the exchange of gases with the atmosphere. Air bubblers and small pumps can be used to keep small areas ice free, but do not allow them to mix the lower 40 degree puddle of water with the colder top layers. They cost less to run than deicers, but do not work when the air temperature drops below the teens for extended periods of time. Any small air or water pump that creates a flow of water across the pond that disturbs the bottom puddle of warm water will eventually lead to a fish kill.

Interesting Relationships with other Sciences (chemistry, biology, earth science, medicine etc.)

Water is a unique substance. At about 40 degrees F. it is denser than water warmer or colder, so it settles to the bottom of the pond. It forms a puddle that if left undisturbed will not mix with the colder water above it. The ice floating on the top of the pond insulates the lower water from even colder air above it.

Other activities related to this activity

For instance, while you are making fermented pickles, you may need to use density if you do not want to deal with numbers or ratios. Even if you use ratios, sometimes your pickles can be spoiled, that’s, you pickles become soft, slippery or slimy. As you know, you need some vegetables such as cucumber, green pepper or tomatoes and etc. to make fermented pickles. Also, water, salt, and vinegar are required. You need to add correct ratio salt to water. There is a very easy way to do that. The easy way to get the correct ratio: Take a cup filled water, start to add salt (unionized), and continue to add salt until an egg (can be raw or hard boiled) start to float in salt water. If the egg floats, that means salt is enough. You do not need to add more salt. As a result, here density plays a role. In other words, since density of salt water is more than egg, the egg floats. As you see, density is almost everywhere in our life.

Note: In above example, the sections can be broadened and more information can be added.

Investigating the Effectiveness of Digital Interactive Multimedia Package in Astronomy to Promote Scientific Temper

A Case Study of the Tertiary Level Students in India

G. P. Pimpale[1] and R. V. Vadnera [2]

[1]Department of Physics
H.P.T. Arts and R.Y.K. Science College,
Nasik 422005, Maharashtra, India.
(gpimpale@gmail.com)

[2] School of Continuing Education,
Yashwantrao Chavan Maharashtra Open University, Nasik 422 222, Maharashtra, India.
(raj_vad@yahoo.com)

(*Submitted* April 2011)

Abstract

A criterion for a good scientific material is that it should not only impart knowledge but must go beyond that and should develop scientific attitude among the users. This paper reports on an investigation of the effectiveness of the interactive multimedia package in Astronomy, already developed by the authors, to inculcate scientific values among the tertiary level students in India. To quantify the change in behavior of target students, a tool was designed by the authors based on the Likert's method. Standard statistical tests were employed for the analysis. The investigation revealed that the students, who were exposed to the multimedia package, did significantly well compared to those who were given only the print material. A retention test was also conducted which indicated statistically significant retention of scientific temper.

1. Introduction

The night sky full of stars is a treasure of celestial beauty. There will be hardly anybody on this blue planet who has not cast glance towards the night sky. But stars form only a small subset of the billions of celestial objects. Astronomy is a branch of science which deals with the study of these celestial objects. During his tenure as a lecturer of physics for 28 years, one of the authors (GP) has keenly observed that though the students take interest in this branch at amateur level, they do not to seek career in this domain. Therefore the authors thought of developing a digital interactive multimedia package in astronomy for tertiary level students to motivate them. Such package has already been developed by the authors using the principles of e-learning and has already been reported. It is accepted widely that a good scientific material must help to inculcate scientific values among the users. Hence the package developed was tested statistically to check whether it satisfied this criterion.

2. Background

A good number of online courses on astronomy are available on the Internet for the undergraduate students. However the fees charged for these online courses are too exorbitant for the tertiary level students in a developing country like India. It is quite unlikely that students with limited resources in developing countries will enroll for such online courses. Hence the challenge of promotion of scientific temper cannot be met using such courses. It is a matter of concern that astronomy -- probably the oldest branch of Science -- is associated by general public with many superstitions and misconceptions. Not much effort has been made to go beyond the knowledge of the subject and to test whether the knowledge has been instrumental in eradication of superstitions. This makes the duty of an astronomy teacher many-fold. A teacher of astronomy should not only impart ideas in the subject but should also make conscious and persistent efforts to remove superstitions related to the subject. As an umbel attempt in this direction, the authors have already developed and reported a customized package for the tertiary level students in India and in the present paper, they report on the investigation for effectiveness of the said package to promote the scientific temper. The scientific temper essentially means "an attitude which involves application of logic and avoidance of bias and preconceived notions". (On- line resource 7).

3. Objectives

In order to check the promotion of scientific temper, a tool has been developed by the authors, which is explained later in this paper. The objectives of the present paper are as follows:

1. To investigate whether the tertiary level package developed for astronomy is effective in promoting the scientific temper of the target group.
2. To test whether the print material prepared by the authors is effective in inculcating the scientific values in the target group.
3. To compare the effectiveness of the print material as compared to the multimedia package to create the scientific temper in the target group.
4. To test the effectiveness of the package for retention of the scientific temper of target students.

4. Methodology

4.1 Background study

To begin with, a review of various DVDs, CDs and video cassettes (Video resources 1 to 24) on astronomy was taken. It was observed that most of the audio-visual material is excellent but is prepared with western students in mind and is not suitable culturally for the target group in India. Also, it was observed that most of the material is not in tune with the philosophy and methodology of self-learning material. Therefore the authors contemplated of making a need-based package for Indian students which can serve as an e-learning material for them.

4.2 Developing the package

Before making the final package, a pilot package was made. It was made by considering the educational background of the tertiary level students in India. The pattern adopted in making the package was in tune with the self-learning material developed by the Indira Gandhi National Open University, New Delhi which is a nation-wide mega open university in India. A group of tertiary level students in Nasik, India was then exposed to this pilot package. They were given a tool in the form of an opinionnaire. The students were asked to register their true opinion. Their suggestions and comments were incorporated and the package was modified accordingly.

During preparation of the text, standard books **(**Mitton 1991; Moore 2003; Moche 1993; Pasacheff 1990) on astronomy were consulted. A large number of diagrams, pictures and video clips (On-line resources 1 to 7) were included in this package to make it fascinating. Soft-ware packages like Flash, Microsoft Word, Photoshop and Front Page were also used.

4.3 Development of print material

In India, astronomy has not been included in the tertiary level formal education in most of the universities. In some universities it is included as an optional course. Therefore the number of tertiary level students undergoing astronomy course is very small. Hence no 'text books' are available in the market. Therefore the researchers prepared a booklet explaining fundamental ideas which are required for tertiary level students. This booklet was prepared for the control group.

4.4 Formation of samples

The population for the present study was the tertiary level students in the city of Nasik, India. Two unbiased samples were obtained from this population of target students using random number generation technique.

Each sample consisted of 31 students. One of the groups was arbitrarily named as the experimental group and the other one as the control group.

4.5 Development of tool

The tool developed was in the form of an opinionnaire. It consisted of 16 statements based on some common superstitions in the society. They were pertaining mainly to astronomy. A superstition, as defined in the Oxford Advanced Learner's Dictionary (Hornby 2005) is "The belief that particular events bring good or bad luck". Superstitions are considered to be contrary to the scientific temper.

The opinionnaire was made according to the Likert method (Best & Kahn 2004). For each statement, five options were given. They included "Strongly Disagree", "Disagree", "Undecided", "Agree" and "Strongly Agree". Each option was given weightage as (+2), (+1), (0), (-1) and (-2) respectively. All these statements were made in such a manner that disagreement to a statement implied rational thinking.

4.6 Testing of samples

Firstly, both the groups -- experimental and control – were administered pre-test. The test was useful to obtain the profile of their rational thinking. The experimental group was then administered the multimedia package. Sixteen computers were arranged for this session which lasted for four hours. At the same time, the control group was given print material to study. Then, both the groups were asked to take the post-test.

It is always worthwhile to check whether the impact made by the treatment is temporary or it lasts for a sufficiently long period. To check this, one month was allowed to pass and then a retention test was conducted for the experimental group.

To quantify the promotion (or demotion) in scientific temper, following method was used. The tool consisted of 16 statements. According to the Likert's scale, as stated earlier, if a student strongly disagrees with a statement he / she gets maximum (+2) marks in that question. It means the maximum score possible for the questionnaire in the tool was 32.

The authors have defined an index called as Scientific Temper Index (S.T.I.) as given below:

Score obtained by a student

$$\text{STI} = (\text{Net score}/32) \times 100$$

Thus the maximum value (+100) of STI corresponds to totally rational thinking while the minimum value (-100) implies complete subscription to superstitions. Using this formula, STI values for each student were calculated for the pre, post and retention tests of experimental group. Similarly the values were calculated for the pre and post tests of control group.

5. Results and discussions

5.1 Testing the scores for normality

Before statistical tests are applied, the data should be checked for the normality. If it satisfies the condition, a parametric test like the 't - test' in statistics could be used for the data analysis. If it does not, non-parametric test is required. To check whether the data fits in the normal distribution, a probability graph is plotted for the data. Montgomery (1992) has given this method and has been widely used in statistical domain. He has remarked that a probability plot is an alternative to the histogram that can be used to determine the shape, centre and spread of the distribution. He has further indicated the advantage of such plot. According to him, in such case, it is not necessary to divide the range of the variable into class intervals, and it often produces reasonable results for moderately small samples. This is not possible with a histogram.

To get the probability plot, following steps were followed.

a) A special "paper" (scale) is used for such graph. One can use such "papers" for normal, exponential and several other distributions. It has been stated by Everitt (1998) that such probability paper is structured in such a way that the values in the cumulative frequency distribution of a set of data from a normal distribution fall on a straight line. He has further indicated that it can be used to assess sample data for normality.

In a nutshell, if the given data is plotted on the normal probability paper and if the points lie nearly along a straight line then the data is said to follow normal distribution.

b) First, data-points are arranged in ascending order. This process is called as the ranking of data. The serial number is called as rank and is denoted by j. The corresponding score is called strength and is shown by Xj. Xj is plotted on the X axis.

c) Then the sample cumulative frequencies are obtained. The plotting positions of points, P_j corresponding to these frequencies are obtained as given below.

$P_j = (j – ½)/ n$, where n means sample size.

Values of P_j are multiplied by 100 to get percentage and are plotted on the Y axis. This terminology is used in the following graphs.

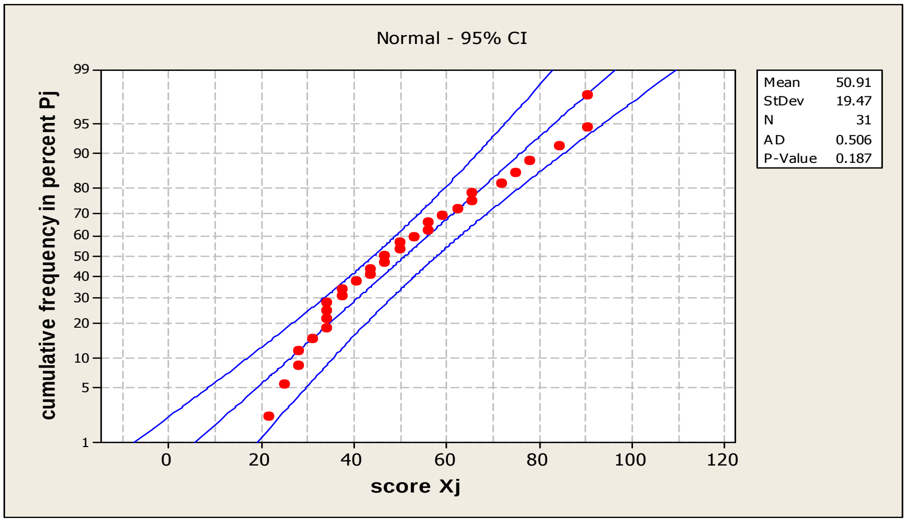


Figure 1. Probability plot for the pre-test of experimental group

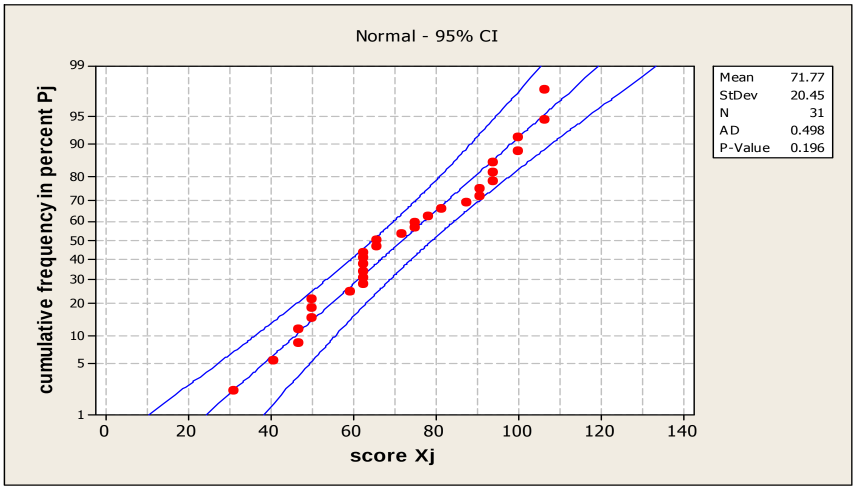


Figure 2. Probability plot for the post-test of experimental group

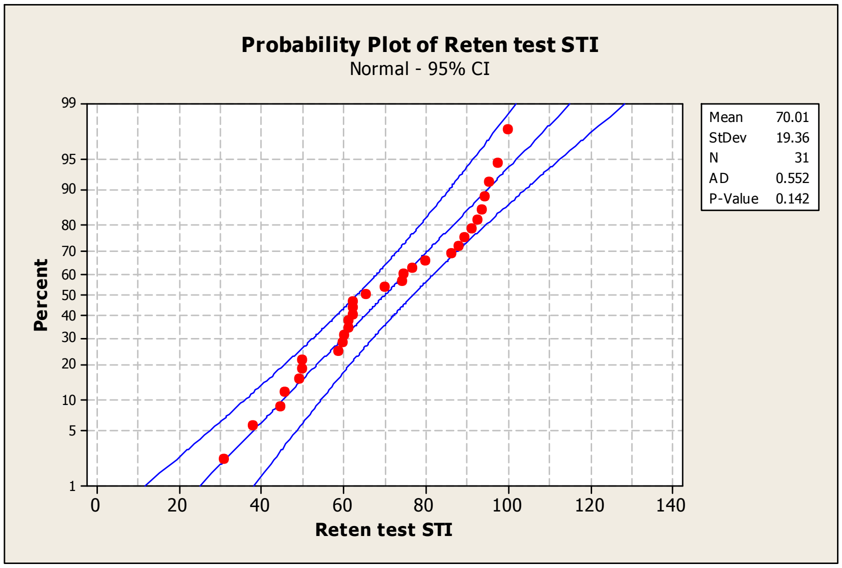


Figure 3. Probability plot for the retention test of experimental group

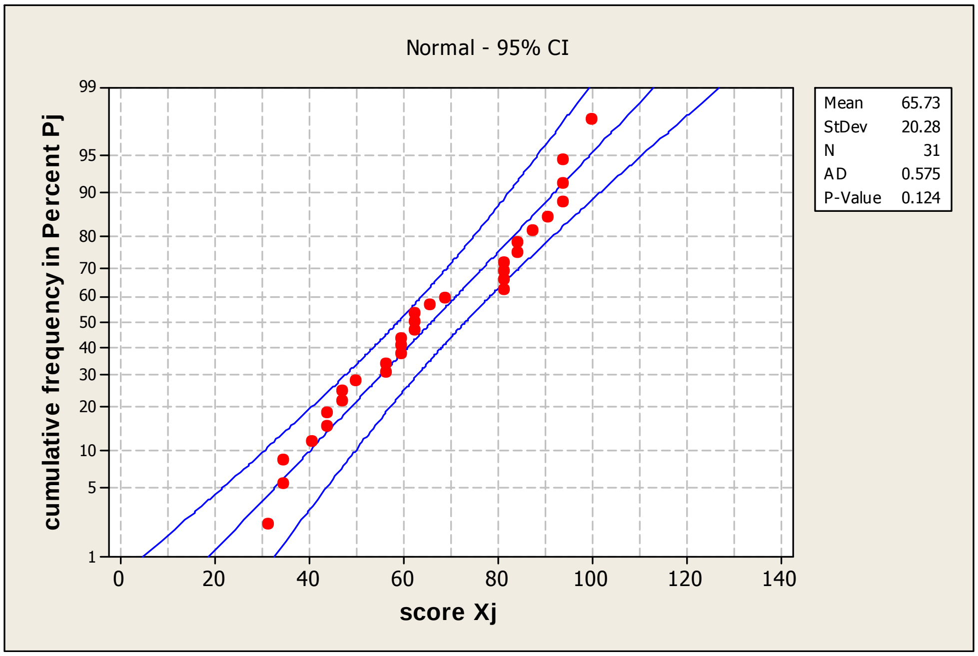


Figure 4. Probability plot for the pre-test of control group

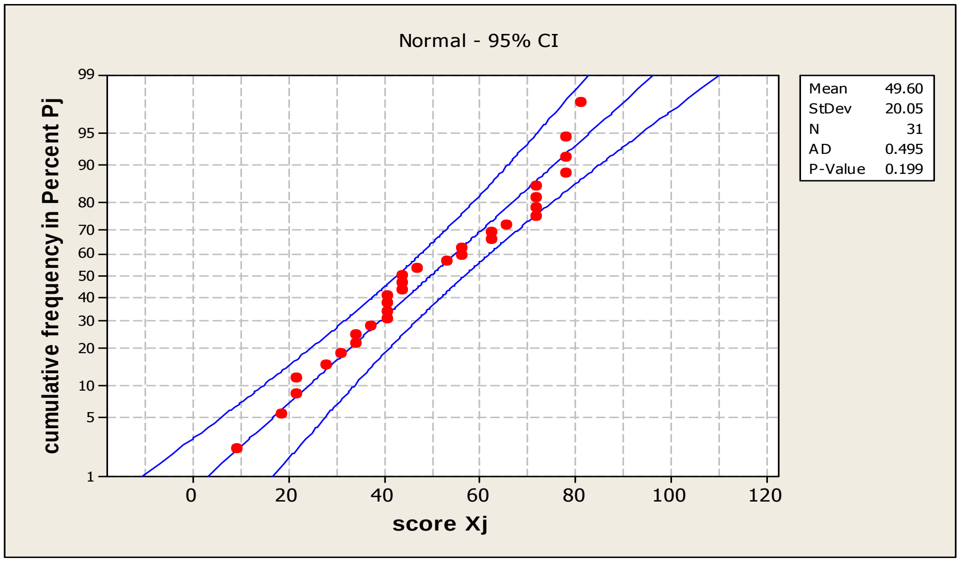


Figure 5. Probability plot for the post-test of control group

Figures 1 to 5 show plots of P_j against Xj on the normal probability paper. It is seen that the data points lie nearly along a straight line for the Figures 1, 2, 4 and 5. It implies that the normality assumption is valid in each case and one can proceed further for the parametric test. Figure 3 indicates that the distribution of data points is not normal. Hence non-parametric test need to be used in that case.

5.2 Comparison of pre and post tests of experimental group

Table 1 Paired t – test for Pre and Post tests of Experimental Group

Test	**N**	**Mean**	**S. D.**	**t- value**
Post	31	71.77	20.45	10.69
Pre	31	50.91	19.47	

Table No.1 shows that the mean score for post-test is much greater than the corresponding value for pre-test. If the t - value equals or exceeds 2.58, it is concluded that the difference between means is significant at the 0.01 level. The calculated value of t for the data obtained by us is 10.69. It is greater than 2.58. Hence it can be inferred that there is a significant increase (at 0.01 level of significance) in the Scientific Temper Index of Experimental group after it was exposed to the Package.

5.3 Comparison of Pre and Post Tests of Control Group

Table 2 Paired t – test for Pre and Post tests of Control Group

Test	**N**	**Mean**	**S. D.**	**t-value**
Post	31	65.73	20.28	6.47
Pre	31	49.60	20.05	

Table No. 2 shows that the mean score for post-test is much greater that the corresponding value for pre-test. If the t - value equals or exceeds 2.58, one can conclude that the difference between means is significant at the 0.01 level. The calculated value of t for the data generated is 6.47. It is greater than 2.58. Hence it can be inferred that there is significant increase (at 0.01 level of significance) in the STI of control group after it was administered the print material. However, the t-value is less than the case (Table 1) where the multimedia package was used. Hence we may concluded that the package is more effective compared to the print material**.**

5.4 Comparison of the experimental and control groups

Table 3 Two Sample t – test for the difference between Mean values of Scores in Post tests of Experimental and Control Group

Group	**N**	**Mean**	**S. D.**	**t**
Experimental	31	20.87	13.90	2.10
Control	31	16.13	10.90	

Table No. 3 shows two sample t – test applied for the difference between the Mean values obtained for both the groups. If the t - value equals or exceeds 1.96, we can conclude that the difference between means is significant at the 0.05 level. The calculated value of t for the data obtained by us is 2.13. It is greater than 1.96. Hence it can be inferred that there is significant increase (at 0.05 level of significance) in the scientific temper for the Experimental group. This clearly shows that the package developed by the authors is effective.

5.5 Comparison of Post and Retention Tests

From figure 3 it is observed that the scores are non-normal. Hence a non-parametric test is required. In the present case, the scores are related to each other i.e. they are matched-pairs in the data. Therefore the Wilcoxon matched-pairs test is appropriate here.

Table 4 given below shows that the test statistic z = (-4.20) is lesser than the table value

(- 2.58). Hence the null hypothesis is accepted at 0.01 level of significance and it is concluded that there is no significant difference between the performance of the experimental group in the post test and the retention test. It means that there is statistically significant retention of the scientific temper for the target students even after one month of exposure of the digital multimedia package developed by the authors.

Table 4 – Paired t-test for the Post and the Retention Tests of Experimental Group

N	n	T	U	σ_T	z value	Table value
31	23	0	138	32.88	(- 4.20)	(- 2.58)

6. Conclusions

The experimental group was exposed to the digital multimedia package while the control group was given only the print material. From analysis of the results obtained, we arrive at the following conclusions:

1. From the scores of pre and post tests of the experimental group it was observed that there is statistically significant promotion in the scientific attitude among the target students. Thus the package has been successful in inculcating the scientific values in the target students.

2. From the pre and post tests of the control group it was observed that there is significant promotion in the scientific temper. Thus the print material written by the authors has proved to be useful in inculcating scientific values. Further it can be noted that the t -value calculated in case of the experimental group is much greater than the t -value calculated for the control group. Hence it can be inferred that the multimedia package is more effective compared to the print material.

3. From the performance in post-tests of the experimental group and the control group, it is observed that there is significant increase in scientific temper of the experimental group. Therefore we can conclude that the package is effective to inculcate scientific values.

4. From the post-test and retention test conducted for the experimental group, it was found that there is statistically significant retention of the scientific attitude among the target group. Thus the package is found to be effective to create lasting impression on the minds of users.

Thus the results obtained using standard statistical test clearly indicates that the difference between achievements of the target groups is not due to the sampling error but because of the treatment given to the experimental group. In other words, it can be concluded that the package developed by the authors is effective to inculcate scientific attitude among the students of target group**.**

Acknowledgements

The author (Pimpale) would like to make special mention of financial assistance given by the Indira Gandhi National Open University, New Delhi, India for this project. He is also very much thankful to Prof. Ajit Kembhavi (Director, Inter-University Centre for Astronomy and Astrophysics – IUCAA, Pune, India) for his constant encouragement and guidance. Thanks are also due to the library staff of IUCAA and to Shri M. B. Kulkarni from Nasik, India for his help.

References

Books

Best, J. W., & Kahn, J. V. 2004, *Research in education*, New Delhi: Prentice Hall of India Pvt. Ltd.

Everitt, B.S. (Ed.). 1998, *The Cambridge dictionary of statistics*. Cambridge, U.K.: Cambridge University Press.

Hornby A.S. (Ed.). 2005, *Oxford Advanced Learner's Dictionary of Current English*, Oxford: Oxford University Press.

Mitton, J. 1991, *A concise dictionary of astronomy*, Oxford: Oxford University Press.
Moche, D. l. 1993, *Astronomy: A Self Teaching Guide,* New York: John Wiley and Sons.
Montgomery, D.C. 1992, *Introduction to statistical quality control*, Brisbane: John Wiley & Sons.

Moore, P. 2003, *Philip's atlas of the universe*, London: Octopus Publishing Group.

Pasacheff, J., & Percy, J. (Ed.) 1990, *The Teaching of Astronomy,* Cambridge, U.K.: Cambridge University Press.

Video resources

1. *Astrophysics on disc* (AAS CD-ROM Series, Vol.8), American Astronomical Society, USA, 1997.
2. *Celestial shadow play*. Vigyan Prasar, New Delhi, India, 1999.
3. *Clear skies!* Swinburne University of Technology, Australia, 1998.
4. *Cosmic dust*. John-Wiley and Sons, New Jersey, 1978.
5. *Destination Mars* (Explore Your World Series). Shloka Publications Pvt. Ltd., Mumbai, India, 2005.
6. *Exploring our solar system*. Astronomical Society of the Pacific, San Francisco, USA, 1997.
7. *Eyewitness encyclopedia of space and the universe*. DK Multimedia, UK, 1996.
8. *Galaxy* (Mars Software). Arc Media.com Inc., Toronto, Canada, 1998.
9. *Galaxy guide: A journey through the constellations*. Parallax Multimedia Inc., CA, USA, 1996.
10. *Guide to astronomy village: Investigating the universe version 1.0*: National Aeronautics and Space Administration, USA, 1996.
11. *India in space*. Vigyan Prasar, New Delhi, India, 2004.
12. *Mars* (Mars Software). Arc Media Inc., Toronto, Canada, 1998.

13. *Realsky*. (Available on 9 CD-ROMs). Astronomical Society of the Pacific, USA, 1997.

14. *Redshift 2: The ultimate virtual observatory*. Expert Software Inc., USA, 1997.

15. *Red-Shift 3: Explore the universe*. Expert Software Inc., USA, 1998.

16. *Search for extraterrestrial intelligence: Listening for life in the cosmos*. John Wiley and Sons, Inc., New Jersey, 1987.

17. *Solar system* (Explore Your World Series). Shloka Publications Pvt. Ltd., Mumbai, 2005.

18. *Solar system and beyond*. CD Titles Inc., USA, 1996.

19. *Space exploration* (Explore Your World Series). Shloka Publications Pvt. Ltd., Mumbai, India, 2005.

20. *Story of astronomy*. MacDonald, London, 1972.

21. *The Sun and Moon* (Mars Software). Arc Media Inc., Toronto, Canada 1998.

22. *The wonders of the universe: An interactive voyage into space*. D-Data Inc., New York, USA, 1997.

23. *TheSky6*. Software Bisque Inc., CO, USA, 2004.

24. *Universe*. Discovery Communications Inc., New York, 2000.

On-line resources

1. European Space Agency. www.esa.int. Accessed in February 2011.
2. Hubble Space Telescope. www.hubblesite.org. Accessed in January 2011.
3. Hubble Space Telescope. www.spacetelescope.org. Accessed in March 2011.
4. Inter-University Center for Astronomy and Astrophysics, Pune, India. www.iucaa.ernet.in. Accessed in March 2011.

5. National Aeronautics and Space Administration Education Center. www.spacecenter.org/distancelearning.htm. Accessed in February 2011.
6. National Aeronautics and Space Administration. www.nasa.gov. Accessed in March 2011.
7. Wikipedia. http://en.wikipedia.org/wiki/scientific_temper. Accessed in March 2011.

A Brief History of Gravitation: Copernicus to Newton *

Somnath Datta
656, “Snehalata”, 13th Main, 4th Stage, T K Layout, Mysore 570009, India
datta.som@gmail.com; http://sites.google.com/site/physicsforpleasure

(Submitted 12-06-2012)

Abstract

The article traces the contributions of the pioneers of the theory of gravitation, in particular the heliocentric model of Nicolaus Copernicus, planetary data collected by Tycho Brahe, Johannes Kepler’s analysis of Tycho’s data leading to the formulation of his three laws of planetary motion, and the “falling of apple” episode that gave Newton a sudden flash of a larger vision that unified the orbital motion of the moon, the orbital motion of the planets and the falling of terrestrial objects downward, into one single law of universal gravitation. Based on the model of the heliocentric universe the geocentric paths of Venus and Mars have been constructed in two ways, using a geometrical method, and plotting the relevant parametric equations. The orbital radius R and the time period T of each planet’s revolution around the sun has been calculated from the observed value of the angle of its maximum deviation from the sun and the measured value of its synodic period. Kepler’s laws of planetary motion have been reviewed and the third law has been checked using the calculated values of T and R. The role played by the 3rd Law of planetary motion in shaping Newton’s law of universal gravitation has been highlighted. How the inverse square nature of the law of gravitation relates the orbital motion of the moon to the falling of an apple has been worked out in mathematical details.

1 Newton and the Apple and the Moon

We have all heard the story that Newton was led to the law of universal gravitation while

---

[*] This article is part of a book on Mechanics, published by *Pearson (2012)* by the author.

watching an apple fall to the ground. This story has become as much a landmark in the history of science as the discovery that followed it. We shall piece together relevant parts of the story to make a story of our own and make a picture of the mind of a genius that saw in such an insignificant event a glimpse of a much larger scheme of the universe that works behind the motion of the moon around the earth, the motion of the planets around the sun, as much as in the falling of an apple, unifying all such diverse and unrelated phenomena into one single law of nature[1, 2, 3, 4, 5, 6].

The period was 1665-1666, the years of plague, when many public institutions were closed and Isaac Newton, now 23 years old, had to leave Cambridge to take shelter in his mother's farm Woolsthrope Manor near Grantham in Lincolnshire. One moonlit evening he was sitting in the garden. His mind was immersed in a deep thought, seeking answer to the question, "what force makes the moon go round the earth?" He looked up to the sky and saw the moon and thought of the force of gravity (of the earth) extending to its orbit. An apple fell from a tree nearby. Spurred by this incident, an idea came to his mind that the same force that the earth exerted on the apple making it fall straight down might also be exerted on the moon making it go round. His conjecture:

Conjecture 1 *The moon is a falling body, just like an apple, falling under the earth's force of gravity.*

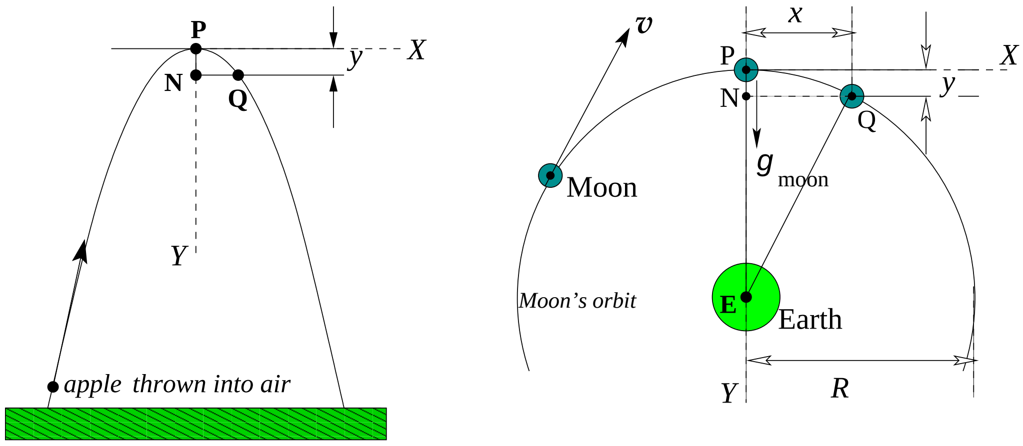
We shall explain how Newton calculated the rate of falling of the moon. Let us consider an apple which has been thrown into the air from the ground, as shown in Fig.1(a). The trajectory is a parabola. At the top of the trajectory its tangent is a horizontal line. The fall of the apple in time t is the distance y, measured vertically downward from the horizontal tangent, and is given by the well known formula

$$y = \frac{1}{2}gt^2. \tag{1}$$

Newton applied the same principle to the "falling moon". However, the trajectory of the moon in this case is not a parabola, but a circle. The surface of the earth, lying underneath the moon, curves into a sphere as the moon travels along its orbit, whereas for an apple or a cricket ball, whose range of flight is negligible compared to the radius of the earth, the ground underneath is nearly a flat surface. Nevertheless, Newton calculated the distance y by which the moon would be falling in a small time $t \ll T$, where T the period of one complete revolution of the moon around the Earth. He probably used simple geometry and Pythgoras's theorem for his calculations.

Fig. 1(b) depicts (part of) the circular trajectory of the moon around the earth. The speed of the moon in the orbit is $v = \omega R$ where ω is the angular velocity of revolution of the moon around the earth, and R is the radius of the moon's orbit.

Let us get some crucial data first. The radius of the moon's orbit, as calculated by the Greek astronomer Hipparchus in 130 B.C., was approximately 3.8×10^8 m, which Newton should have used for his calculations. The



(a)

(b)

Figure 1: *Trajectories of a projected apple in (a), and the Moon in (b).*

period of the moon's revolution around the earth is $T = 27.3$ days $\approx 2.4 \times 10^6$ s. From these two figures we can calculate the orbital velocity v of the moon $v = \omega R = \frac{2\pi}{T} \times R = \frac{2\times\pi\times3.8\times10^8}{2.4\times10^6} \approx 10^3$ m/s $= 1$ km/s.

At a certain time $t = 0$ the moon is at the point P. We have drawn the Cartesian X and Y axes through P such the the X axis is tangential to the circular path pointing in the direction of the moon's motion, and the Y axis is pointing down radially to the centre of the circle. If there had been no Earth, the moon would be following a straight path, along the X axis. The Earth's gravity bends the path downward.

Let P and Q be two locations of the moon differing by a very small time interval δt (say, 1 second.) The coordinates of Q are (x, y). $x \approx v\,\delta t = \omega R\,\delta t$. y is the distance by which the centre of the moon "drops" vertically downward in time δt from the straight line path along the X axis. For the very small time δt this drop y is very small compared to the distance x the moon travels. For example if we take $\delta t = 1$ s, then $x = v\,\delta t = 1000$ m, whereas $y \approx 1.3$ mm, as we shall soon find out. In the following estimate we shall therefore set $y \ll x$.

Note that $\overline{EN} = R - y$. By Pythagoras's theorem $\overline{EN}^2 + \overline{NQ}^2 = \overline{EQ}^2$. In terms of the coordinates,

$$(R-y)^2 + x^2 = R^2.$$

Cancelling R^2 on both sides,

$$-2Ry + y^2 + x^2 = 0.$$

Or, neglecting y^2 compared to x^2,

$$y \approx \tfrac{x^2}{2R} = \tfrac{(\omega R\,\delta t)^2}{2R} = \tfrac{1}{2}(\omega^2 R)\,\delta t^2. \qquad (2)$$

Comparing Eq. (2) with (1) we find that in every tiny interval of time δt the moon drops perpendicular to the tangent drawn to its trajectory and the acceleration of this fall is:

$$g_{\text{moon}} = \omega^2 R = \frac{\omega^2 R^2}{R} = \frac{v^2}{R}. \qquad (3)$$

This is also the familiar expression for what we often refer to as the "centripetal acceleration". Taking the values of v and R already given, Newton obtained the acceleration of the moon to be

$$g_{\text{moon}} = \frac{10^6}{3.8 \times 10^8} = 0.26 \times 10^{-2}\,\text{m/s}^2. \qquad (4)$$

The distance through which the moon falls in the tiny interval δt, say 1 second, is then

$$y = \frac{1}{2}\,g_{\text{moon}}\,\delta t^2 = 0.13 \times 10^{-2}\,\text{m}. \qquad (5)$$

This acceleration given in (4) is too small compared to the the acceleration due to gravity near the surface of the earth, given as $g = 9.8\,\text{m/s}^2$, which is also the acceleration of the apple. The fall distance given in (5) is also incredibly small compared to a fall distance of 4.9 m in one second for an apple. Such small values puzzled Newton. He left the problem of "falling moon" for the time being and diverted his mind to seek answer to a larger question "what forces are acting on the planets making them follow the orbits as described by Kepler's laws of planetary motion?"

2 Heliocentric Model of Copernicus

2.1 Motion of planets as seen from Earth - Geocentric view of the Greek school

The theory of universal gravitation did not descend on Newton's mind in one stroke with the falling of an apple. Newton arrived at this theory primarily by analyzing Kepler's laws of planetary motion. Kepler had earlier formulated these laws by a meticulous analysis of the data on the position of Mars and other planets observed and recorded over a period of thirty five years by Tycho Brahe. These historical anecdotes are important and should be part of one's understanding of gravitation. We shall try to present an elementary sketch of what had happened before Newton[7].

The ancients had keenly observed the pattern of the motion of the heavenly objects in the sky, and had invented a scheme of their motion around the Earth. The Greek philosophers, from Aristotle to Plato, from Ptolemy to his followers, had drawn up an earth-centric model of the universe, known as the *geocentric* universe, in which the Earth was considered to be fixed in space, and constituted the centre of the universe. The heavenly objects, namely the sun, the moon, the planets and the stars, were hypothesised to be moving in *perfect circles*, on crystal spheres, in perfect harmony, because a circle was considered to be a perfect curve, and because for the heavenly bodies only the perfection of circular motion was permitted.

This simplistic idea came into conflict with the apparent motion of the planets. Seen in the background of the stars the planets were moving non-uniformly. They moved eastward nearly straight for much of the time, in what is now referred to as the *direct motion.* However, at certain points they slowed down, reversed the motion westward, made a loop-the-loop, then preceeded eastward, as before. This reverse motion was called *retrograde motion* [8, 9].

We have shown this pattern in Fig.3a for Venus and in Fig.5a for Mars. In both figures we have labelled the background stars as "fixed stars".

The ancients had also observed that the brightness of the planets was not constant but changed with time. The retrograde motion and the varying brightness pointed to a complex motion of the planets, quite different from a simple minded uniform circular motion, in which the distances of the planets from the earth were continuously changing.

The Greek philosophers, strongly rooted in the faith that only circular motion was permissible for heavenly bodies, had tried to explain away the above mentioned anomalous movement of the planets by hypothesizing the planetary motion to be a combination of two or more circular motions. We have illustrated this idea in Fig. 2. With the earth E as the centre, the planet P moved on a *smaller* circle, called *Epicycle*, the centre of which moved on a *bigger* circle called *Deferent* (fixed on the crystal sphere), as shown in (a). If this construction did not fit the observation, they added further epicycles, as shown in Fig.(b). [10, 11]

2.2 Motion of planets as seen from Earth - Heliocentric Explanation of Copernicus

A different model of the universe was suggested by a Polish astronomer Nicolaus Copernicus in 1543 in his book *De revolutionibus orbium coelestium (On the revolutions of the Celestial Spheres).* In the Copernican system the Sun was the centre of the universe and assumed to be immovable. The stars were also fixed on the "immovable celestial sphere" with its centre on the sun. The planets, including the Earth, moved around the sun in a *uniform circular motion.* This model was called the *Heliocentric universe* (Helio=the Sun). Another feature of Copernicus's proposal was that the Earth rotated about its axis, once a day, as it moved along its orbit around the sun.

Copernicus had cited the Greek philoso-

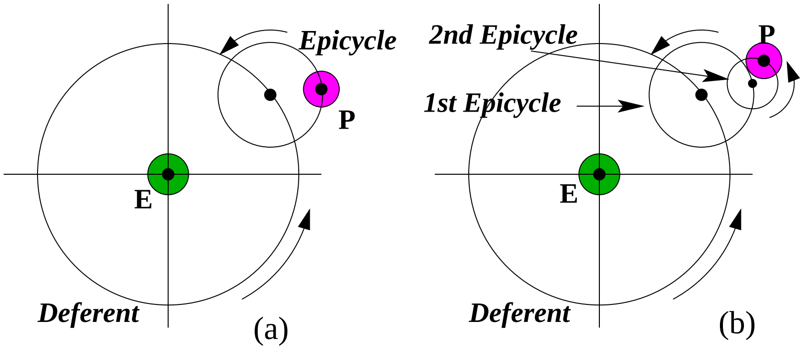


Figure 2: *Epicycle on Deferent*

pher Aristarchus as the source of his central heliocentric idea.

This new model came in direct conflict with the pre-existing Ptolemic view, the geocentric universe, which was also the most natural and obvious thing to believe, and which had the support of the Roman Catholic Church. Copernicus, himself a cleric under the Catholic church, in order to avoid any controversy, suggested that this model was for mathematical convenience only, and was not necessarily the truth. The mathematical calculations needed to predict the position of the planets in the sky, as in the Ptolemic scheme, could only be simplified if one used this model.

2.3 Geocentric path of Venus from the Copernican model

We shall follow the hint given by Copernicus, use the heliocentric model as a starting point, and reconstruct the Ptolemic paths of the planets Venus and Mars, each moving on its own epicycle around a deferent, and causing the "loop-the-loop" retrograde motion. We shall refer to this path as the *geocentric path*, or *g-path* for abbreviation.

For the geometric constructions that will follow, it will be convenient to distinguish between the *inferior planets*, or, the *inner planets*, having orbital radii less than that of the Earth, and the *superior planets* or the *outer planets*, having orbital radii larger than that of the Earth. Venus belongs to the first category, and Mars to the second. We shall measure planetary distances in Astronomical Unit (AU). *One AU is equal to the mean radius of the Earth's orbit around the Sun,* and is equal to 1.496×10^{11} m.

In this subsection we confine ourselves to Venus, and illustrate our construction of the g-path of the planet in Fig. 3. The Earth, the Sun and Venus have been represented by the letters E,S and V respectively.

Fig. 3(b) shows the Copernican picture of the motion of E and V around S. Here S is the centre of the universe. It is fixed and is the origin of the Cartesian coordinate system.

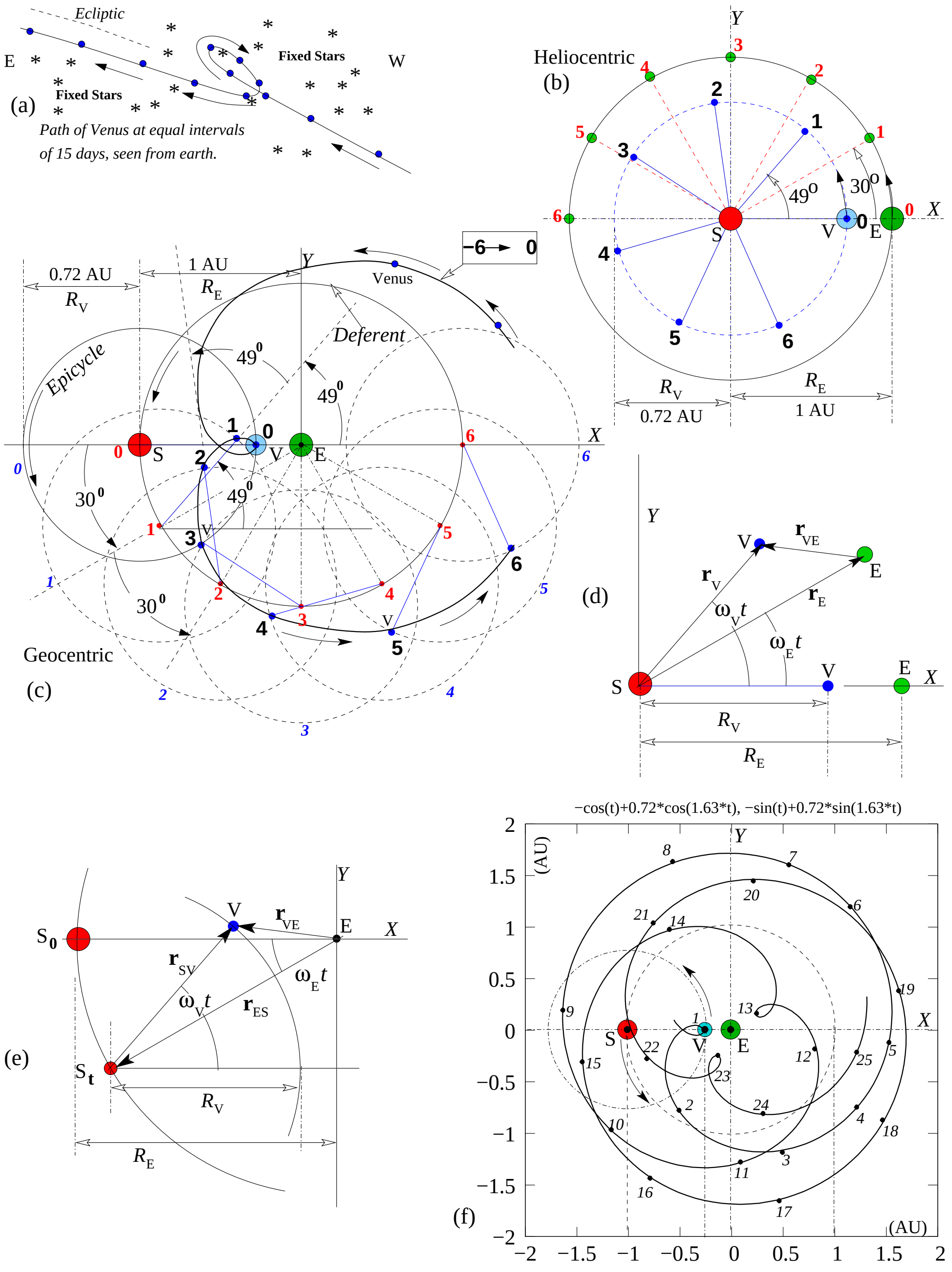


Figure 3: *Path of Venus. (a) Seen against background stars; (b) Heliocentric paths of V and E; (c) Geometrical construction of the g-path; (d) and (e) Radius vectors R_E, R_V, R_{VE}, etc; (f) Gnuplot of the g-path.*

Figs. 3(c),(f) show the g-path of V. Here E is the centre of the universe and the origin of the Cartesian axes X and Y. Seen from E, the Sun S is moving on the *Ecliptic*, which in this case is the *Deferent.* And V, which is actually moving around S, is seen to be moving on the *Epicycle* which is a moving circle with its centre always lying on the deferent.

We shall use the following data [12] for our calculations and plotting. The radii of the orbits of the Earth and Venus around the Sun are $R_E = 1$ AU and $R_V = 0.72$ AU respectively. The period of one complete revolution around the Sun is $T_E = 365.25$ days for the Earth and $T_V = 224.7$ days for Venus.

The orbital angular velocities of E and V follow from their periods. $\omega_E = \frac{2\pi}{12}$ rad/month, and $\omega_V = \frac{2\pi}{12} \times \frac{T_E}{T_V} = \frac{2\pi}{12} \times \frac{365.25}{224.7} \approx 1.63 \times \frac{2\pi}{12}$ rad/month. Converting into degrees, $\omega_E = 30^0$ per month, and $\omega_V \approx 1.63 \times 30^0 \approx 49^0$ per month. We shall adopt one month as the unit of time.

In Fig. 3b we have shown two concentric circles, of radii R_V and R_E on which E and V are revolving around the centre S in the *anticlockwise* direction. This is the Copernican view.

The locations of V at equal time intervals of *one month* are represented by large boldface sans serif numerals **0,1,2,3,4,...**. Similarly normal size numerals 0,1,2,3,4,... indicate the locations of E in Fig. (b) and the locations of S with respect to E in Fig (c) at the same time intervals.

Let us now get a *qualitative* understanding of the geocentric motion on the basis of the Copernican hypothesis (Fig. c). The point S is always located at a distance R_{E} from E. As seen from E, it is moving in a circle of radius R_{E} anticlockwise. This circle is called the *Ecliptic*. For our purpose this circle is the *Deferent*.

The point V is always moving around S in a circle of radius R_{V}, according to the Copernican model. In the geocentric picture S is moving, and therefore V is moving in a circle of the same radius about the moving point E. This moving circle is then identified with the *Epicycle*.

In Fig (c) we have shown the epicycle at seven instants of time $t = 0, 1, 2, 3, 4, 5, 6$, labelling them by slanted numerals *0,1,2,3,4,5,6*.

Let us now consider the instant $t = 0$. In Fig (b) V and E are both on the right side of the centre S. Therefore, in Fig (c) S and V are on the left of the centre E and (0, **0**) mark the locations of (S,V).

After one month, i.e., at $t = 1$, V moves to **1** and E moves to 1 in Fig. (b). In Fig (c) E remains stationary at the origin, whereas V is riding on the epicycle. The centre of the epicycle has moved by angle 30^0 to its new location 1, whereas V has moved by angle 49^0 to **1**.

Note that the line $\overrightarrow{1E}$ in (c) is parallel to the line $\overrightarrow{S1}$ in (b), and the line $\overrightarrow{1\mathbf{1}}$ in (c) is parallel to the line $\overrightarrow{S\mathbf{1}}$ in (b).

At the end of two months, i.e., at $t = 2$, V moves to **2** and E moves to 2 in Fig. (b). In Fig (c) the centre of the epicycle has moved further by another angle of 30^0 to its new location 2, whereas V has moved further by another angle of 49^0 to the location **2**. The line $\overrightarrow{2E}$ in (c) is parallel to the line $\overrightarrow{S2}$ in (b), and the line $\overrightarrow{2\mathbf{2}}$ in (c) is parallel to the line $\overrightarrow{S\mathbf{2}}$ in (b).

Proceeding in this way we find the positions of V at $t = 3, 4, 5, 6$, shown as **3,4,5,6**. Joining the points **0,1, ... ,6** by a smooth curve we have completed the construction of the g-path **0-1-2-3-4-5-6** traced by Venus in six months, from $t = 0$ to $t = 6$.

We can extrapolate the path to the past, from $t = 0$ to $t = -6$, by making an inversion of the original curve **0-1-2-3-4-5-6** and then adding to it. This extrapolated curve is labelled as $\boxed{\mathbf{-6}\rightarrow\mathbf{0}}$.

We have thus obtained the g-path of Venus, spread over one full earth year, from $t = -6$ to $t = +6$ (time measured in months) on the basis of the heliocentric model of Copernicus. The loop-the-loop cusp is prominent at $t = 0$, i.e., around the point **0** in Fig.(c).

The same path can be obtained using coordinate geometry and "gnuplot". For this we need the transformation equation that will convert the Copernican orbit into a Ptolemian orbit.

Referrring to Fig.(d) the radius vectors of V and E, are $\mathbf{r}_V$ and $\mathbf{r}_E$. The radius vector of V relative to E is $\mathbf{r}_{VE} = \mathbf{r}_V - \mathbf{r}_E$. Then

$$\begin{aligned}
\mathbf{r}_V &= R_V(\cos\omega_V t\,\mathbf{i} + \sin\omega_V t\,\mathbf{j}) && (a)\\
\mathbf{r}_E &= R_E(\cos\omega_E t\,\mathbf{i} + \sin\omega_E t\,\mathbf{j}) && (b)\\
\mathbf{r}_{VE} &= (R_V\cos\omega_V t - R_E\cos\omega_E t)\mathbf{i} + (R_V\sin\omega_V t - R_E\sin\omega_E t)\mathbf{j}. && (c)
\end{aligned} \qquad (6)$$

We have shown the graphical construction of the vector $\mathbf{r}_{VE}$ in two different ways. In Fig. (d) the vectors $\mathbf{r}_V$ and $\mathbf{r}_E$ are drawn according to Eq. (6 a,b). The straight line joining E to V is the relative displacement vector $\mathbf{r}_{VE}$.

In Fig. (e) $\mathbf{r}_{ES}$ is the displacement of the sun S with respect to the "fixed" earth E, and $\mathbf{r}_{SV} = \mathbf{r}_V$ is the displacement of Venus V with respect to the "moving Sun". Adding these two vectors we get back $\mathbf{r}_{VE} = \mathbf{r}_{ES} + \mathbf{r}_{SV}$.

$$\begin{aligned}
\mathbf{r}_{ES} &= R_E\left[\cos(\pi + \omega_E t)\,\mathbf{i} + \sin(\pi + \omega_E t)\,\mathbf{j}\right] = -R_E(\cos\omega_E t\,\mathbf{i} + \sin\omega_E t\,\mathbf{j}) && (a)\\
\mathbf{r}_{SV} &= R_V(\cos\omega_V t\,\mathbf{i} + \sin\omega_V t\,\mathbf{j}) && (b)\\
\mathbf{r}_{VE} &= (R_V\cos\omega_V t - R_E\cos\omega_E t)\mathbf{i} + (R_V\sin\omega_V t - R_E\sin\omega_E t)\mathbf{j}. && (c)
\end{aligned} \qquad (7)$$

We can now plot the g-path of Venus, given by the radius vector $\mathbf{r}_{VE}$, at different times. Taking the values of R_E, R_V and ω_V, ω_E obtained at the beginning of this subsection, we write the parametric equation of the path, with t as the parameter.

$$\begin{aligned} x &= R_V \cos \omega_V t - R_E \cos \omega_E t &&= 0.72 \cos(1.63 \times \tfrac{2\pi t}{12}) - \cos \tfrac{2\pi t}{12} \\ & &&= 0.72 \cos(1.63\tau) - \cos \tau. \\ y &= R_V \sin \omega_V t - R_E \sin \omega_E t &&= 0.72 \sin(1.63 \times \tfrac{2\pi t}{12}) - \sin \tfrac{2\pi t}{12} \\ & &&= 0.72 \sin(1.63\tau) - \sin \tau. \end{aligned} \tag{8}$$

In the last equalities we have chosen a new parameter $\tau = \frac{2\pi t}{12}$. We have made a plot of the above curve in Fig. 3(f) from $\tau = -\pi/2$ to $\tau = 9\pi$, covering 4.75 years. The numerals 1,2,3,..., 25 written alongside the path are in increasing order of time, but not placed at equal time intervals. The "loop-the-loop cusps" appear at the points 1,13,23. These are the points where retrograde motion of the planet appears to take place.

The g-path we have just plotted is not what is seen from the Earth. It is seen by a stationary observer sitting on the Z axis (i.e., the axis passing through the origin E in Figs. c and f) above the plane of the Ecliptic . An earthbound observer sees the projection of the g-path on the "celestial sphere". Noting that the plane of the orbit of Venus makes an angle of 3.4^0 with the plane of the Ecliptic [16] (though in our drawing we have taken them to be coplanar) one should be able to show that this projection is similar to the path shown in Fig. (a).

2.4 Geocentric path of Mars from the Copernican model

Mars has a special place in our narrative. By a painstaking analysis of the observation data of Mars, taken earlier by Tycho Brahe, Kepler was able to obtain his laws of planetary motion.

Let us obtain the necessary data for Mars. Radius of the orbit $R_M = 1.524$ AU. Period of one complete revolution around the Sun T_M= 686.98 days. The orbital angular velocity of Mars: $\omega_M = \frac{2\pi}{12} \times \frac{T_E}{T_M} = \frac{2\pi}{12} \times \frac{365.25}{686.98} \approx 0.53 \times \frac{2\pi}{12}$ rad/month. Converting into degrees, $\omega_M \approx 0.53 \times 30^0 \approx 16^0$ per month.

We have illustrated the construction of the g-path of Mars, both by geometrical construction and by plotting of the parametric equation, in Fig. 5. The Earth is represented by E and Mars by M. This construction is similar to the one for Venus with one important difference. Venus is an inner planet having its orbit inside that of the Earth, and Mars is an outer planet having orbit outside Earth's. For the interior planets the ecliptic (larger circle) is the Deferent and the orbit (smaller circle) is the Epicycle. The roles get interchanged when we go to the outer planets. Now the orbit of the planet (larger circle) is the Deferent, and the Ecliptic (smaller circle) is the epicycle. Since this requires some clarification, and can be confusing, we shall first show a simpler construction of a part of the orbit of Mars in Fig. 4.

Fig. 4(a) shows the heliocentric motion of the Earth and Mars, and their locations at equal intervals of one month, with the Sun S fixed at the origin, all drawn to scale. The Earth and Mars are moving on their respective circular orbits around the Sun, of radii R_E and R_M and covering angles 30^0 per month and 16^0 per month respectively. As in the case of Venus we adopt one month as the unit of time. At $t = 0$, E and M are in conjunction, i.e., they lie on one straight line passing through S. We denote their locations as E_0 and M_0 respectively. At $t = 1$, E has moved by 30^0 to E_1 and M has moved by 16^0 to M_1. Continuing this way we get the locations $(\mathrm{E}_2, \mathrm{M}_2), \cdots, (\mathrm{E}_6, \mathrm{M}_6)$, corresponding to $t = 2, \cdots, 6$.

Going backward in time we get the locations $(\mathrm{E}_{-1}, \mathrm{M}_{-1}), \cdots, (\mathrm{E}_{-6}, \mathrm{M}_{-6})$, corresponding to $t = -1, \cdots, -6$. Note that the Earth comes back to the same location after 12 months, and therefore, the point E_6 coincides with point E_{-6}, and we have merged the two points with the label $\mathrm{E}_{6,-6}$. The displacements of Mars relative to the Earth at the times $t = n$ are given by the vectors $\mathbf{r}_n$, stretching from $\mathrm{E_n}$ to $\mathrm{M_n}$; $n = -6, \cdots + 6$.

We have brought these vectors to a common staring point at the location E in Fig. 4(b) The displacements of M relative to E at equal intervals of one month are now clearly seen.

In Fig. 4(c) we have joined the tips of these vectors with a smooth curve. This curve is the g-path over a period of one year, spread equally before and after the conjunction time $t = 0$.

We now come to a more methodical construction and plot of the g-path in Fig. 5. Since this construction is similar to the one for Venus, we shall avoid some of the details. Fig. 5(b) gives the Copernican picture of the positions of E and M at equal intervals of 1 month, with the Sun at the centre, and E and M going around it.

Fig. 5(b) shows the Copernican picture of the motion of E and M around S. Here S is the centre of the universe. It is fixed and is the origin of the Cartesian coordinate system.

Fig. 5(c) has E fixed at the origin of the Cartesian coordinates, as we are analyzing the motion of M relative to E. S goes on the smaller circle of radius R_E labeled *Ecliptic* with angular velocity ω_E, and M on the bigger circle of radius R_M labeled *Orb* with angular velocity ω_M. At $t = 0$, S is at S_0 and M is at M_0.

After time t, S has moved on the Ecliptic, through an angle $\omega_E t$, to the location $\mathrm{S_t}$. And M, riding on the Orb, has moved through an angle $\omega_M t$ to $\mathrm{M_t}$. Let $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ represent the displacements $\overrightarrow{\mathrm{ES_t}}, \overrightarrow{\mathrm{S_tM_t}}, \overrightarrow{\mathrm{EM_t}}$ respectively. Then $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{c}$

Here $\boldsymbol{a}$ is the displacement of S relative to E, as the former moves through the angle $\omega_E t$ on a circle of smaller radius R_E. Similarly, the vector $\boldsymbol{b}$ is the displacement of M relative to S, as the former moves through the angle $\omega_M t$ on a circle of larger radius R_M.

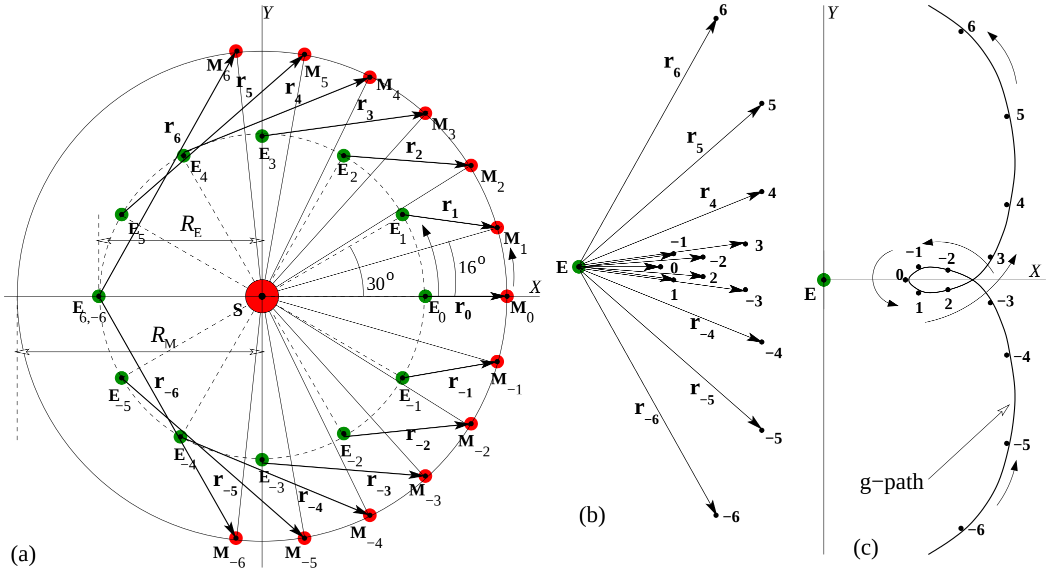


Figure 4: *Simple construction the g-path of Mars. (a) Heliocentric motion of E and M; and the relative displacemnt* $\{\mathbf{r}_n; n = -6, -5, \ldots, -1, 0, 1, \ldots, 5, 6\}$ *of M with respective to E over one year at 1 month intervals; (b) Relative displacement vectors* $\{\mathbf{r}_n\}$ *drawn from E; (c) Joining the tips of the vectors* $\{\mathbf{r}_n\}$ *with a smooth curve to get the g-path.*

Now, independent of S,E and M, the vector $\boldsymbol{b}$ can be looked upon as a displacement, in the anticlockwise direction, on a circle of larger radius R_M, and $\boldsymbol{a}$ as a displacement, in the same direction, on a circle of smaller radius R_E. Also $\boldsymbol{c} = \boldsymbol{b} + \boldsymbol{a}$. We have illustrated this in Fig 5(d).

We come to the *conclusion* that the *net displacement* of M, represented by the vector $\boldsymbol{c}$ is *same as a displacement on a bigger circle (deferent)*, represented by the vector $\boldsymbol{b}$, *followed by a displacement on a smaller circle (epicycle)*, represented by the vector $\boldsymbol{a}$.

In summary, E is fixed. An imaginary point I (shown in Fig 5d) is moving on the deferent of radius R_M *around E. Around this point I the planet M is moving on an epicycle of radius equal to the radius of the Earth* R_E.

Using the same graphical method employed for Venus we have done a graphical construction of the g-path of Mars in Fig 5(e).

To obtain the parametric equation for the g-path let us first note that:

$$\begin{aligned}
\boldsymbol{a} &= \mathbf{r}_{ES} = -R_E(\cos\omega_E t\,\mathbf{i} + \sin\omega_E t\,\mathbf{j}) \qquad \text{(see Eq.7a)} && (a)\\
\boldsymbol{b} &= \mathbf{r}_{SM} = R_M(\cos\omega_M t\,\mathbf{i} + \sin\omega_M t\,\mathbf{j}) && (b) \qquad (9)\\
\boldsymbol{c} &= \mathbf{r}_{EM} = (R_M\cos\omega_M t - R_E\cos\omega_E t)\mathbf{i} + (R_M\sin\omega_M t - R_E\sin\omega_E t)\mathbf{j}. && (c)
\end{aligned}$$

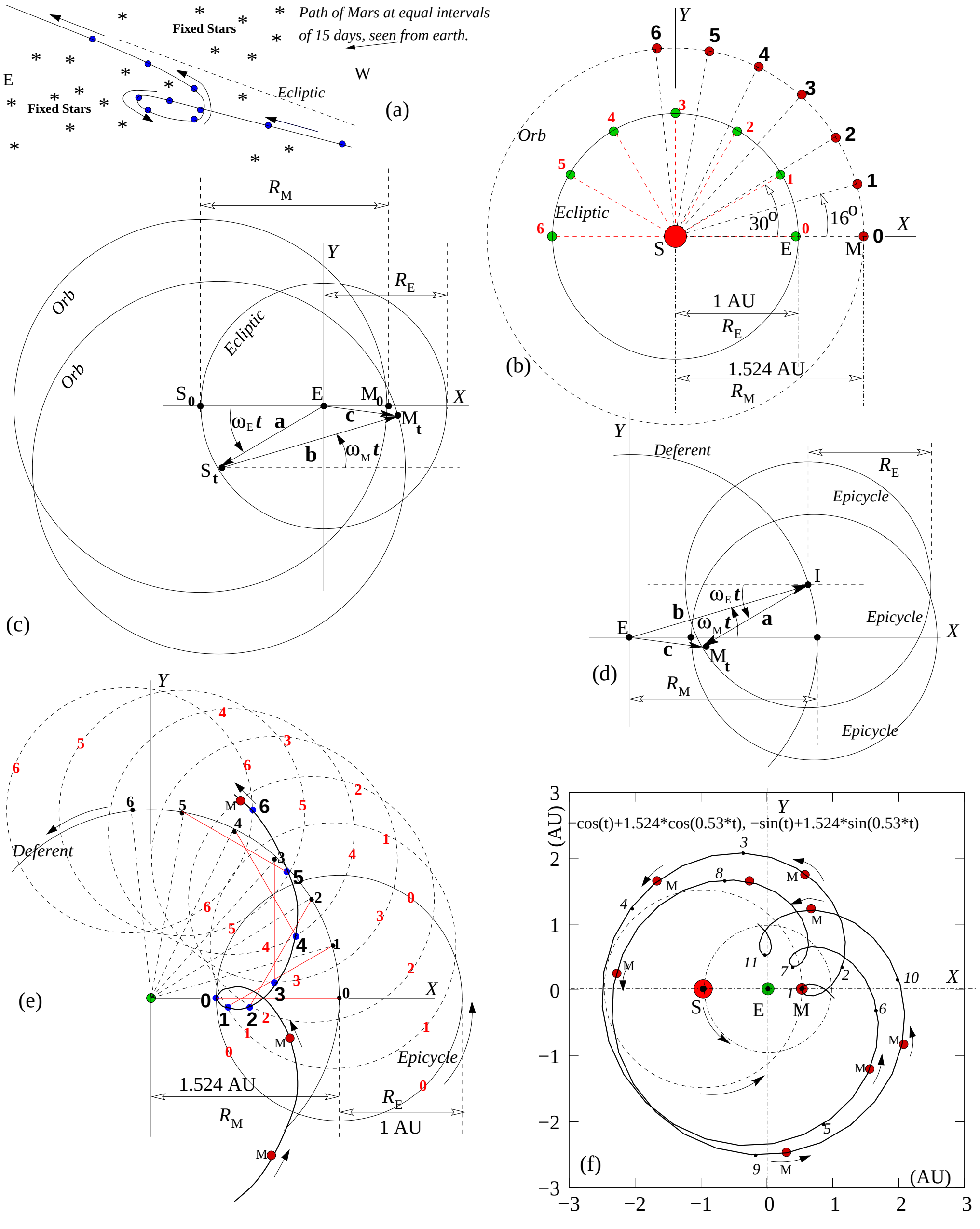


Figure 5: *Path of Mars. (a) Seen against background stars; (b) Heliocentric paths of M and E; (c) and (d) Identifying the deferent and the Epicycle; (e) Geometrical construction of the g-path; (f) Gnuplot of the g-path.*

We now write the parametric equation of the g-path as

$$\begin{aligned} x &= -R_E\cos\omega_E t + R_M\cos\omega_M t &&= -\cos\tfrac{2\pi t}{12} + 1.524\cos(.53\times\tfrac{2\pi t}{12}) \\ & &&= -\cos\tau + 1.524\cos(.53\tau). \\ y &= -R_E\sin\omega_E t + R_M\sin\omega_M t &&= -\sin\tfrac{2\pi t}{12} + 1.524\sin(.53\times\tfrac{2\pi t}{12}) \\ & &&= -\sin\tau + 1.524\sin(.53\tau). \end{aligned} \qquad (10)$$

The parametric gnuplot of g-path is shown in Fig. 5(f). It has been drawn over the range $\tau = \pi/2$ to $\tau = 9\pi$, i.e., covering 4.75 years. However, (as in the case of Venus) this is not what is seen from Earth. An earthbound observer sees the projection of the g-path on the "celestial sphere". Noting that the plane of the orbit of Mars makes an angle of 1.9^0 with the plane of the Ecliptic [16] one should be able to show that this projection is similar to the path shown in Fig 5(a).

2.5 Calculation of the periods of the planets by Copernicus

Copernicus had also obtained the distances of the planets from the Sun and the time periods of their orbital motion. The ancient astronomers, starting from Ptolemy, had obtained the same or similar data. This should not be surprising since in the ancient world astrology, rather than astronomy, held sway, and it was of paramount importance to predict the time of appearance of a planet at a specified location in the sky, for astrological predictions.

We shall take a quick look at the trigonometrical methods which might have been applied by Copernicus to obtain the periods and the orbital radii of the planets.

First the period. The interval of time T_P in which a planet completes one full orbit around the Sun is the period of revolution of the planet. From our observation station, the Earth, we cannot determine T_P directly. We can, however, determine the *Synodic Period* τ of a planet by direct observation, from which one can obtain T_P.

We have explained synodic period in Fig. 6.

Let us take the planet P to be an *outer planet*, say Jupiter. There is a moment when when the Sun S and the planet P are in *opposition* with respect to the Earth. This means that they lie on the same straight line as E, but located on opposite sides of E, as seen in the configuration SE_1P_1 in Fig. 6(a). We can identify this moment by noting the date when P crosses the celestial *meridian* at *midnight.*

Now, both E and P are revolving around the Sun, but E is revolving faster than P (it should have been a common knowledge of the ancient astronomers that the angular motion of the sun around the ecliptic was faster than the angular motion of the outer planets on the celestial sphere.) After some time τ, S and P will come back to opposition once

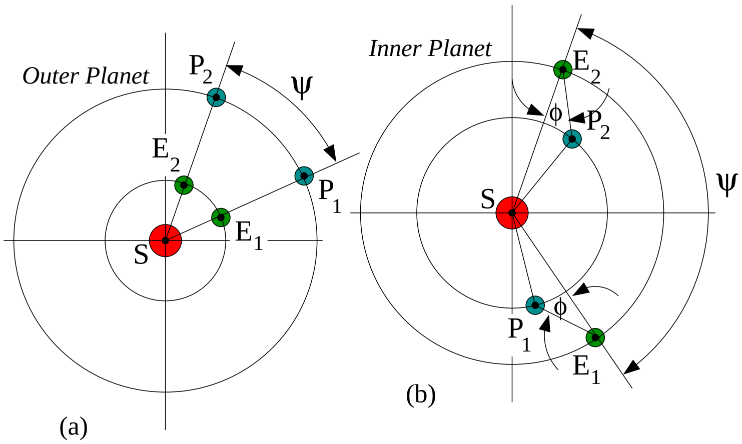


Figure 6: *Explaining Synodic Period. (a) Inner planet; (b) Outer planet.*

again, to the new configuration SE_2P_2. This time τ is the synodic period of the planet.

This will happen because in this time τ the planet will move through an angular displacement ψ, whereas E will go through an extra angle 2π, i.e., go through a total angular displacement $2\pi + \psi$. (This is similar to the movement of the minute hand over the hour hand in a clock.)

The angular velocity of E relative to P is $\omega_{rel} = \omega_E - \omega_P$. Then

$$\begin{aligned} \omega_{rel}\tau = 2\pi. \quad &\Rightarrow \quad \frac{2\pi}{T_E} - \frac{2\pi}{T_P} = \frac{2\pi}{\tau}. \\ &\Rightarrow \quad \frac{1}{T_E} - \frac{1}{T_P} = \frac{1}{\tau}. \end{aligned} \tag{11}$$

Inserting the vaue of τ obtained from measurement, and $T_E = 365.26$ days in Eq. (11) we get the required period T_P.

In the case of the *inner planets*, they will never be in opposition. We could have used their conjunction instead, i.e., position of the planet in the direction of the Sun along the line joining E and S. However, it is not possible to view the planet when it is in conjunction. Therefore we can offset the planet from the ES line by a certain angle ϕ. For example we view Venus some day when it makes angle $\phi = 30^0$ with the ES line (which will be, say 2 hours after sunset), and wait till it again makes the *same* angle $\phi = 30^0$ with the ES line. The period of waiting, τ, is the synodic period of the planet.

In this case $\omega_P > \omega_E$, and $\omega_{rel} = \omega_P - \omega_E$. Therefore,

$$\begin{aligned} \omega_{rel}\tau = 2\pi. \quad &\Rightarrow \quad \frac{2\pi}{T_P} - \frac{2\pi}{T_E} = \frac{2\pi}{\tau}. \\ &\Rightarrow \quad \frac{1}{T_P} - \frac{1}{T_E} = \frac{1}{\tau}. \end{aligned} \tag{12}$$

As in the previous case we obtain T_P from the known values of T_E and τ.

We now list below the Synodic Periods and the Time Periods of the planets as recorded by Copernicus.

Table 1: COPERNICAN ESTIMATE OF TIME PERIODS OF PLANETS (IN YEARS) AND COMPARISON WITH MODERN VALUES*.

	0	1	2	3	4	5	6
1	Category →	inner	inner		outer	outer	outer
2	Planet →	**Mercury**	**Venus**	**Earth**	**Mars**	**Jupiter**	**Saturn**
3	τ (days) →	115.88	538.92	-	779.04	398.96	378.09
	τ (years) →	0.317	1.475	-	2.133	1.092	1.035
4	Formula:	Eq.(12) ↓	Eq.(12) ↓		Eq.(11) ↓	Eq.(11) ↓	Eq.(11) ↓
		0.24	0.596		1.885	11.869	29.57
5	**Copernican →**	**0.24**	**0.615**	**1**	**1.882**	**11.87**	**29.44**
6	Modern →	0.24	0.615	1.00	1.881	11.862	29.457

*Rows 3, 5 and 6 are taken from A.P.French, *op. cit.*

2.6 Calculation of the orbital radii of the planets by Copernicus

Copernicus calculated the orbital radii of the five planets (other than the Earth) known to the ancients. We shall obtain these values in Astronomical Units (AU), from direct observation of what one may call the *angle of maximum deviation*, denoted by θ_m.

We first take up the case of the *inner* planets. There are two planets in this category, Mercury and Venus. *Mercury* is the innermost planet. *Venus* comes next, its orbit placed between those of Mercury and the Earth.

We assume that to a first approximation the motion of the inner planet, as seen from the Earth, is along an epicycle of radius R_P, riding on a deferent of radius R_E. Here the subscripts $_P$ and $_S$ stand for the Planet and the Earth respectively. We have illustrated this in Fig. 7(a). The Sun S is at the centre of the epicycle, moving around E on the Deferent.

The planet P makes an angle θ with the Sun when it is at any arbitrary point P. However, there are two locations on the deferent, A and B, at which this angle has the maximum value θ_m. This angle is the angle of

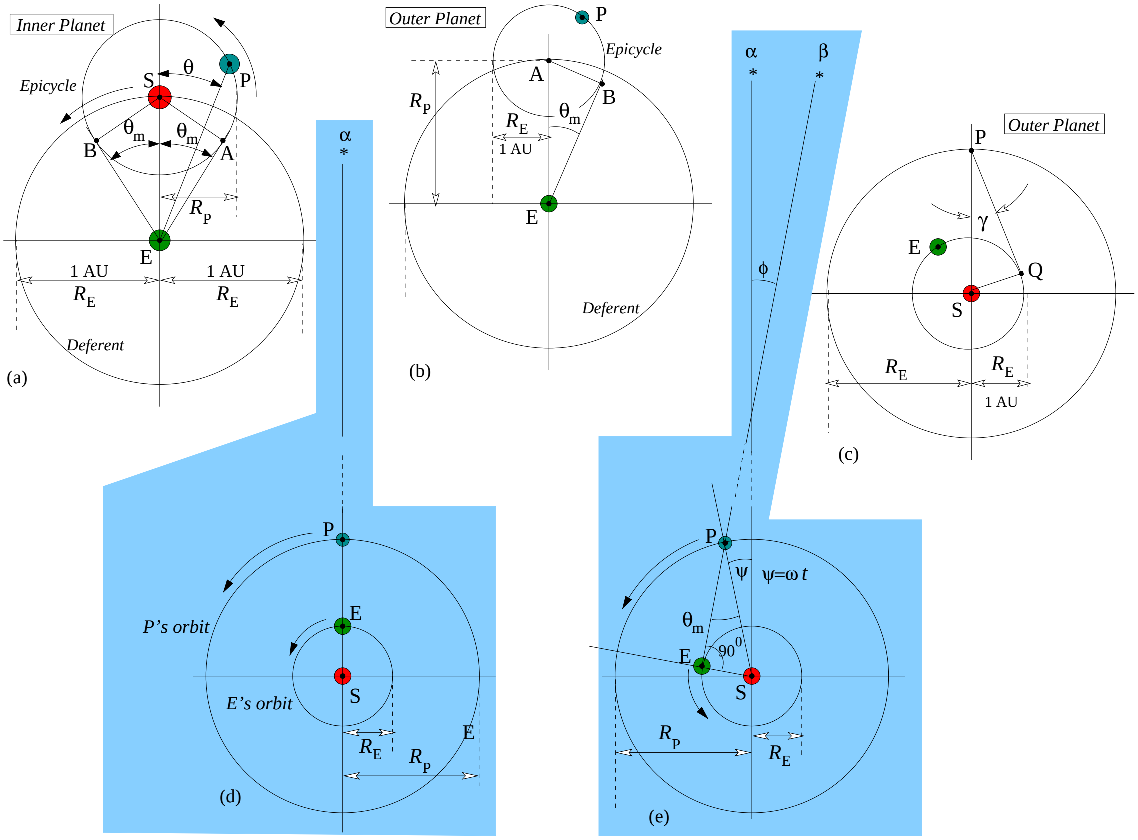


Figure 7: *Determination of the orbit radii of planets. (a) Inner planet; (b)-(e) Outer planet.*

maximum deviation.

Imagine that the planet is located at A. From the geometry of Fig. 7(a) the radius of the planet's orbit is

$$\begin{aligned} R_P &= \overline{\mathrm{SA}} = \overline{\mathrm{ES}} \sin\theta_m = R_E \sin\theta_m \\ &= \sin\theta_m, \ \text{(in AU)}, \end{aligned} \tag{13}$$

since $R_E = 1$ AU.

Let us turn to Venus as a special case. Venus is called the the *morning star* if seen in the morning and *evening star* [1] [14] , if seen in the evening (say, about 50 days later). Let us watch Venus in the evening. It can be seen as a bright object in the sky. Venus will set some time after Sunset. The time gap between the Sunset and Venus-set changes with time, but at a certain time of the year it reaches a maximum value, say τ_m (in hours). This time gap can be translated into θ_m It is obvious that $\theta_m = \frac{T_m}{24} \times 360^0$.

Alternatively, we can measure θ_m by mea-

---

[1] "The Greeks thought of the two as separate stars, Phosphorus and Hesperus untill the time of Pythagorus in the sixth century BC". See Wikipedia.

suring the maximum time lag τ_m between Venus-rise followed by Sunrise and convert it into θ_m applying the same formula.

The angle θ_m is *half of the angle that the angle $\angle AEB$ that the whole epicycle subtends at the centre of the deferent.* This should be clear from from Fig. 7(a). We shall apply this principle for the outer planets.

Let us now come to one of the *outer* planets (Mars, Jupiter, Saturn.) We have shown the planet in Fig. 7(b). In this case the deferent is the planet's orbit, of radius R_P, and the epicycle is the Earth's orbit of radius R_E. (See *conclusion* on page 12.) As in the case of the inner planets, we take θ_m as *half of the angle that the whole epicycle subtends at the centre of the deferent.* How to find the angle θ_m from observation?

The angle θ_m is half of the angle that the Earth's orbit subtends at the centre of P. To show this we have drawn a heliocentric (Copernican) view of S,E and P in Fig. 7(c). The outer circle is the orbit of P, the inner circle is the orbit of E, the Sun is at the centre S. The straight line PQ is a tangent to the Earth's orbit, so that the angle γ is half of the angle that the Earth's orbit subtends at the centre of P. Now compare the triangles $\triangle$ABE and $\triangle$SQP appearing in Figures (b) and (c) respectively. They are congruent, because they are both right angled triangles, AE = SP = R_P, and AB = SQ = R_E. Hence, $\angle$AEB = $\angle$SPQ = γ. Q.E.D.

One can now think of the following procedure. Let us find the location of P against a marker α on the celestial sphere (it can be a star, a nebulae, or some other astronomical object many light years away so that it can be taken to be permanently fixed on the celestial sphere) when S and P are *in opposition* (see page 14), as shown in Fig. 7(d).

We have drawn Figs. (d) and (e) on a light shaded background to demarcate their upper parts from the domains of Figs. (a)-(c).

A look at Table 1 shows that the time period of a planet's revolution around the Sun increases (and its angular velocity decreases) with increasing radius of its orbit. This means that the angular velocity ω_P of P is less than the angular velocity ω_E of E (the latter is 30^0 per month.) After some time t, E and P go to new locations as shown in Fig.(e), such that the angle $\angle$PES is a *right angle*, so that PE is tangent to the Earth's orbit and the angle $\angle$EPS is same as θ_m. (This will happen, for instance, on the day the planet crosses the meridian at sunset.) At this time, when viewed from Earth, P is seen against another marker β on the celestial sphere.

From the time periods of the planets listed in Table 1 we can find out ω_P. We have also measured t. Therefore we can find the angle $\psi = \omega_P t$ by which P has moved from the straight line $\widehat{\mathrm{S}\alpha}$, as shown in Fig.(e). Let ϕ be the angle between the straight lines $\widehat{\mathrm{S}\alpha}$ and $\widehat{\mathrm{S}\beta}$, as measured from Earth. Then $\theta_m = \phi + \psi = \phi + \omega_P t$. We now return to Fig.(b). The radius of the orbit we want to measure is given as

$$\begin{aligned} R_P &= \overline{\mathrm{EA}} = \overline{\mathrm{AB}} \csc\theta_m = R_E \csc\theta_m \\ &= \csc\theta_m, \text{ (in AU)}. \end{aligned} \tag{14}$$

We have tabulated in Table 2, the values of θ_m that were probably known to Copernicus, and the values of orbital radii he had obtained.

Table 2: Copernican estimate of orbital radii of planets* (in A.U.), and comparison with modern values.

	0	1	2	3	4	5	6
1	Category →	inner	inner		outer	outer	outer
2	Planet →	**Mercury**	**Venus**	**Earth**	**Mars**	**Jupiter**	**Saturn**
3	θ_m (deg) →	22.5	46		41	11	6
4	Formula: $R_P =$	$\sin\theta_m \downarrow$ 0.382	$\sin\theta_m \downarrow$ 0.719		$\csc\theta_m \downarrow$ 1.524	$\csc\theta_m \downarrow$ 5.24	$\csc\theta_m \downarrow$ 9.57
5	**Copernican →**	**0.376**	**0.719**	**1.000**	**1.520**	**5.219**	**9.174**
6	Modern →	0.3871	0.7233	1.0000	1.5237	5.2028	9.5389

*Data in rows 3, 5 and 6 are taken from A.P.French, *op. cit.*. The values of θ_m listed in Row 3 are attributed to the ancient Greek Philosopher Ptolemy.

3 Kepler's Struggle with Mars

One important landmark in the path to the discovery of the law of universal gravitation was the arrival of a Danish astronomer named Tycho Brahe (1546-1601) who had made his observatory near Copenhagen with the patronage of the king of Denmark, but later moved to Prague to continue his study of the planets. Paradoxically the model of the universe as propagated by Tycho (the Tychonic model) was similar to the Platonic model with suitable modifications.

Johannes Kepler, a German astronomer (1571-1630) with extraordinary mathematical skills, was invited by Tycho to work with him in Prague. However, Kepler could not fall in line with the Tychonic model, but had faith in the Copernican system. After Tycho's death Kepler dedicated much of his life in analyzing the tables of planetary positions Tycho had left behind, after obtaining them with difficulty from the unwilling hands of his heirs.

Kepler's life is a saga of the indomitable human spirit, of the difficult battle a single individual fights against all odds and adversities, to follow the star of his conviction with the power of a superlative mathematical mind, and comes out unvanquished. He lived at a time when religious dogma reigned over reason, and any independent thinking differing from the official tenets of the church was met with religious persecution, humiliation and death. Elderly women living alone were charged with witchcraft and burnt alive at the stake. It is said that Kepler's discoveries prompted a war, in which thousands of innocent people died, including Kepler's wife and son.

Kepler had a dream, described in his science fiction *Somnium*, in which he told the story of space travelers going to the Moon, and watching Earth-rise from the lunar surface. Such an imagination was considered to be outrageous and heretic (as it came into conflict with the Earth-centred universe). It is said that his mother had to pay a heavy price for this heresy. She was carried away in a laundry basket in the middle of night.

Kepler's earlier work, when he was teaching secondary school mathematics, in Graz, Austria, was a discovery, which he called *Cosmic Mystery*. When teaching a class of bored students, his mind drifted to a different world, trying to find an order among the orbits of the six planets known to the world at that time, the radii of which had been found out by Copernicus. He was visited by a revelation that the orbits of the planets could be fitted into the geometrical solids of Pythagorus, in which he also found the answer why there existed only six planets. His geometrical construction is an outstanding artwork of sheer delight, and should be seen and studied by students of mathematics, physics and art, by all those who love to discover beauty in geometrical shapes.

We quote Feynman. "He quickly devised a model in which the six invisible spheres that regulated the orbits of the six planets then known were fitted on either side of each of the five perfect solids of antiquity (solids having all sides the same: the tetrahedron, cube, octahedron, dodecahedron, and isocahedron), nested one inside the other. Sure enough, by arranging the solids in the right order, the diameters of the spheres came out to be in almost the same ratios as those of the orbits of the planets.

"Kepler's model explained why there were six, and only six planets - because there were five and only five perfect solids"

Kepler's discovery was published in his book *Mysterium cosmographicum* in 1596. However, this discovery lost part of its relevance after the discovery of extra planets Uranus, Neptune and Pluto.

Kepler left Graz, anticipating the horror of religious persecutions that would follow, and came to work under Tycho in Prague as we have already mentioned. Here he concentrated his study on the position of Mars based on thirty five years of data collected by Tycho. "What real motion of the Earth and Mars about the Sun could explain the apparent motion of Mars in the sky, including its retrograde loop through the background constellations, as recorded by Tycho?". This was the raging question in his mind and he left no stone unturned to seek its answer. Tycho had earlier wanted Kepler to study the motion of Mars, because compared to other planets Mars appeared to move in the most anomalous manner, difficult to explain with the help of circle over circle.

Tycho's data consisted of measurements of the angle between Mars and the "fixed stars" at different times. These measurements were valid with respect to an observatory fixed on the earth. Like his predecessors Kepler had faith in circular paths for the planets, because a circle was considered to possess the perfect geometrical shape, and such perfections only deserved to be the attributes of heavenly bodies. Therefore Kepler, transformed the angles from Tycho's table of data into another set of angles that would be valid with respect to a frame of reference fixed at the centre of the sun. Assuming that the earth was spinning about its axis, and was also revolving around the sun in a circle, he applied his superb mathematical skill to perform the required transformation of the angles [2] and made "seventy attempts" to fit them into the assumed circles. However, somewhere there was an error of "eight minutes" of arc (there are sixty minutes of arc in one degree), which could not be patched up with the imagined circular motion.

When all such attempts failed Kepler could see the light at the end of the tunnel, with the realization that an ellipse rather than a circle will fit Tycho's data beautifully. That was the greatest revolution in the history of astronomy, and this discovery is known as Kepler's First Law of Planetary motion.

Kepler also realized that the prevailing notion that a planet moved with uniform speed along the orbit will be inconsistent with the available data. The data indicated that the planet moved faster when it is near the Sun, and slower as its distance from the Sun increased. In this process of varying speed, one thing remained constant. It is the areal velocity, by which we mean the rate at which the planet sweeps out area around the Sun. This realization had occurred to Kepler earlier but came to be known as Kepler's Second Law of Planetary motion.

We shall explain these two laws with the help of Fig.8.

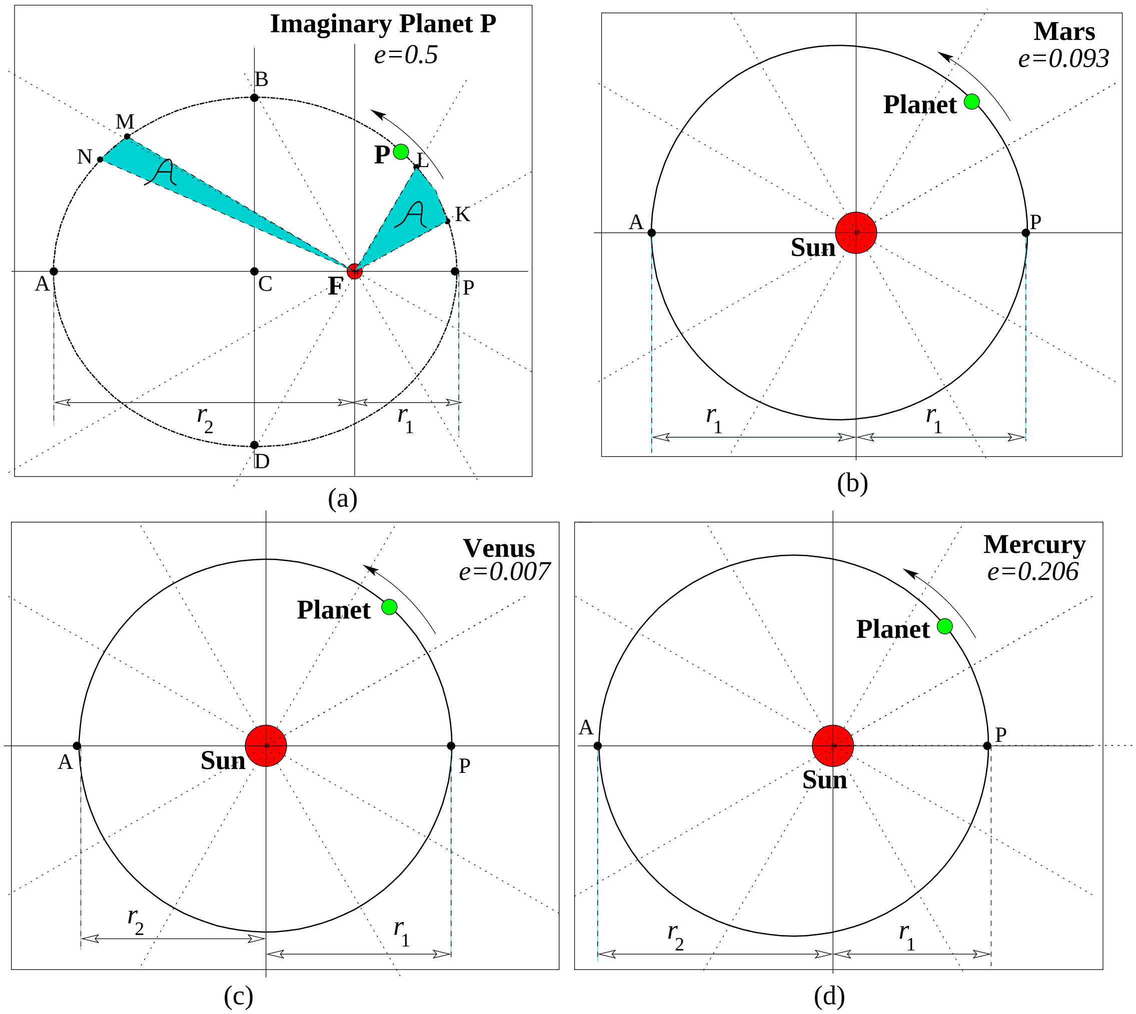
In Fig. (a) we have shown an imaginary object P moving around the Sun. We call such an object a Planet. The path that the planet follows is an ellipse. The ellipse has a characteristic point F called its Focus, where the Sun will be always residing. In other words, the planet always moves in an ellipse in such a way that the Sun is always at the focus F. This is the *first law.*

Suppose the planet P moves from K to L in a given time interval τ. After some time P goes to another point M, and moves from M to N in the *same* time interval τ. The sector KL makes a certain area $\mathcal{A}$ at the focus F. Then the sector MN will make the *same* area $\mathcal{A}$ at the focus F. In other words the planet will sweep out equal areas at the focus F at equal intervals of time. This is the *second law.*

Let us introduce the term "eccentricity" e

---

[2] This is the reverse of we have done in the previous subsection, in which we drew the g-path of Mars on the basis of heliocentric circular orbits. Tycho's data gave the projection of the true g-path on the celestial sphere, as in Fig. 5a. From this, we guess, Kepler constructed the true g-path using his genius, and then "transformed" that into the heliocentric paths of the Earth and Mars.



(a) (b) (c) (d)

Figure 8: *Orbits of some planets around the Sun*

at this point without going into its definition. For an ellipse e is a number that gives a measure of how far the rounded figure is elongated in one direction (flattend in the other) compared to a circle. For an ellipse e is less than one. A circle is a very special case of an ellipse with $e = 0$.

Let us take another look at the elliptical path of the hypothetical planet P in Fig. (a). As the planet moves around the Sun, which is fixed permanently at F, its distance from F is continuously changing. This distance is a minimum, and equal to r_1 when it is the point P, called the *perihelion* of the planet, and maximum, and equal to r_2 when it is at the point A, called the *aphelion* of the planet.

Its dimension in the X direction is equal to $2a$, and is referred to as its *major axis* (the distance a is called the *semi-major axis*). And its dimension in the Y direction is equal to $2b$, referred to as its *minor axis* (the distance b is called the *semi-minor axis*).

The relationship between e and the other parameters is given as

$$r_1 = (1-e)a; \quad r_2 = (1+e)a; \quad \frac{b}{a} = \sqrt{1-e^2}. \tag{15}$$

It is seen that when $e \to 0$ the ellipse returns to a circle with $a = b$, $r_1 = r_2 = a$.

The hypothetical planet in Fig (a) moves in an ellipse of eccentricity $e = 0.5$. In Fig (b), (c) and (d) we have shown *true plots* of the orbits of Mars, Venus and Mercury (using gnuplot), having $e = 0.093, 0.007, 0.206$ respectively. It is seen that the orbit of Venus looks almost like a circle[15]. There is a marked departure from circle of the orbit of Mars and further departure for the orbit of Mercury. In both cases, even if the orbit looks nearly circular, there is a marked shift in the position of the Sun from the "centre". It is this anomaly, in the case of Mars, that showed up in the form of an error of eight minuites of arc when Kepler tried to patch up Tycho's data with a circle.

4 Kepler's Third Law - Key to Inverse Square

Let us make a formal statement of the three laws of planetary motion discovered by Johannes Kepler. We have already explained with diagram what the first two laws are. We shall now write all the laws together to summarize Kepler's most important work on planetary motion.

Kepler's Laws of Planetary Motion

1st Law. All planets move in elliptical paths.

2nd Law. A straight line drawn from the Sun to a planet sweeps out equal areas in equal times.

3rd Law. Let T represent the time period of one complete revolution of a planet around the Sun, and let a represent the semi-major axis of its (elliptical) orbit. Then the ratio $\frac{T^2}{a^3}$ is the same for all planets. In other words $T^2 \propto a^3$.

Let us spend a little time understanding the 3rd law, which says

$$T^2 = c\,a^3 \tag{16}$$

where c is a constant, same for all planets. For convenience we shall assume that the planetary orbit is (approximately) a circle, and replace a by the average distance of the planet from the Sun, which we shall call the average radius and represent by R. For Earth $T = 1$ Earth-year, and $R = 1$ AU. Therefore if time is measured in Earth-year, and radius in AU, then the constant c in Eq. (16) should come out to be equal to 1. We shall now make a table for T^2 and R^3 for some planets, to test Kepler's third law. We shall take the values of T and R, as obtained by Copernicus, and as tabulated in Tables 1 and 2, although Kepler had used different values, and had obtained better agreement.

TABLE 3 : R AND T FOR THE PLANETS, AND THE VALUE OF c.

	0	1	2	3	4	5
	Planet ↓	R	R^3	T	T^2	$c = \frac{T^2}{R^3}$
		AU		E-year		
1	**Mercury**	0.376	0.53	0.24	.0576	1.087
2	**Venus**	0.719	.372	0.615	.378	1.016
3	**Earth**	1	1	1	1	1
4	**Mars**	1.520	3.51	1.882	3.54	1.008
5	**Jupiter**	5.219	142.15	11.87	140.90	0.991
6	**Saturn**	9.174	772.10	29.44	866.71	1.123

The average of the numbers given in column 5 comes out to be c=1.037. The variation of T with R can now be represented the empirical relation

$$T = \sqrt{c}\, R^{\frac{3}{2}} = 1.02\, R^{\frac{3}{2}}, \tag{17}$$

where R is given in AU and T in Earth-year. We have plotted the above relationship in Fig. 9, and shown the approximate locations of the six planets on the plot. We have represented the inner planets Mercury and Venus by the lower case letters m and v, the Earth by E, and the outer planets Mars, Jupter and Saturn by the upper case letters M, J and S respectively.

We shall review how Kepler's 3rd law led Newton to his discovery. As before, we shall approximate the planetary orbit by a circle of (average) radius R. Now we introduce a different constant K and write the third law in the form of the following relationship.

$$R^3 = KT^2, \tag{18}$$

Then (18) means that

$$\frac{R^3}{T^2} = K. \tag{19}$$

Instead of T we shall use the angular velocity ω.

$$\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega}. \tag{20}$$

Then from (19) and (20)

$$\frac{R^3\omega^2}{4\pi^2} = K \tag{21}$$

Now, the centripetal acceleration, as calculated by Newton, is given by, using the symbol a to denote this acceleration (this a should not be confused with the semi-major axis).

$$a = \omega^2 R. \tag{22}$$

Then from (21) and (22)

$$\frac{R^2 a}{4\pi^2} = K_s. \quad \text{Or, } a = \frac{4\pi^2 K_s}{R^2} = \frac{\mathcal{K}_s}{R^2}. \tag{23}$$

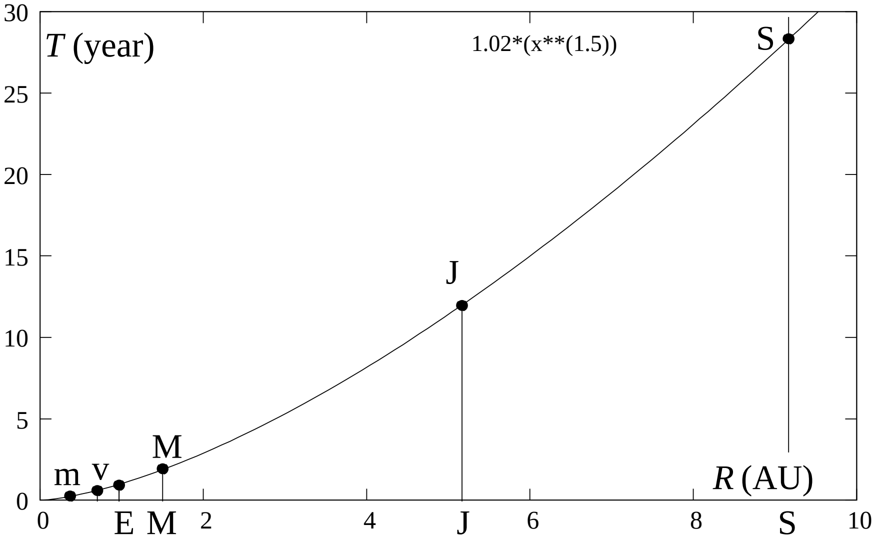


Figure 9: *Relationship between R and T for the six planets*

In the above we have "normalized" constant K_s to $\mathcal{K}_s \equiv 4\pi^2 K_s$. We have also added subscript s to mean that the proportionality constant is now associated with the force of gravitation emanating from the Sun.

Note the important inference we have derived from Kepler's third law: *The acceleration of a planet "freely falling" under the gravitational pull of the Sun, is inversely proportional to the square its distance from the Sun.*

The gravitational force F_s of the Sun on the planet whose mass is m is then

$$F_s = ma = \frac{\mathcal{K}_s m}{R^2}. \qquad (24)$$

The force of the Sun's gravity F_s acting on a planet, like its acceleration a, is inversely proportional to the square of its distance of the planet.

If the above force law is truly universal, then the same relation should also apply to objects moving under the gravitational pull of the Earth, as Newton surmised. Hence Newton conjectured

Conjecture 2 *The force of gravity F_e on a particle of mass m under the Earth's gravitation is*

$$F_e = mg = \frac{\mathcal{K}_e m}{R^2}. \qquad (25)$$

where g and $\mathcal{K}_e$ replace a and $\mathcal{K}_s$ respectively. The subscript e now implies "Earth", and g stands for the acceleration due to Earth's gravity. This acceleration is then

$$g = \frac{\mathcal{K}_e}{R^2}. \qquad (26)$$

Now,

$$\begin{aligned} R_{\text{apple}} &= \text{radius of the earth} \\ &= 6,371\,\text{km}. \\ R_{\text{moon}} &= \text{radius of the moon's orbit} \\ &= 384,000\,\text{km}. \end{aligned} \tag{27}$$

so that

$$\frac{R_{\text{moon}}}{R_{\text{apple}}} = 60.3 \tag{28}$$

Hence, by Eqs. (26) and (28), and noting that g_{apple} = acceleration due to gravity near the surface of the earth = 9.81 m/s^2, we get

$$\frac{g_{\text{moon}}}{g_{\text{apple}}} = \left(\frac{R_{\text{apple}}}{R_{\text{moon}}}\right)^2 = \left(\frac{1}{60.3}\right)^2 \approx \frac{1}{3636} \tag{29}$$

It now follows that

$$g_{\text{moon}} = \frac{9.81}{3636} \approx 2.7 \times 10^{-3}\text{m/s}^2. \tag{30}$$

Newton now found complete agreement with his estimate given in Eq. (3). It was a historic triumph of Newton's uncanny vision and crowning of the Law of Universal Gravitation.

5 The Law of Universal Gravitation

There is another element in Newton's discovery. This is about the constant $\mathcal{K}$. The gravitaional force F_e between the Earth and the Moon is proportional to the (inertial) mass m of the Moon (see Eq. 25). Similarly, the gravitational force F_s between the Sun and the Earth is proportional to the (inertial) mass m of the Earth (see Eq. 24). It made sense to Newton that the gravitational force between two objects A and B must be reciprocal. If A pulls B with a force F then B should also pull A with the same force F. (This comes under a wider principle called Newton's Third Law of Motion.) And this force must be proportional to the "material content" of A and also proportional to the material content of B. One may call this unspecified "material" quantity the *gravitational masses* of A and B. Call them $m_g(A)$ and $m_g(B)$ respectively.

If A is the Sun, and B the Earth, then Eq. (24) shows that $m_g(B)$ is to be identified with the (inertial) mass m of the Earth.

Similarly, if A is the Earth, and B the Moon (or the apple), then Eq. (25) shows that $m_g(B)$ is to be identified with the (inertial) mass m of the Moon (or the apple).

Therefore let us recognize the *gravitational mass and the inertial mass to be identical*, and when we say mass, we may mean either of them. In the following we shall represent the mass by a capital italic, e.g., M.

Newton therefore formulated his *Theory of Universal Gravitation* as follows.

Conjecture 3 *Two particles A and B, having masses M_A and M_B, when separated by a distance r, attract each other along the line AB joining them with a gravitational force F_g which is proportional to M_A and M_B, and inversely proportional to the square of the distance r between them.*

$$F_g \propto \frac{M_A\, M_B}{r^2}. \tag{31}$$

There is a constant of proportionality G, called *Gravitational constant.* We can now write the above equation more completely as

$$F_g = G\frac{M_A\, M_B}{r^2}. \tag{32}$$

In the SI system mass is measured in kg, distance in meters and force in newtons. In that case

$$G = 6.67 \times 10^{-11}\,\mathrm{N.m^2/kg^2}. \tag{33}$$

The value og G was first measured by Henry Cavendish many years after Newton.

In writing the force law, either in (31)or in (32), we have assumed the objects A and B to be *point particles*, as illustrated in Fig. 10a. In Fig. 10b we have shown two large size objects A and B, of which A can represent the sun and B a planet. We would like to find the gravitational force between these two objects on the basis of the force law given in (32). This can be achieved by treating A and B to be composed of a very large number of particles. The force between every particle in A and every particle in B is given by Eq. (32). We *add such forces between pairs of particles*, and obtain the force F_g between A and B. This force takes a very special and simple form when the distribution of matter in each of A and B is spherically symmetrical, as Newton proved himself.

Lemma: 1 *Let there be two spheres with centres at A and B containing masses M_A and M_B respectively distributed in a spherically symmetric manner, and let r be the distance between their centres. Then the force between these two spheres is given by Eq. (32). In other words, the gravitational force between two spherically symmetric distributions of matter A and B is exactly same as the force between two point particles coinciding with the centres of these spheres, and carrying the entire masses of A and B respectively, provided that there is no overlapping of these spheres.*

In order to prove the above theorem, we will need to introduce the concept of gravitational field, which we will defer for another occasion.

Let us now consider two LARGE non-symmetrical massive objects A and B (for example, A can be the Himalaya mountain, and B the Alps mountain) as shown in Fig. 10c we have shown. What is the force between them?

We can again begin with Eq. (32), add forces between pairs of particles, as in the previous example, to calculate the force between A and B. However, in this case the answer is not so simple. One may be tempted to say that the force is still given by Eq. (32) where r is the distance between the centres of mass of A and B. But that would be WRONG!! The force in this case will be very complicated. The force here will NOT be a pure inverse square force [17].

However, if the distance r between the "centres" of A and B is *very large* compared to the size of the objects, the force is almost the same as given by the inverse-square-law.

Consider, for example an asteroid at a distance of $r = 400$ million kilometers from the sun. It may be just a rocky mountain of irregular shape having a maximum dimension of, say 100 km. The diameter of the sun is about 1.4 million km. Therefore the force

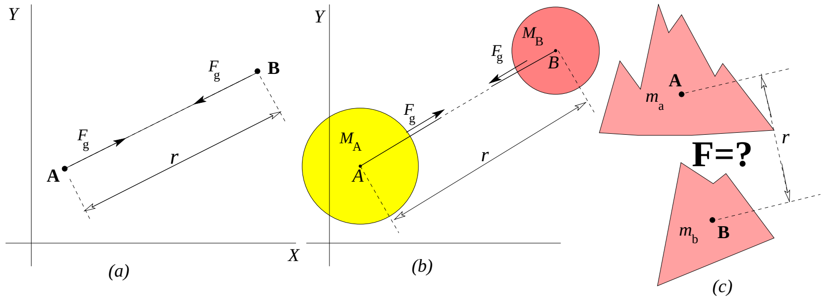


Figure 10: *Mutual forces of gravitation between (a) two point masses, (b) between two spherically symmetrical masses, (c) between two non symmetrical masses?*

with which the sun pulls the asteroid is almost exactly the same as given by the inverse square law. The same is true for the gravitational force of the earth acting on an artificial satellite, like the sky lab with its elaborate solar panels and housing colonies.

From the account we have given it may appear that Newton's role in the discovery of the inverse-square-law of gravitation may have been partly eclipsed by the pioneering work of Kepler. Newton himself acknowledged his debt to his predecessors when he said, "If I have seen further than others, it is because I was standing on the shoulders of giants".

We should pay due respect to the greatest genius of physics by recounting how he contributed to our understanding of the theory of gravitation in another way, characteristic of his greatness. Newton was the inventor of calculus (it is said that the German mathematician Gottfried Leibniz also invented calculus about the same time.) Newton had realized that it is impossible to absorb and apply the principles of mechanics without calculus. He showed us that all the laws of planetary motion, discovered by Kepler by analyzing Tycho's data, can be reconstructed from the inverse-square-law of gravitation and his second law of motion, with the use of calculus. Also it is his genius that recognized the common thread between the planetary motion and the motion of objects near and far from Earth under its own gravitational influence, making the inverse-square-law truly universal.

Acknowledgement

The author thanks Professor S.R.Madhu Rao for clarification of some astronomical concepts and terms. The original manuscript has been corrected and improved upon by the valuable observations received from Professor A.N.Maheshwari and Professor S.R.Madhu Rao and from the author's ex-students Ms Rajeshwari Kalyan and Dr. John Philip. The author is immensely grateful to all of them.

References

[1] The author is not an expert in the history of science. He has collected materials from the resources listed below and organized them and expressed them in his own style and language. A serious reader should read the historical accounts first-hand in the mentioned references to get the facts more reliably and authentically.

[2] There are several websites where the "apple anecdote" has been narrated in some details. Two of them are (1) "*Newton, the Apple, and Gravity, Gravitation*" http://mb-soft.com/public2/newton.html;
(2) "*The core of truth behind Sir Isaac Newton's apple*" http://www.independent.co.uk/news /science/the-core-of-truth-behind- ....

[3] *P.S.S.C.Physics*, 2nd ed, D.C.Heath and Co, Indian Edn, NCERT (1967), pp.368-371

[4] A.P.French, *Newtonian Mechanics*, Norton & Co., London (1971). Best resource about the early history. Gives mathematical details and facts on the measurements of planetary distances and planetary time periods. Read Ch. 2 (pp. 74-78) and Ch. 8 (pp. 245-258).

[5] George Gamow, *Gravitation*, Doubleday & Co. New York, pp.37-43. Gives an account of how Newton found out the free fall acceleration of the moon.

[6] D. Goodstein and J. Goodstein, *Feynman's Lost Lecture, The Motion of Planets around the Sun*, Vintage (1997). See Ch 1: *From Copernicus to Newton*, pp. 21-43. We have borrowed the phrase "war on Mars" from this book.

[7] Carl Sagan, *Cosmos*, Random House, New York (1980). Best narration of the history. Chapter III, pp.51-68, has the most vivid accounts of the works of Copernicus, Tycho and Kepler and a poignant narration of the trials and tribulations of Kepler.

[8] http://www.lasalle.edu/-smithsc/Astronomy/retrograd.html. This site gives a finest animation of the retrograde motion of Mars against the background constellations, and another animation of the epicycle-deferent motion of an unspecified planet. Also gives a Copernian explanation of the retrograde motion on the basis of his heliocentric model.

[9] http://csep10.phys.utk.edu/astr161 /lect/retrograde/retrograde.html. This site also gives an animation of the apparent motion of planets on the celestial sphere.

[10] http://csep10.phys.utk.edu/astr161 /lect/retrograde/aristotle.html. This site gives an animation of the hypothesised planetary motion on an Epicycle riding on a Deferent. Also gives an account of the early model of Ptolemy visualizing planetary motion in terms of

two epicycles, i.e., "Epicycle on Epicycle on Deferent". We have illustrated a similar motion in Fig. 2b. .

[11] This is similar to the familar problem in calculus. Construct any function $f(x)$ of x by making a power series of $1, x, x^2, x^3$, called Taylor series. Alternatively, construct any odd function $f(x)$ of x of periodicity L by making a Fourier series of sine functions: $\sin(n\pi x/L),\ n = 0, 1, 2, \cdots$.

[12] See Wikipedia. Also p. 582 of A.P.French, *op.cit.*

[13] This is the reverse of what we have done in the previous subsection, in which we drew the g-path of Mars on the basis of heliocentric circular orbits. Tycho's data gave the projection of the true g-path on the celestial sphere, as in Fig. 5a. From this, we guess, Kepler constructed the true g-path using his genius, and then "transformed" that into the heliocentric paths of the Earth and Mars.

[14] "The Greeks thought of the two as separate stars, Phosphorus and Hesperus untill the time of Pythagorus in the sixth century BC". See Wikipedia.

[15] It is said that Kepler could not have discovered his laws if Tycho had asked him to analyze the motion of Venus, instead of Mars

[16] *National Geographic Picture Atlas of Our Universe* (1986). See pp. 39,43

[17] The expression for the gravitational force F with which A and B are pulling each other will be in the form of an infinite series: $F = \frac{\kappa}{r^2} + \frac{\alpha}{r^3} + \frac{\beta}{r^4} + \ldots$, called *multipole expansion*. The coefficients $\alpha, \beta, \ldots$ will depend on the orientations of A and B. When r is very large compared to the dimensions of A and B, the 2nd term becomes very small compared to the first, the 3rd term very small compared to the second,..., etc. In that case $F \approx \frac{\kappa}{r^2}$, i.e., the force becomes almost the same as given by the inverse-square-law. The multipole expansion for the electrostatic field originating from a non-spherical charge distribution, which is also based on inverse square law, is discussed in D.J.Griffiths, *Introduction to Classical Electrodynamics*, 3rd Edn, Pearson Education/Prentce Hall, New Delhi (2006). See pp.164ff.

Microcontroller based low cost strain measurement in a single ended cantilever beam using a plastic optical fibre

K Samrajyam and B Sobha

Department of Physics
National Institute of technology
Warangal 506004, India.

(*Submitted* 21-06-2012)

Abstract

This paper reports on the use of a Plastic optical fiber(POF) for strain sensing in a single ended cantilever experiment. The beam bending is caused by a stepper motor controller. The results show that the POF sensor exhibits good linearity in with the applied stress. The POF strain response is twice that of strain gauge response This technique has an advantage for measuring micro strain in small deflection steps of 0.4 mm/sec very precisely. Thus POF strain sensor is more useful for the measurement of strain of a structure with a low cost as well with more accuracy.

1. Introduction

Plastic optical fiber (POF) sensors have been known to be useful for measurement of several physical variables such as displacement, strain, force , stress, load, temperature, and pressure [1,2,3,4]. Hence they have found wide ranging applications in several areas such as telecommunications, structural health monitoring, biomedical applications, chemical and environmental monitoring etc. Amongst the several advantages of POFs include small dimensions, high mechanical strength, durability in harsh chemical and environmental conditions and low cost per meter.

The small dimensions of the fibre and high mechanical strength enable them to be used as embedded sensors for structural members such as composite materials[5]. They are known to provide a large elastic strain range and are more flexible than silica optical fibres. Also these are more durable in harsh chemical and environmental conditions and have high sensitivity to environmental factors. Hence they are used often for chemical and environmental monitoring. Due to their characteristics such as lightweight, non-conductivity and greater flexibility to bending they are more preferable in different fields. In addition they are insensitive to electromagnetic radiation (especially in the vicinity of power generators in construction sites) and require less expensive components. POF sensors are available at low cost and require simple solid-state devices like light-emitting diodes (LED) and photodiodes. For low-cost sensing systems, POFs are especially advantageous due to their excellent flexibility, easy manipulation, great numerical aperture, large diameter, and the fact that plastic is able to withstand smaller bend radii than glass. The advantage of using POFs is that the properties of

POFs, that have increased their popularity and competitiveness for telecommunications, are exactly those that are important for optical sensors based on optical fibers[6,7,8].

Theoretical background

The theoretical analysis of the deflections of an isotropic beam subjected to loading is well discussed [9, 10]. Fig 1 shows a cantilever beam subjected to an out of plane end-load. If L is the distance between the load and the fixed end, W= applied load, E is the Young's modulus of the beam material , M is the bending moment and I is the second moment of area of the cross section of the beam, y is the distance of the plane from the neutral axis, x is longitudinal distance from the applied load then the beam deflection δ at load end can be expressed by equation (1)

$$\delta=((-Wx^3/6) + Ax+ B)(1/EI)) \qquad (1)$$

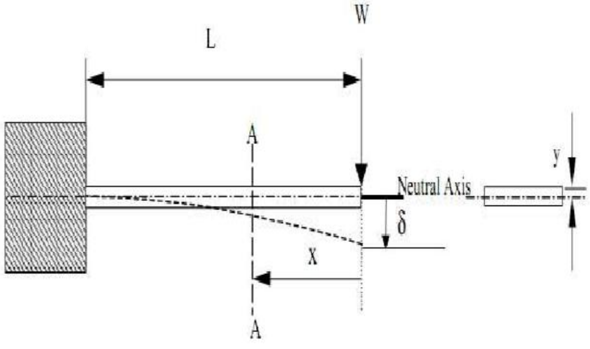


Figure 1. A cantilever beam subjected to an out-of-plane end load.

The boundary conditions for the present problem are defined by equation (2)

$$A = Wx^2/2 \text{ and } B = - (WL^3)/3 \qquad (2)$$

Substitution of the boundary conditions into equation (1) above gives deflection δ as given in equation (3)

$$\delta = ((- Wx^3/6) + (WL^2/2)x - (WL^3)/3)) (1/EI)) \qquad (3)$$

The deflection δ at the free end is given by equation (4) below by putting x=0 in equation (3)

$$\delta_{x=0} = - ((WL^3)/(3EI)) \qquad (4)$$

It can be shown that the flexural (in-plane) strain can be expressed by equation (5) in terms of the W , x, E, I as

$$\varepsilon(W,x) = -((Wx)/EI) .y \qquad (5)$$

Eliminating the load term W from equations (3) and (4) gives equation (6) which the strain at the free end

$$\varepsilon(\delta_{x=0} ,x) = ((3xy)/L^3)(\delta_{x=0}) \qquad (6)$$

Since the values of x, y and L are not varied in this study (i.e. the position of the POF sensor and distance between the fixed end and the location of the applied load was unchanged throughout the test), the flexural strain, ε, can be directly related to the deflection at the loaded end of the beam, $\delta_{x=0}$. Using equation (6) as a first approximation, it is possible to estimate the flexural strain in a beam subjected to a transverse load. In a cantilever beam configuration, it is evident that the bending moment (and by inference the curvature, 1/R) varies along the length of the beam. The response of the POF sensor therefore represents an integrated value over the length of the sensitized region (i.e. the gauge length of the sensor).

Sensors Preparation:

The surface of the POF sensor was abraded using a fresh blade causing the removal of a predetermined length of the cross section. (we refer to this as gauge sensitization length, approximate range = 2 to 3 cm.) This process of abrading increases the amount of light lost in the fibre during its bending process resulting in better accuracy. The specimens used were spring steel beams having length =300 mm, width = 30 mm and thickness = 1 mm. The plastic optical fiber was nearer to the fixed end of the cantilever beam using cyanoacrylate adhesive, and also strain gauge was pasted closer to the plastic optical fiber on the cantilever beam. This is shown in photograph at fig 4a Specified precaution were taken in mounting the strain gauges to the steel specimen.

The measurement system

The measurement system uses a standard multi-mode plastic optical fibre (POF) having a core diameter of 980 µm made with PMMA resin. The core is surrounded by a fluorinated polymer cladding of diameter 1000 µm. Light of wavelength (λ=630 nm) produced by a RED LED is used to inject a modulated light beam into the plastic optic fibre . The fibre is inserted into a highly sensitive phototransistor which receives the light from the source end. (refer figs 2a and 2b)

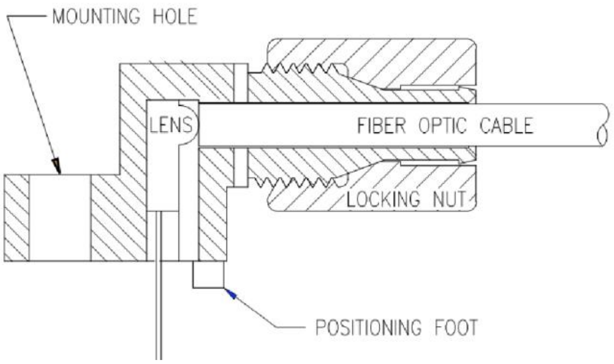


Fig 2a) The plastic optic fibre mounted in LED

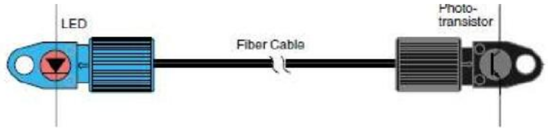


Fig 2(b) The source and receiver along with the plastic optic fibre

The output light is received by NPN phototransistor. The mounting arrangement at the source side is shown in the figure 2a. A trans-impedance amplifier was used to convert the photo transistor current in to equivalent voltage. The above voltage and the voltage from a Wheatstone bridge circuit (having two strain gauges of resistance 350 ohms each) are logged into a PC using a data acquisition card. The schematic circuit for the same is shown in Fig 3 in the next page.

Experimental Procedure:

To measure the strain of a cantilever beam the free end of the cantilever beam is deflected using a microcontroller controlled stepper motor (Model no. ESA 51E,The Electro systems associates Pvt Ltd, Bangalore, India) through a pulley arrangement. The deflection rate was set at 0.04 mm/sec The total deflection caused was 80 mm.)

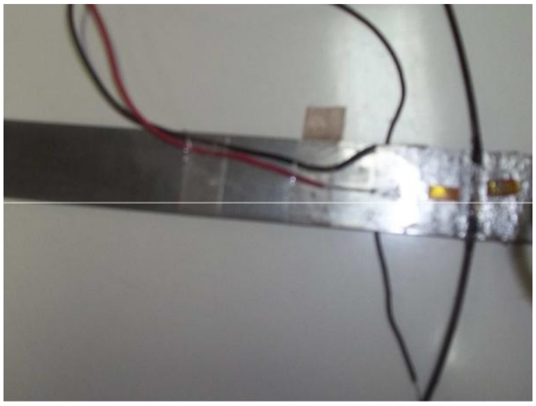
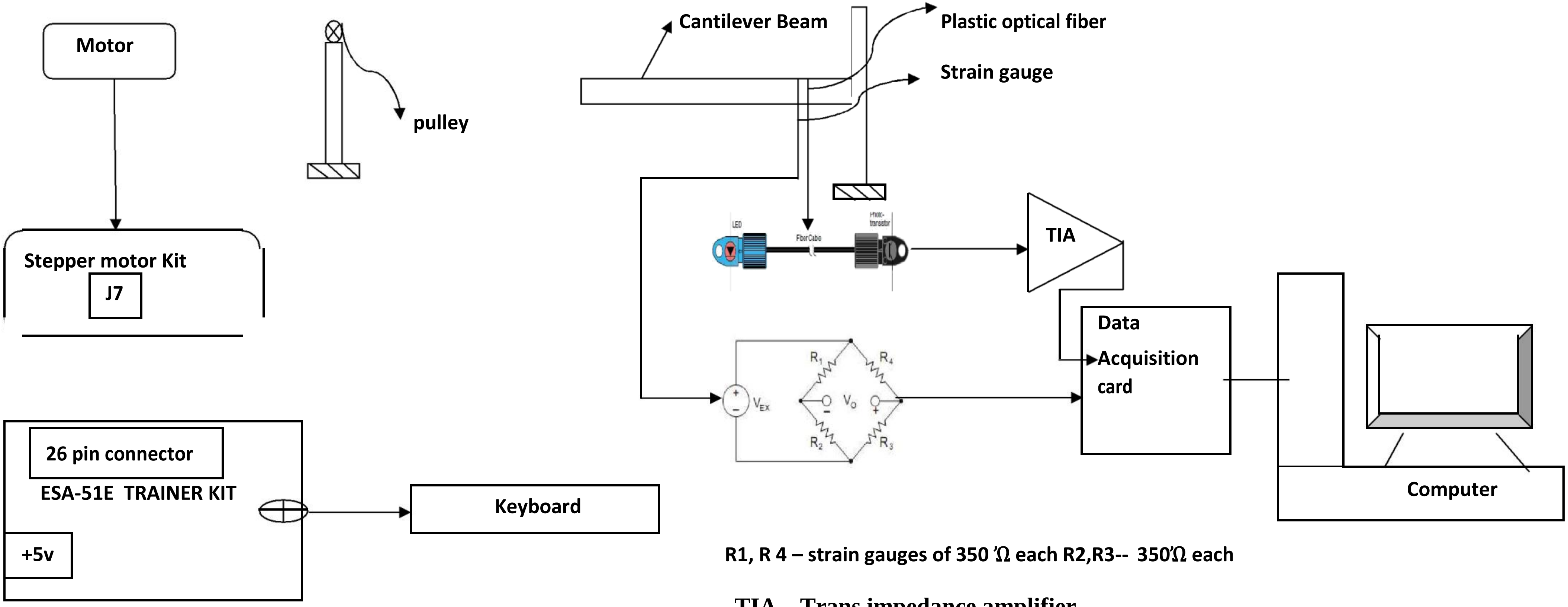


Fig 4a) Sensors (the plastic optical fibre and strain gauges) can be seen mounted on the string steel specimen.



R1, R 4 – strain gauges of 350 'Ω each R2,R3-- 350'Ω each

TIA-- Trans impedance amplifier

Fig 3 : Schematic circuit diagram for data acquisition

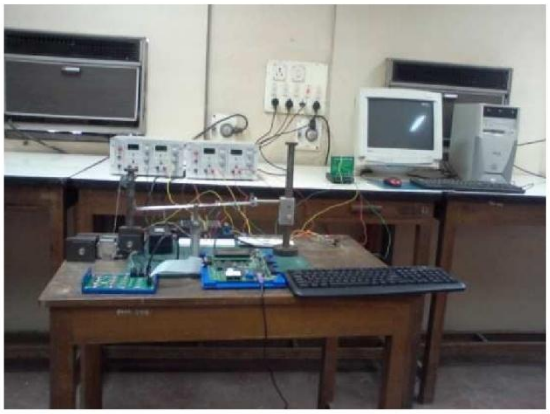


Fig 4 b) Photograph showing the experimental set up.

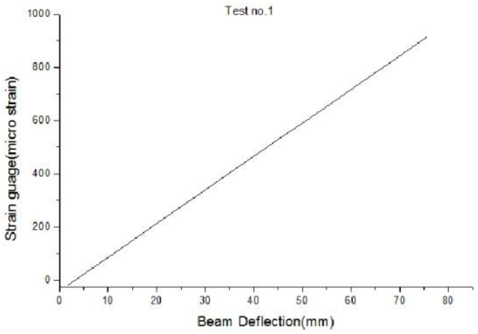


Fig 5 Graph shows between Beam deflection (mm) v/s strain measured by strain guage (microstrain (με))

Results and Discussion

Fig 5 shows a plot drawn between beam deflection(mm) v/s strain as recorded by strain gauge (hereafter (με) indicated in the rest of the text will refer to microstrain) calculated from the theoretical formula no (6) outlined in the above sections

---

$$\varepsilon(\delta_{x=0}, x) = ((3xy)/L^3)(\delta_{x=0})$$

Figs 6a) and 7a) shows the combined experimental plot of the data plotted between time in sec on x-axis vs POF strain(με) and strain guage on y axis.

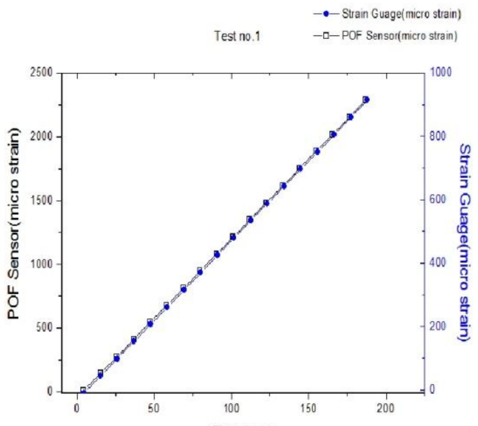


Fig 6 a)Plot between Time (s)vs Pof strain(με) vs strain guage strain (με)

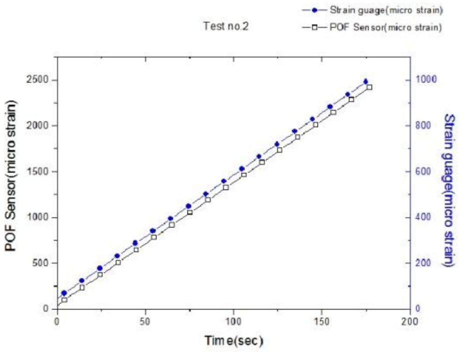


Fig 7a) Time vs POF strain(με) /strain gauge strain(με)

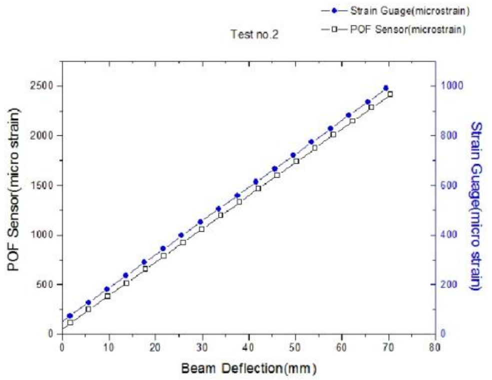


Fig 6b) Plot between beam deflection(mm) vs POF strain(με)/ strain gauge strain(με).

In Fig 6b) and 7b) a second plot is also quantities are calculated using equation 7 given below:

POF strain= calibration factor* (Raw POF signal-Initial POF Reading) (7)

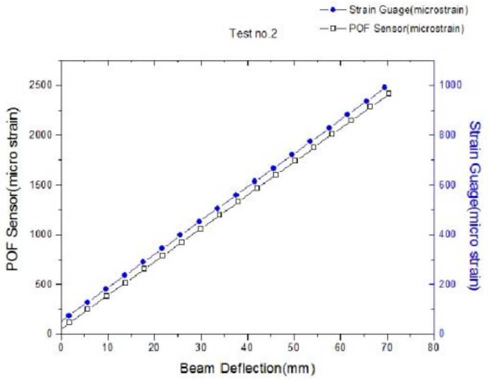


Fig 7b) Plot between beam deflection (mm) vs POF strain(με) / strain guage strain (με)

This helps us in estimation of the calibration factor given in equation (8)

Calibration factor =
change in strain guage reading
/change in POF intensity (8)

The strain response of the POF can be obtained from equation 7 above. It is observed that the strain response of POF is more (almost double) when compared to the strain gauge response.

Conclusions :

a) The strain measured by the POF sensor for the same beam deflection is more and almost twice that measured using strain guage circuit.

A total 15 number of tests were done for the measurement of strain using POF sensor and good agreement is observed in support of the above observation

b) The POF (plastic optical fibre) being of less cost and accompanied by other advantages we feel the method suggested here can be adopted for laboratory experiments to students of physics at graduate level

References :

[1] Golnabi, H. , " Design and Operation of Different Optical Fiber Sensors for Displacement Measurements." , 1999 ,*Rev. Sci. Instrum.*, *70*, pp2875-2879

[2] Golnabi, H. , "Simulation of Interferometric Sensors for Pressure and Temperature Measurements" . 2000 , *Rev. Sci. Instrum.*, *71*, pp 1608-1613

[3] Jiang, C.; Kuzyk, M.G.; Ding, J.L.; Johns, W.E.; Welker, D.J. Fabrication and Mechanical Behavior of Dye-Doped Polymer Optical Fiber. *J. Appl. Phys.* 2002, *92*, 4-11

[4] Sugita, T., "Optical Time-Domain Reflectometry of Bent Plastic Optical Fibers.", 2001 , *Appl. Opt.* , *40*, pp897-905.

[5] R.O.Claus, K.D Bennett ,A.M Vengsarkar, K.A.Murphy , " Embedded optical fiber sensors for material evaluation. " , 1989, *J. Nondestructive Evaluation* ,8 ,pp 135-146

Pedagogical Framework of Elementary Mechanics

Comparable to Elementary Electromagnetism

Introductory Approach without Reliance on Equation of Motion

Yukio Kobayashi
Department of Information Systems Science, Faculty of Engineering,
Soka University, 1-236 Tangi-cho, Hachioji-shi, Tokyo 192-8577, Japan
koba@t.soka.ac.jp

(*Submitted 26-09-2009*)

Abstract

A pedagogical framework of elementary mechanics is developed from temporal and spatial viewpoints. In contrast to conventional physics courses, the equation of motion is not a starting point for elucidating mechanical phenomena but an equation that summarizes three propositions on (1) the change of the linear momentum of a particle caused by impulse, (2) the change in the kinetic energy of a particle caused by the work done on the particle by applied force, and (3) the change of the angular momentum of a particle caused by torque. This construction is comparable to the formulation in the elementary course of electromagnetism in which Maxwell's equations are not a starting point but, instead, summarize the four laws of electromagnetic fields. The formulation that derives the above three propositions from the equation of motion cannot necessarily help a student understand the mechanisms of mechanical phenomena. For students, the acquisition of temporal and spatial viewpoints in regard to mechanical phenomena and the understanding of the essence of the intensity of motion are important in stimulating physical thought.

1. Introduction

The law of motion is a principle of causality in that the change of motion is induced by an applied force. The state of motion of a particle at any time t is specified by its position r and velocity *v*. According to the law of motion, the acceleration of a particle is determined under the influence of an applied force. If the acceleration is known, the future state of a particle can be predicted on the basis of kinematics. The causality of motion is summarized in a mathematical expression called the equation of motion. In undergraduate elementary physics courses, students learn some applications of the equation of motion. Certainly, they can write the equation of motion for the respective phenomena and solve it formally. However, from a pedagogical viewpoint, there are some problems in regard to the equation of motion.

First, some students regard the equation of motion as only a formula stating that mass times acceleration equals force. Thus, they do not necessarily understand the causality of motion. Some students cannot distinguish between an applied force and *m*a (the product of mass and acceleration). Other students regard *m*a as a physical quantity similar to linear momentum *m*v (the product of mass and velocity) although it is not. In addition, some students think that the aim of mechanics is solving the equations of motion as differential equations instead of elucidating natural phenomena. The processes of obtaining the velocity and position of a particle are only exercises in kinematics.

Second, acceleration is the second derivative of position with respect to time and, thus, is more difficult to realize intuitively than is velocity. In undergraduate advanced courses in science and engineering, generalized momentum, rather than acceleration, is related with generalized force.[1] Therefore, the framework of elementary mechanics can be constructed without reliance on acceleration.

Third, the conventional approach to elementary mechanics is in contrast with the elementary course of electromagnetism. Coulomb's law, Ampere's law, and Faraday's law are introduced through experimental results, and the laws of electromagnetic fields are then summarized in Maxwell's equations.[2] In contrast, in elementary mechanics, the equation of motion is a starting point for elucidating mechanical phenomena. The equation of motion called Newton's equation is comparable to Maxwell's equations. An approach to elucidate electromagnetic phenomena on the basis of Maxwell's equations is difficult for some students in undergraduate elementary courses on electromagnetism. For example, there are also several approaches in the theory of probability: probability based on axioms and probability as a limit of the relative frequency. The probability axioms are difficult for students to understand. In a graduate electromagnetic wave engineering course, however, electromagnetic phenomena are elucidated axiomatically on the basis of Maxwell's equations. In addition, in an undergraduate elementary course on mechanics, the formulation that derives experimental results from the equation of motion cannot necessarily help a student understand the mechanisms of mechanical phenomena. Indeed, Newton did not express the law of motion as the equation of motion. The so-called Newton's equation was enunciated by Euler[3]. Although the use of Newton's name is not entirely accurate here, it is used for contrast with Maxwell's in this paper.

Motivated by the considerations reported above, the present article will attempt to change the order in which the basic notions of elementary mechanics are introduced. The starting point is not the equation of motion but the three propositions on the causality of motion that are described later. The principle of these propositions is that the mechanisms of the same mechanical phenomenon are explained from both the temporal and spatial viewpoints. These viewpoints are the foundation of the exploration of physical phenomena. The motion of a particle in a uniform gravitational field is examined here. This is the most familiar mechanical phenomenon in daily life. Motion in a uniform gravitational field is a key to the essential problem of "how quantities for describing the intensity of motion are defined." Surveying the history of physics, Galileo and Descartes conducted an elaborate examination of the motion of a particle in a uniform gravitational field.[4] Some textbooks, however, treat this phenomenon as only an example. In the next section, the motion of a particle in a uniform gravitational field is discussed quantitatively from a kinetic viewpoint.

Defining the intensity of motion of a particle is an essential problem. If the intensity of motion is not defined quantitatively, it cannot be used for representing the law of motion. In this article, the physical meaning of the intensity of motion is clarified by examining the features of a particle thrown upward and those of a particle thrown downward. Based on these features from a temporal viewpoint, the intensity of motion of a particle can be described by the linear momentum, and, from a spatial viewpoint, it can be described by the kinetic energy. In addition, an angular momentum is convenient for describing the intensity of rotation of a particle from a temporal viewpoint. The intensity of the motion of a particle changes as time passes or as the particle moves. Linear momentum, kinetic energy, and angular momentum are all at the same level of physical quantities for representing the intensity of motion. In the conventional framework of elementary mechanics, these quantities are formally introduced by integrating the equation of motion with respect to time or displacement. In contrast to this approach, after introducing the three quantities on the basis of both kinematics and the concept of mass, the three propositions are constructed on (1) the change of the linear momentum of a particle caused by impulse, (2) the change of the kinetic energy of a particle caused by the work done on the displacement of the particle by applied force, and (3) the change of the angular momentum of a particle caused by torque. The common factor of the three propositions is the equation of motion; in other words, this equation summarizes these propositions in the same manner as Maxwell's equations summarize the laws of electromagnetic fields. The temporal viewpoint can be translated into the spatial viewpoint; that is, propositions (1) and (2) are two different representations of the same phenomenon. The temporal viewpoint of rotational motion is described in proposition (3), while the spatial viewpoint is described in proposition (2) in the same manner as translational motion.

For students, the acquisition of temporal and spatial viewpoints in regard to mechanical phenomena and the understanding of the essence of the intensity of motion are important for stimulating physical thought. The most important achievement is not to solve typical problems formally through the equation of motion but to understand the causal relationship that the intensity of motion is changed by the temporal or spatial action of force. Temporal and spatial points of view are also essential for learning about electromagnetic phenomena and wave phenomena. In linguistics, there are two viewpoints: the change of language throughout time and dialect with a regional difference.

2. Causality of motion

The propositions on the causality of motion of a particle can be constructed without reliance on the equation of motion by recognizing the two physical meanings of mass through the kinetic observations of a particle in a uniform gravitational field. Here, the shape and volume of a particle are disregarded, and, thus, a particle is treated as a mass point. After introducing impulse, work, and torque for describing the effects of force on a particle and linear momentum, kinetic energy, and angular momentum for describing the intensity of motion of a particle, the causality of motion is considered.

A. The temporal and spatial effects of force on a particle

In a uniform gravitational field, a particle thrown downward is accelerated vertically downward, while a particle thrown upward is decelerated vertically upward. The change of the velocity of the particle is caused by gravity, which attracts the particle toward the center of the earth. Impulse and work for describing the temporal and spatial consecutive effects of force are defined by observing the change in the velocity of a particle caused by gravity. The longer the time spent in falling or rising, the greater the change in velocity. Thus, impulse is defined as (Force acting on a particle) X (Time of force acting) for force other than gravity as well. Essentially, the concept of impulse is applicable to any mechanical phenomenon, although collision problems often deal with impulse. Looking at the same phenomenon differently, we have the greater the displacement of the particle, greater is the change of velocity. Thus, work done on a particle by applied force is defined as (Force acting on a particle) X (Component of the displacement of the particle in the direction of force) for forces other than gravity as well.

Torque is defined to describe the rotational motion of a particle as follows. Greater the force, greater is the work done on a particle by a force during rotation, even when the rotational angle or the distance between the particle and the center of rotation is the same. The greater the distance between the particle and the center of rotation, the greater the work, even if the applied force or the rotational angle is the same. The greater the rotational angle, the greater the work, even if the applied force or the distance between the particle and the center of rotation is the same. Thus, work done on a particle by applied force during rotation is defined as (Component of force in the direction of rotation) X (Displacement of a particle during rotation). The displacement of a particle during rotation is represented as (Distance between a particle and the center of rotation) X (Rotational angle), and, thus, work done on a particle by applied force during rotation is (Distance between a particle and the center of rotation) X (Component of force in the direction of rotation) X (Rotational angle). Thus, torque is defined as (Distance between a particle and the center of rotation) X (Component of force in the direction of rotation), which changes the velocity of a particle by rotating it. In the advanced courses of mechanics, the component of force in the direction of rotation and the rotational angle are regarded as generalized force and a generalized coordinate[1] respectively, which are important quantities in the advanced courses of electromagnetism as well.

These three quantities describing the effects of force, impulse, work done on a particle, and torque, are related to linear momentum, kinetic energy, and angular momentum, respectively, as discussed in the following.

B. Quantitative description of the intensity of the motion of a particle

To describe the intensity of the motion quantitatively, its physical meaning is essential. Let us again observe the motion of a particle in a uniform gravitational field. This phenomenon is familiar to students and, thus, suitable for discussion. From a temporal viewpoint, a particle thrown upward gradually slows down and finally stops over time. The intensity of the motion is greater if the time spent before a particle stops is longer. If the speed is at time t and the particle stops at time t', $t' - t$ is proportional to $|v|$ from a simple calculation based on kinematics. The same is also true for a particle thrown downward. It is convenient to consider velocity a vector quantity instead of considering speed a scalar quantity to distinguish between rising and falling. Thus, velocity is a temporal determining factor of the intensity of motion. From a spatial viewpoint, a particle thrown upward gradually slows down and finally stops as the particle rises. The intensity of motion is greater if the displacement before the particle stops is greater. If the speed is $|v|$ at height z and the particle stops at height z', $z' - z$ is proportional to $(1/2)|v|^2$ from a simple calculation based on kinematics. Thus, (1/2) X (velocity)^2 is a spatial determining factor of the intensity of motion. The two quantities, velocity and (1/2) X (velocity)^2, in reverse proportion to acceleration, are essential factors because acceleration is common to the representations of $t' - t$ and $z' – z$. Thus, velocity and (1/2) X (velocity)^2 also dominate the intensity of motion of a particle under the influence of forces other than gravity.

It is pedagogically important for students to infer the intensity of motion from the experimental results. Let us ask students the following questions: *Exercise*. If a heavier particle and a lighter one are both thrown upward from the same position at the same time, which moves with a greater intensity of motion? Although a heavier particle is attracted toward the center of the earth by a greater force of gravity, in a vacuum, both time and displacement are the same as those of a lighter particle until the particles stop. A heavier particle rises with a greater intensity of motion sufficient to overcome the greater force of gravity than does a lighter particle. Thus, there are two physical meanings of mass: one is the property that a particle is accelerated by gravity, and the other is the property in which a particle retains its velocity. The latter is called inertia, which is a property that does not depend on the applied force and is not restricted to the state in which no applied force acts on a particle. In any case, the greater the mass of a particle, greater is the intensity of motion. In some cases, gravitational mass and inertial mass are distinguished. For beginning students, however, these technical terms should be avoided from a pedagogical point of view because daily experience makes it clearly evident that a heavier particle is less accelerated and decelerated than a lighter one.

Let us turn to the rotational motion of a particle and consider rotation with a constant angular acceleration about any axis passing through a fixed reference point. Even if the angular velocity is the same, the velocity is greater at a greater distance from the center of rotation. If the time before a particle stops is the same, the greater the area swept, the greater the intensity of rotation. The rate at which an area is swept is represented by the product of the distance between the particle and the center of rotation and the velocity of the particle. Therefore, the rate at which an area is swept is a temporal determining factor of the intensity of rotation. From a spatial point of view, a particle gradually slows down and finally stops as it rotates. The intensity of rotation is greater if the displacement before the particle stops is greater. Using similar reasoning to that for the temporal viewpoint, $(1/2) \times (\text{velocity})^2$ is a spatial determining factor of the intensity of rotation.

According to the considerations discussed above, the three physical quantities, i.e., linear momentum (mass X velocity), kinetic energy [(1/2) X mass X (velocity)^2], and angular momentum [(distance between the particle and the center of rotation) X mass X velocity], describe the intensity of motion. Originally, velocity had only been a kinetic quantity defined as displacement per unit time. Mechanics enlarged the concept by giving velocity the physical meaning of linear momentum per unit mass.

Next, let us consider quantitatively the changes of linear momentum, kinetic energy, and angular momentum to relate the force acting on the particle with the change of the intensity of motion, i.e., the cause/effect relationship between them. From the definitions of acceleration and velocity, $d\mathbf{v} = \boldsymbol{a}dt$ and $d\mathbf{r} = \boldsymbol{v}dt$ are obtained, where **r** and **a** are the position relative to the origin and the acceleration of a particle, respectively. The changes of the three physical quantities are expressed as $d(m\boldsymbol{v}) = m\boldsymbol{a}dt$, $d[(1/2)mv^2] = md\boldsymbol{v}\cdot\mathbf{v} = m\boldsymbol{a}\cdot\boldsymbol{v}dt = m\mathbf{a}\cdot d\boldsymbol{r}$, $d(\boldsymbol{r} \times m\boldsymbol{v}) = \boldsymbol{r} \times md\boldsymbol{v} + d\boldsymbol{r} \times m\boldsymbol{v} = \boldsymbol{r} \times m\boldsymbol{a}dt + \boldsymbol{v}dt \times m\boldsymbol{v} = \boldsymbol{r} \times m\boldsymbol{a}dt$. The quantities $d(m\boldsymbol{v})$, $d[(1/2)mv^2]$, and $d(\mathbf{r} \times m\boldsymbol{v})$ have a common factor, $m\boldsymbol{a}$. By regarding the amount of force necessary for the accelerated motion with acceleration **a** as $m\boldsymbol{a}$, a method of measuring force is established. Here, it is not necessary to consider the origins of the applied forces, such as a gravitational field, a hand, and a rope. A force with the amount of $m\mathbf{a}$ is acting on a particle. In other words, $m\mathbf{a} = \mathbf{F}$ indicates that a particle with mass m is accelerated by the net applied force **F** with an amount of $m\boldsymbol{a}$. The equals sign relates the effect that a particle is accelerated with the reason that applied force is acting on the particle. From temporal point of view, quantitatively, the intensity of motion of a particle is changed by the applied force during the time when the force is applied to the particle. From spatial point of view, quantitatively, the intensity of motion of a particle is changed by the applied force in the displacement where the force is applied to the particle. Remembering the definitions of impulse, work done on a particle, and torque (or the moment of impulse) expressed as $\boldsymbol{F}\,dt$, $\boldsymbol{F}\cdot d\boldsymbol{r}$, and $\mathbf{r} \times \boldsymbol{F}\,dt$, respectively, it is reasonable to measure the net applied force by $m\boldsymbol{a}$. Thus, three propositions are obtained: (1) The change of the linear momentum of a particle is equal to the impulse during the time interval that the applied force is acting on the particle: $d(mv)= F\,dt$. (2) The change of the kinetic energy of a particle is equal to the work expended by the applied force acting on the particle: $d[(1/2)mv^2]= \boldsymbol{F}\cdot d\boldsymbol{r}$. (3) The change of the angular momentum of a particle is equal to the torque during the time interval that the applied force is acting on the particle: $d(\boldsymbol{r} \times m\boldsymbol{v}) = \boldsymbol{r} \times \boldsymbol{F}\,dt$.

These three propositions are comparable to the laws of electromagnetism, such as Coulomb's law, Ampere's law, and Faraday's law, on the basis of experimental results. The physical quantities of mass and gravitational field correspond to the electric charge and electromagnetic field, respectively. The equation of motion can be regarded as a goal of elementary mechanics in the manner of Maxwell's equations in elementary electromagnetism. For students, the physical meanings of the statements of the three propositions are easier to understand than the equation of motion, which expresses that mass times acceleration is equal to applied force. Students can learn these propositions through the interesting equation poems contrived by Prentis.[6] In the present approach to elementary mechanics, these three propositions, rather than the equation of motion, are the starting points. Students can solve typical exercises on mechanical phenomena from the temporal or spatial point of view on the basis of the three propositions. The keys to these exercises through the equation of motion cannot necessarily help students learn the causality of motion and the temporal and spatial points of view on mechanical phenomena.

C. Remarks on the equation of motion

The starting points of the present approach are the three propositions on the causality of motion. The equation of motion, however, is assumed in these propositions as reported above and, thus, is not disregarded in this approach. Remarks on the equation of motion are presented below.

1. Inertial mass is defined through proposition (1) by subjecting different masses to the same force and measuring their changes of velocity in the present approach in accordance with the conventional method. Here, significant remarks are necessary for the equal sign of the equation of motion. In the equation describing causality, it is convenient to set the effect (output) on the left and the cause (input) on the right in the manner form of function $y = f(x)$. For example, $0 = \Sigma \boldsymbol{F}_i$ implies zero acceleration in the equation of motion, while $\Sigma \boldsymbol{F}_i = 0$ implies that the amount of combined forces is zero. In contrast, force is measured by the amount of $m\boldsymbol{a}$. The action of force is observed only by the acceleration of a particle because force is invisible. The form of $\boldsymbol{F} = d\boldsymbol{p}/dt$ is similar to the definition of velocity, $\boldsymbol{v} = d\boldsymbol{r}/dt$, where $\boldsymbol{p}$ is the linear momentum. The law of motion is the only key to the transformation of the qualitative representation into the quantitative one. Thus, if the inertial mass is defined through proposition (1), $\boldsymbol{F} = m\boldsymbol{a}$ can be regarded as the quantitative definition of force. The equations $m\boldsymbol{a} = \boldsymbol{F}$ and $\boldsymbol{F} = m\mathbf{a}$ seem to be only the exchange of the respective sides because the same symbol $\boldsymbol{F}$ is used. The meaning of the symbol, however, depends on the equations. The meaning of the equals sign of $m\boldsymbol{a} = \boldsymbol{F}$ is different from that of $\boldsymbol{F} = -kx\boldsymbol{i}$ describing Hooke's law, where $\boldsymbol{i}$ is a unit vector in the x-direction and k is the spring constant. The symbol $\boldsymbol{F}$ denotes the representation of applied forces, such as gravity, restoring force, and resisting force in $m\boldsymbol{a} = \boldsymbol{F}$, while it is only a designation of the product of mass and acceleration in $\boldsymbol{F} = m\boldsymbol{a}$, as shown by the symbol $\boldsymbol{p}$ in the definition of linear momentum $\boldsymbol{p} = m\boldsymbol{v}$. Therefore, $m\boldsymbol{a} = \boldsymbol{F}$ and $\boldsymbol{F} = m\boldsymbol{a}$ are different equations. The problem is whether it is the law or the definition that the same equation represents.

2. The equation of motion describes the causality of motion, that is, the relationship between the change of velocity and the applied force. The motion of a particle in a uniform gravitational field also obeys the equation of motion. However, the expression of gravity mg is sometimes interpreted as mass times gravitational acceleration. This interpretation indicates that the right-hand side is only a rewritten form of the left-hand side. Thus, $ma = mg$ is similar to "$2x = 10$ if $x = 5$". Some textbooks[5] fall into the circular reasoning that acceleration is g by solving the equation of motion after representing the applied force as mg with gravitational acceleration. The following concept is essential to the problem. The acceleration observed on earth is constant, and, thus, the force acting on a particle $\boldsymbol{F}$ is also constant. By solving the equation of motion $m\mathbf{g} = \boldsymbol{F}$, where gravitational acceleration is denoted by $\boldsymbol{g}$ and gravity $\boldsymbol{F}$ is unknown, $\boldsymbol{F}$ is represented by $m\boldsymbol{g}$. According to the meaning of the equation of motion, it is more appropriate to understand that the velocity of a particle is changed by a gravitational field $\boldsymbol{g}$, which denotes gravity acting

on a standard particle with unit mass. This point of view is also essential to learning about electromagnetic fields.

3. When the motion of a particle is observed in an accelerated reference framework, the equation of motion is more convenient than the proposition on the change of the linear momentum of a particle caused by impulse. The reason is that the problem is whether the force acting on a particle varies with the reference framework. Thus, the equation of motion $m(d\boldsymbol{v}/dt)= \boldsymbol{F}$ can be used rather than $d(m\boldsymbol{v})= \boldsymbol{F}\ dt$. In experiments in a noninertial frame, inertial force is included on the right-hand side of the equation of motion.

3. Discussion

The present approach treats equally propositions (1) and (2) shown in the previous section and is thus similar, as shown in the following, to Mach's interpretation[7]: Both natural and simple assumptions that the velocity of a particle is determined by the time it spends in falling and the distance traveled during falling are equivalent. The representations of both of these laws are given by empirical science. Therefore, $f\ t = m\mathrm{v}$ and $f\ s = (1/2)mv^2$ are equivalently effective as a starting point, where $f\ t$ is an impulse and $f\ s$ is the work done on a particle with a distance of s. In the 17th Century,[4] the controversy had continued for more than half a century between the proposition that the same motion is proportional to the product of mass and velocity and the proposition that it is proportional to the product of mass and the square of velocity. To describe the intensity of the motion of a particle, Descartes considered the product of mass and velocity, while Leibniz considered the product of mass and the square of velocity. Today, it is believed that both propositions are the temporal and spatial points of view of the same motion. Let us consider the relationship between propositions (1) and (2). $d\ [(1/2)mv^2]= \boldsymbol{F}\cdot d\boldsymbol{r}$ can be derived by the inner product of $d(m\boldsymbol{v})= \boldsymbol{F}\ dt$ and $\boldsymbol{v}d\mathrm{t} = d\boldsymbol{r}$. In the present approach, this derivation is regarded as the translation of the temporal point of view into the spatial point of view. Velocity $\boldsymbol{v}$ has a mathematical meaning of a mapping that time dt is input and displacement $d\boldsymbol{r}$ is output. Thus, $d\boldsymbol{r} = \boldsymbol{v}dt$ is the translation of time into displacement. The same mechanical phenomenon can be explained from each point of view. In the conventional approach, however, the proposition that the work done by the applied force is equal to the change of the kinetic energy of a particle is only a key to the solution by the reason that some exercises can easily be solved by applying this proposition rather than directly using the equation of motion.

In some textbooks[5], kinetic energy is formally introduced by eliminating time from the equations of displacement and velocity represented as the function of time. Thus, students cannot necessarily understand the physical meaning of the idea that kinetic energy describes the intensity of motion of a particle from a spatial point of view. The work done on a particle in a uniform gravitational field changes the kinetic energy of the particle. This in turn implies that a particle with kinetic energy can dissipate the intensity of motion by restoring the kinetic energy to the gravitational field. To understand this mechanism, it is advisable to emphasize that $\boldsymbol{g}$ denotes the gravitational field comparable to electromagnetic fields rather than only gravitational acceleration.

As reported in the previous section, solving exercises through the equation of motion does not necessarily help students learn the causality of motion and the temporal and spatial points of view on mechanical phenomena. Reif[8] also pointed out: "Instruction must ensure that students can adequately interpret any concept or principle before they use it to perform problem-solving tasks." In the present approach, the propositions on the causality of motion are constructed after introducing the concepts of force and the intensity of motion. This procedure is consistent with Reif's pedagogical suggestion. Reif emphasized that the ability to use knowledge depends on how well it is hierarchically organized. Mechanics deals with the motions of systems and the interactions between them and achieves its predictive power by laws of mechanics that specify the relationships between motion and interactions. Knowledge about the relationships between motion and interactions can be elaborated into the three basic laws of linear momentum, energy, and angular momentum. In addition, Reif indicated that this hierarchical structure of mechanics remains pertinent in more advanced courses, in which the laws of mechanics would also include Lagrangian and Hamiltonian formulations. The present approach is a specific formulation consistent with Reif's suggestions. The three laws playing principal roles are first constructed from daily experiences in the previous section.

There is also a new view for elementary electromagnetism. Imai[9] presented a new framework of electromagnetic theory based on the principles of the conservation of energy and momentum. There still remain some questionable points on fundamental physical quantities, although these days it is generally believed that the classical theory of electromagnetism is already well established. From a pedagogical point of view, similar circumstances are also true for elementary mechanics. The process of introducing linear momentum, kinetic energy, and angular momentum is essential. Momentum and energy are the physical quantities describing the intensity of motion from temporal and spatial points of view, respectively, although Imai has not mentioned this fact explicitly in the new framework of electromagnetism.

Finally, let us consider a related problem of thermodynamics. The first law of thermodynamics is essentially a statement of the principle of the conservation of energy for thermodynamical systems.[10] Some students may wonder why, in mechanics, the principle of the conservation of energy for dynamical systems can be derived from the equation of motion, while, in thermodynamics, the principle of the conservation of energy for thermodynamical systems must be assumed. The reason for this difference needs to be explained. In the conventional approach to elementary mechanics, there is no satisfactory explanation because the equation of motion is a starting point. In the framework of the present approach, however, proposition (2) shown in the previous section is a starting point and a hypothesis in the manner of the first law of thermodynamics. In practical terms, the equations of motion of the particles cannot be described in a thermodynamical system. It is only believed that the validity of these equations from the points of view of many experimental results can be explained by the principle of the conservation of energy. Based on this view, the equation of motion is not necessarily a practical starting point. Therefore, in a limited sense, the present approach without reliance on the equation of motion is comparable with thermodynamics.

4. Summary

In contrast to the traditional approach to elementary mechanics, the present approach can explain the physical meaning of the intensity of motion of a particle systematically and transparently. In the first place, the three physical quantities describing the intensity of motion are introduced naturally by observing the motion of a particle in a uniform gravitational field and a simple rotational motion. The propositions on these three quantities result in the equation of motion. It is assumed that the equation of motion is also valid for mechanical phenomena other than the above motions. In the advanced courses of mechanics, extension to the representation with the generalized coordinates is achieved. The construction of elementary mechanics and its expansion to analytical mechanics is summarized in a Chart.

It is advisable for beginners to understand the physical meaning of the intensity of motion of a particle from both the temporal and spatial points of view before learning keys to the typical exercises by formally applying the equation of motion[11]. In the advanced courses of mechanics, however, the equation of motion is essential, because students already understand the three propositions. The equation of motion is a differential equation that relates the change of velocity of a particle with the forces acting on the particle. By solving this differential equation formally, the positions and the velocities can be obtained at every moment. After understanding the physical meaning of the causality of motion on the basis of the three propositions, it is sufficient to calculate the positions and the velocities of particles numerically from the equation of motion in molecular dynamics and chaotic motion, for example. This approach is similar to that used in a graduate-level electromagnetic wave engineering course, in which electromagnetic phenomena are elucidated axiomatically on the basis of Maxwell's equations.

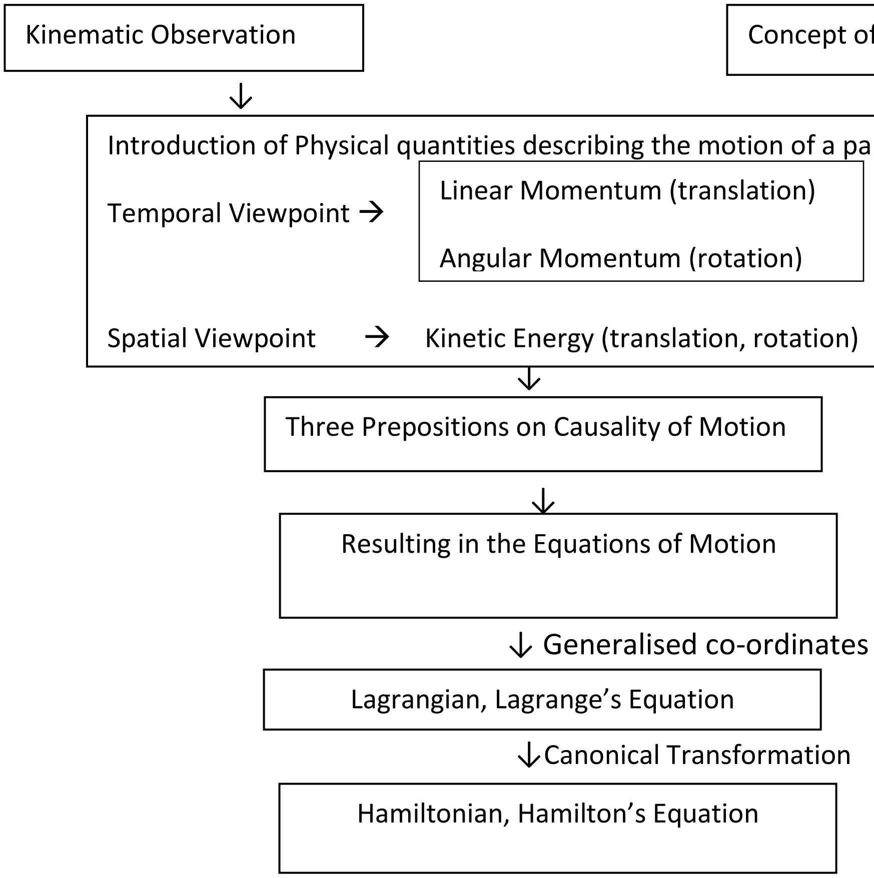


Chart: Construction of elementary mechanics and its expansion to analytical mechanics.

References

1 H. Goldstein, *Classical Mechanics*. 4th ed. Amsterdam, Addison-Wesley, (1978).

2 E. M. Purcell, *Electricity and Magnetism*. 2nd ed. Berkeley Physics Course, New York, McGraw-Hill, (1985).

3 V. V. Raman, The Physics Teacher 10, (1972), 136 - 137.

4 N. R. Hanson, *Patterns of Discovery*. Cambridge, Cambridge University Press, (1958).

5 C. Kittel, W. D. Knight, and M. A. Ruderman, *Mechanics*. 2nd ed. Berkeley Physics Course, New York, McGraw-Hill, (1973).

6 J. Prentis, *Am. J. Phys*. 64, (1996), 532 -

538.
7 E. Mach, *Die Mechanik in ihrer Entwicklung — historisch-kritisch dargestellt* 9. Verlag F. A. Brockhaus, Auflage Wiesbaden, (1933). (in German)
8 F. Reif, Am. J. Phys. 63, (1995), 17 - 32.
9 I. Imai, J. Phys. Soc. Jpn. 60, (1991), 4100 - 4118.
10 E. Fermi, *Thermodynamics*. New York, Dover, (1956).
11 Y. Kobayashi, *Mechanics Station*. Tokyo, Morikita, (2002). (in Japanese)

Physics Through Problem Solving - XXIII
Classical Lagrangian and Hamiltonian

Ahmed Sayeed
Department of Physics
University of Pune
Pune - 411007
sayeed@physics.unipune.ac.in

Submitted on 4-Jun-2012

In this article we shall discuss some problems where we find the Hamiltonian of a system starting from its from Lagrangian using Legendre transformation. We also discuss when the Hamiltonian can be taken as $H = T + V$, that is, as the sum of kinetic and potential energy functions, with no need for Legendre transformation.

In classical mechanics we learn that Lagrangian and Hamiltonian equations of motion are two equivalent formulations of Newtonian equations of motion, though much more convenient in many ways, especially in advanced applications. This means the Lagrangian function $L(\mathbf{q}, \dot{\mathbf{q}}, t)$ and the Hamiltonian function $H(\mathbf{q}, \mathbf{p}, t)$ of a system contain the same information. (Here $\mathbf{q} \equiv (q_1, q_2, \ldots q_n)$, $\dot{\mathbf{q}} \equiv (\dot{q_1}, \dot{q_2}, \dot{\ldots q_n})$, and $\mathbf{p} \equiv (p_1, p_2, \ldots p_n)$; q_i's being generalized coordinates, $\dot{q_i}$'s generalized velocities and p_i's canonical conjugate momenta. n is the degrees of freedom for the system, and t is time) Thus we should be able to obtain the Hamiltonian from the Lagrangian and the the *vice versa*. The general method for doing the is through Legendre transformations. The functions $L(\mathbf{q}, \dot{\mathbf{q}}, t)$ and $H(\mathbf{q}, \mathbf{p}, t)$ form a Legendre transformation pair - that is, one is the Legendre transform of the other. Mathematically this means:

$$H(\mathbf{q}, \mathbf{p}, t) = \sum_{i=1}^{n} \dot{q}_i p_i - L(\mathbf{q}, \dot{\mathbf{q}}, t) \tag{1}$$

where, the conjugate momenta p_i are *defined* by

$$p_i = \frac{\partial L}{\partial \dot{q}_i} \tag{2}$$

The transformation given by Eq.1 is used to obtain H when we have L. We see that the right hand side is a function of $q_i, \dot{q}_i$ and t, whereas the Hamiltonian on the left hand side is a function of q_i, p_i and t. But this is easily taken care of by eliminating $\dot{q}_i$'s using the Eq. 2, as we shall see in the examples to follow.

The reverse transformation (to obtain L from H) is given by simply rewriting eq. 1:

$$L(\mathbf{q}, \dot{\mathbf{q}}, t) = \sum_{i=1}^{n} \dot{q}_i p_i - H(\mathbf{q}, \mathbf{p}, t) \tag{3}$$

In this case we need to eliminate p_i's on the right hand side, and we can do by using one set of Hamiltonian equations of motion

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \tag{4}$$

This also we shall demonstrate in the examples.

Problem 1: Let us begin with a simple case. The Lagrangian of a simple pendulum is given by $L = \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta$, where m is the mass of the bob, l the length of the string and θ the angle made by the string with the downward vertical. Find the Hamiltonian, and from the Hamiltonian recover the Lagrangian.

Solution: Here we have only one generalized coordinate, θ, and therefore the sum in eq. 1 will consist of only one term. Also the Lagrangian has no explicit time dependence, and therefore Hamiltonian will have no explicit time dependence either. Thus

$$\begin{aligned} H(\theta, p_\theta) &= \dot{\theta}p_\theta - L \\ &= \dot{\theta}p_\theta - \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta \end{aligned} \tag{5}$$

where p_θ is the conjugate momentum for θ, given by eq. 2. That gives us

$$\begin{aligned} p_\theta &= \frac{\partial L}{\partial \dot{\theta}} \\ &= ml^2\dot{\theta} \end{aligned}$$

which can be readily inverted to get $\dot{\theta}$ in terms of p_θ:

$$\dot{\theta} = \frac{p_\theta}{ml^2}$$

which we use in eq. 5 to eliminate $\dot{\theta}$, and finally get Hamiltonian expressed as the function of θ and p_θ:

$$H(\theta, p_\theta) = \frac{1}{2}\frac{p_\theta^2}{ml^2} - mgl\cos\theta \tag{6}$$

This is a good point to discuss one important issue. You might know that usually Hamiltonian is taken as the total energy function $H = T + V$. And we have Lagrangian defined as $L = T - V$, T and V being the kinetic and potential energy functions . So to obtain H from L why not just reverse the sign (plus or minus) of V (when it can be identified), and then eliminate $\dot{\theta}$ for p_θ, instead of going to the trouble of performing Legendre transformation? The reason is that the Legendre transformation definition of H given in eq. 1 is general and always applies, where as the relation $H = T + V$ applies only when (1) the system is conservative, that is L is not an explicit function of time t, (2) The forces acting on the particles of the system can be obtained from a scalar potential V. These conditions are met in many important applications (examples are harmonic oscillator, central force driven motion), and therefore we very often use the relation $H = T + V$. You can easily check that we can follow this procedure to obtain the above Hamiltonian.

But there is one all important case where these conditions are not met - the motion of a charged particle in a magnetic field (even if the field is constant in time). We know that the magnetic force acting on a charged particle *cannot* be obtained as a spatial gradient of a scalar function (i.e. cannot be written as $-\nabla\phi$ where ϕ is some scalar function of coordinates), and it is a function of velocity of the particle. We shall not discuss this topic any further here, because it is a standard topic discussed in detail in every classical mechanics textbook. In the following problem we consider magnetic field to see the necessity of going through a Legendre transformation to obtain the correct Hamiltonian. But before that let us verify that we can get back the Lagrangian for the simple pendulum starting with the Hamiltonian in eq. 6. From eq. 3

$$\begin{aligned} L(\theta, \dot{\theta}) &= \dot{\theta} p_\theta - H(\theta, p_\theta) \\ &= \dot{\theta} p_\theta - \frac{1}{2}\frac{p_\theta^2}{ml^2} + mgl\cos\theta \end{aligned}$$

Now we need to eliminate p_θ for $\dot{\theta}$, which we do using eq. 4. That is, $\dot{\theta} = \partial H/\partial p_\theta = p_\theta/ml^2$, which gives us $p_\theta = ml^2\dot{\theta}$. Using this in the above

$$\begin{aligned} L &= \dot{\theta}\cdot ml^2\dot{\theta} - \frac{1}{2}\frac{(ml^2\dot{\theta})^2}{ml^2} + mgl\cos\theta \\ &= \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta \end{aligned}$$

as expected. Now we move on to the next problem.

Problem 2: The Lagrangian for a charged particle with charge q, mass m moving in a uniform, time-independent magnetic field B in z-direction is given by

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) + \frac{qB}{2}(x\dot{y} - y\dot{x}) \tag{7}$$

Find the Hamiltonian

Solution: Now if we try using the relation $H = T + V$, identifying as "kinetic energy" $T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$, and "potential energy" $V = -\frac{qB}{2}(x\dot{y} - y\dot{x})$ we come up with the Hamiltonian

$$H = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - \frac{qB}{2}(x\dot{y} - y\dot{x})$$

This is wrong! The right answer is

$$H = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$$

Its only the "kinetic energy"!

You might worry that there is no magnetic field in this expression, which cannot be quite right, because we do know that the motion of the charged particle is affected by the magnet is field. But we will see that the field does appear as soon as we put the Hamiltonian in the standard form, that is, by eliminating velocities for respective conjugate momenta. So us work this out. From eq. 1 we get

$$\begin{aligned} H &= \dot{x}p_x + \dot{y}p_y + \dot{z}p_z - L \\ &= \dot{x}p_x + \dot{y}p_y + \dot{z}p_z - \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - \frac{qB}{2}\left(x\dot{y} - y\dot{x}\right) \end{aligned}$$

At this point we normally eliminate the velocities $\dot{x}, \dot{y}, \dot{z}$ for respective conjugate momenta p_x, p_y, p_z. But we can save some algebra by persisting with the velocities for a while. Using $p_x = \partial L/\partial\dot{x} = m\dot{x} - qBy/2$, $p_y = \partial L/\partial\dot{y} = m\dot{x} - qBy/2$, and $p_z = \partial L/\partial\dot{z} = m\dot{z}$, the expression readily simplifies to, as promised

$$H = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) \tag{8}$$

Inverting the expressions for p_x, p_y, p_z above we have

$$\dot{x} = \frac{1}{m}\left(p_x + \frac{qB}{2}y\right) \tag{9}$$

$$\dot{y} = \frac{1}{m}\left(p_y - \frac{qB}{2}x\right) \tag{10}$$

$$\dot{z} = \frac{1}{m}p_z \tag{11}$$

using eqs. 9, 10 and 11 in eq. 8 and we have

$$H = \frac{1}{2m}\left[\left(p_x + \frac{qB}{2}y\right)^2 + \left(p_y - \frac{qB}{2}x\right)^2 + p_z^2\right]$$

Thus the magnetic field does appear in the Hamiltonian.

Our final problem is one involving Lagrangian with explicit time dependence.

Problem 3: Consider a Lagrangian whose length is l at time $t = 0$ and is gradually shortening, so that at time t the length is given by $l - y(t)$, and $y(0) = 0$. Its Lagrangian is given by

$$L(\theta, \dot{\theta}, t) = \frac{1}{2}m\left[(l - y(t))^2\dot{\theta}^2 + \dot{y}(t)^2\right] + mg\left[l - y(t)\right]\cos\theta \tag{12}$$

Find the Hamiltonian.

Solution: Note that y is *not* a coordinate of the pendulum, but some known function of time. A coordinate of the a system is determined by the equation of motion, and the initial conditions for that coordinate. Here that is not the case with y, whose time dependence does not depend on the forces acting on the mass. Also, the Lagrangian is explicitly time dependent, so this is not, in general, a conservative system (though it could be so for some specific function $y(t)$). Thus once again the Hamiltonian is not given by $H = T + V$, and we have to use the Legendre Transformation of eq. 1.

$$\begin{aligned} H(\theta, p_\theta, t) &= \dot{\theta}p_\theta - L(\theta, \dot{\theta}, t) \\ &= \dot{\theta}p_\theta - \frac{1}{2}m\left[(l - y(t))^2\dot{\theta}^2 + \dot{y}(t)^2\right] - mg\left[l - y(t)\right]\cos\theta \end{aligned} \tag{13}$$

The conjugate momentum $p_\theta = \partial L/\partial\dot{\theta} = m(l - y)^2\dot{\theta}$, which gives $\dot{\theta} = p_\theta/m(l - y)^2$. Using this eq. 13 above

$$\begin{aligned} H(\theta, p_\theta, t) &= \frac{p_\theta}{m\left(l - y(t)\right)^2}\cdot p_\theta - \frac{1}{2}m\left[(l - y(t))^2\left(\frac{p_\theta}{m\left(l - y(t)\right)^2}\right)^2 + \dot{y}(t)^2\right] - mg\left[l - y(t)\right]\cos\theta \\ &= \frac{p_\theta^2}{2m\left(l - y(t)\right)^2} - \frac{1}{2}m\dot{y}(t)^2 - mg\left[l - y(t)\right]\cos\theta \end{aligned}$$

You can easily check in this case also the relation $H = T + V$ (identifying $T = \frac{1}{2}m\left[(l - y(t))^2\dot{\theta}^2 + \dot{y}(t)^2\right]$, and $V = -mg\left[l - y(t)\right]\cos\theta$) yields the wrong answer:

$$H(\theta, p_\theta, t) = \frac{p_\theta^2}{2m\left(l - y(t)\right)^2} + \frac{1}{2}m\dot{y}(t)^2 - mg\left[l - y(t)\right]\cos\theta$$

But this reduces to correct Hamiltonian if $\dot{y} = 0$, as it should, because in that case the length of pendulum is fixed and the system is conservative.

Higgs Boson at the LHC

S. Sahoo

Department of Physics, National Institute of Technology,

Durgapur – 713209, West Bengal, India.

sukadevsahoo@yahoo.com

(*Submitted April 2012*)

Abstract

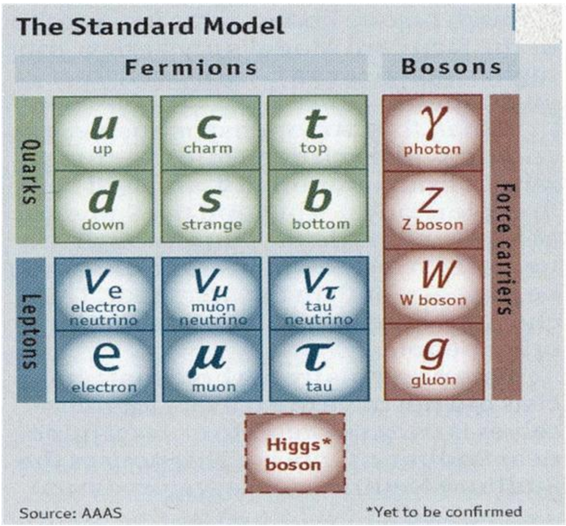
The Higgs boson, considered as the final particle of standard model (SM) of particle physics, has not been confirmed experimentally yet. It is an important particle because it is responsible for the Higgs mechanism by which all particles acquire mass. It has some unique properties which give a special status to it in the table of elementary particles of the SM. In this article, we discuss the recent results about the Higgs at the LHC briefly.

Keywords: Higgs boson, standard model, spontaneous breaking of gauge symmetries, LHC.

1. Introduction

The standard model (SM) [1, 2] of particle physics unifies three fundamental forces: electromagnetic, strong and weak force. These forces (electromagnetic, weak and strong force) are mediated by the gauge bosons: the photon (γ); the W^+, W^- and Z boson; and the gluons respectively. There are 6 types of quarks and 6 types of leptons in the SM. Quarks are called up (u), down (d), charm (c), strange (s), top (t) and bottom (b). Leptons are called electron (e^-), electron-neutrino (ν_e), muon (μ^-), muon-neutrino (ν_μ), tau (τ^-) and tau-neutrino (ν_τ). Fig.1 shows the particle content of the standard model. All the particles except the Higgs boson have been discovered. The final particle of the SM, the Higgs boson (H), has not been confirmed experimentally yet. It is theoretically predicted by Englert, Brout, Higgs, Guralinik, Hagens and Kibble [3–6]. It is very important because it is responsible for the mechanism (Higgs mechanism [3,4]) by which all particles acquire mass. The Higgs mechanism allows the generation of particle masses preserving the gauge symmetry of electroweak interactions.

The standard model suggests that just after the big bang all particles were massless. As time passed on, the universe cooled and temperature fell below a critical value, an invisible field called the 'Higgs field' filled all space [7]. The particle associated with the Higgs field is called the Higgs boson. Although the Higgs field is not directly measurable, accelerators can excite this field and can detect the Higgs boson. So far, experiments using the world's most powerful accelerators have not observed any Higgs bosons, but indirect experimental evidence suggests that it is possible in future. Since the Higgs field is a scalar field, the Higgs boson has no spin, and hence no intrinsic angular momentum. The Higgs boson is also its own antiparticle and is CP-even. One of the important properties of this field is that the Higgs field is exactly the same everywhere whereas the magnetic or gravitational fields vary from place to place. When particles are moving in a uniform Higgs field, they change their velocities i.e. they accelerate. The Higgs field exerts a certain amount of resistance or drag; this is the origin of inertial mass.



Source: AAAS

*Yet to be confirmed

Fig. 1: Particles of the Standard Model

In the SM, the Higgs field consists of two neutral and two charged component fields. Both of the charged components and one of the neutral fields are Goldstone bosons, which act as the longitudinal third-polarization components of the massive W^{+}, W^{-} and Z bosons. The quantum of remaining neutral components corresponds to the massive Higgs boson. In the SM, there is only a single Higgs particle. But supersymmetric extensions of the SM predict the existence of different Higgs particles. The minimal supersymmetric extension of the SM (MSSM) predicts the smallest number (five) of Higgs boson [8–10]: two CP-even neutral Higgs boson h and H, a CP-odd neutral Higgs boson A, and two charged Higgs particles $H^{\pm}$. The lightest neutral Higgs particle h has the same properties as the standard Higgs boson but by virtue of supersymmetry, its mass is below 140 GeV.

Supersymmetry [11–15] is a hypothetical symmetry between fermions and bosons. Unlike traditional symmetries, supersymmetry does not treat bosons and fermions as two different classes of particles. The supersymmetry operation converts bosons into fermions and vice versa. For each particle, it predicts the existence of a superpartner (hence, doubles the SM particle spectrum) which should have the same properties [10] but with a spin different by a unit ½ and also a different mass as supersymmetry must be broken in nature. But super particles are not detected experimentally so far and are expected at the LHC [16] in the coming years. Some particles and their superpartners are shown in Table 1.

Table 1: Some Particles and their Superpartners

Particle	**Spin**	**Superpartner**	**Spin**
Electron	½	Selectron	0
Muon	½	Smuon	0
Tau	½	Stau	0
Neutrino	½	Sneutrino	0
Quark	½	Squark	0
Graviton	2	Gravitino	3/2
Photon	1	Photino	½
Gluon	1	Gluino	½
$W^{\pm}$	1	$Wino^{\pm}$	½

Z^0	1	Zino	½
Higgs	0	Higgsino	½

. 2. Properties of the Higgs Boson

The Higgs boson has some unique properties [10] which give a special status to it in the table of elementary particles of the SM:

(i) Matter particles have spin ½, gauge particles have spin 1 but Higgs boson has spin zero. At present there are no known elementary scalar bosons in nature, although many composite spin-0 particles are known. Since it has integer spin, it is a boson but it does not mediate gauge interactions.

(ii) The Higgs particle interacts with or couples to elementary particles proportionally to their masses: the more massive is the particle, the stronger is its interaction with the Higgs boson [17].

(iii) It does not couple to the neutrinos, which are considered as massless particles.

(iv)The Higgs boson does not couple directly to photons and gluons (in case of gluons, a direct coupling is also absent because the Higgs boson does not carry color quantum numbers). However, couplings can be induced in an indirect way through quantum fluctuations.

(v) According to Heisenberg’s uncertainty principle, the Higgs boson can emit pairs of very heavy particles (for example, top quarks) and immediately absorb them, but these virtual particles can, in the meantime, emit photons or gluons. Higgs-photon-photon and Higgs-gluon-gluon couplings are then generated. However, they are expected to be rather small, as they imply intermediate interactions of the virtual particles to photons and gluons, which have a small intensity.

(vi) The Higgs boson has self-interactions. The magnitude of triple and quartic self-interactions is proportional to the Higgs boson mass (in fact, Higgs mass squared) [18].

3. Higgs Boson from Theoretical Calculations, at the LEP and Tevatron

The existence of Higgs boson is related to the spontaneous breaking of electroweak symmetry and to the generation of elementary particle masses. The Higgs boson is highly unstable and once produced, decays very quickly to either a fermion-antifermion pair or a pair of bosons. By energy conservation, the Higgs mass m_H , must be at least twice that of the particle in the pair to which it decays. The mass of the SM Higgs boson is given by $m_H = \sqrt{2\lambda}$ v, while the vacuum expectation value of the Higgs field v = 246 GeV is fixed by the Fermi coupling but the quartic Higgs self-coupling λ is not pre-determined; hence, the Higgs mass is not predicted. Constraints, at the theoretical as well as experimental level, restrict the value of Higgs mass quite strongly. Mahbubani [19] gives limits on the Higgs mass as 110 GeV $\le m_H \le$ 250 GeV in arbitrary extension of split supersymmetry. We [20] have predicted the mass of Higgs boson as $m_H \approx$ 120 GeV in a flavor-independent potential model. Recently [21], in compactified string/M theories it is predicted that there will be a single SM Higgs boson with a mass in the range 105 GeV ≤ m_H ≤ 129 GeV depending on $\tan\beta$ (the ratio of the Higgs vacuum expectation values in the MSSM). For $\tan\beta$ > 7, the prediction is 122 GeV ≤ m_H ≤ 129 GeV.

The four LEP collaborations, ALEPH, DELPHI, L3 and OPAL were searching for the neutral Higgs boson in the SM. At LEP, the SM Higgs boson would be produced mainly in association with the Z boson through Higgs-strahlung process $e^+e^- \rightarrow H\,Z$. The lower bound on the mass of the SM Higgs boson obtained from LEP data [22] is 114.4 GeV at 95% CL. The SM Higgs boson is also searched at the Fermilab Tevatraon $p\bar{p}$ collider. The two experiments, CDF and DO, exclude two regions: 100 GeV$< m_H <$106 GeV and 147 GeV$< m_H <$179 GeV at 95% CL [23]. There is an excess of data events with respect to the background estimation in the mass range 115 GeV$< m_H <$135 GeV.

4. Higgs Boson at the LHC

The Large Hadron Collider (LHC), the world's largest and highest energy particle accelerator, at CERN is expected to measure the mass of Higgs boson accurately. Since 2010, it is in operation, in the first phase with an energy of 7 TeV, to be extended later to 14 TeV. There are six detectors constructed at the LHC: A Toridal LHC Apparatus (ATLAS), Compact Muon Solenoid (CMS), A Large Ion Collider Experiment (ALICE), Large Hadron Collider beauty Experiment (LHCb), Large Hadron Collider forward Experiment (LHCf), Total Cross section, Elastic Scattering and Diffraction Dissociation at the LHC (TOTEM). Two of them the ATLAS and CMS, are large and general purpose particle detectors. The last two LHCf and TOTEM are very much smaller and are for very specialized research. The two large detectors (ATLAS and CMS) at the LHC are searching for the Higgs boson. They have optimized for the Higgs boson search in a mass range from the LEP limit of 114.4 GeV up to $\approx$700 GeV. At the LHC, the SM Higgs boson production is dominated by gluon-gluon fusion ($gg \rightarrow H$) followed by the vector boson fusion (VBF) and associated production with a vector boson (VH), each of which contributes less than 10% of the total production cross section.

On 13th December, 2011 ATLAS and CMS reported their data collected by each experiment about the status of Higgs boson in a special seminar at CERN [24]. The main result is that the mass of SM Higgs boson, if it exists, is constrained to the range 115–131 GeV (ATLAS) and 115–127 GeV (CMS) at 95% CL (confidence level) with possible hints of evidence within a few GeV of 125 GeV [25]. Prof. Guido Tonelli, spokesman for CMS experiment, says "We have not collected enough evidence for a discovery. There is an excess of events compatible with the hypothesis that it could be a Higgs" [26]. Prof. Fabiola Gianotti, spokesman for ATLAS experiment, says "It could well be something intriguing, but it could be a background fluctuation". More data will be needed to establish the existence of the Higgs boson with confidence.

In February, 2012 the ATLAS and CMS collaborations have got the evidence for the Higgs signal in the mass range 124 –126 GeV [27–29]. Recently [30], in May, 2012 the CMS collaboration have searched the SM Higgs boson using approximately 5 fb^{-1} of 7 TeV pp collisions data at the LHC. Combining the results of different searches they exclude a SM Higgs boson with mass between 127.5 and 600 GeV at 95% CL. They find the most significant result for the SM Higgs boson with mass of about 125 GeV with a local significance of 2.8σ. Similarly, in May, 2012 the ATLAS collaboration [31] have also searched the SM Higgs boson using approximately 4.9 fb^{-1} of 7 TeV pp collisions data at the LHC. A Higgs boson with a mass ranges from 110.0 GeV to 117.5 GeV, 118.4 GeV to 122.7 GeV, and 128.6 GeV to 529.3 GeV is excluded at 95% CL. They find the most significant result around the Higgs boson mass of 126 GeV with a local significance of 2.5σ. The LHC will continue to collect the data regarding the Higgs boson till the end of 2012. The LHCb experiment is also planning to measure the Higgs boson mass. We expect that these experiments will tell us whether the Higgs boson exists or not.

5. Conclusions

The standard model of particle physics describes the strong and electroweak interactions of fermions (spin– ½), gauge bosons (spin–1) and a final vital ingredient – the spin–0 Higgs boson. The Higgs boson is a hypothetical elementary particle which would give the mechanism by which particles acquire mass. It has not been confirmed experimentally yet. If the Higgs boson exists, it is an integral and pervasive component of the material world. Its mass is not specified in the SM. Its mass is constraints as 114 GeV $\le m_H \le$ 1.4 TeV [32,33]. We [20] have predicted the mass of Higgs boson as $m_H \approx$ 120 GeV in a flavor-independent potential model. Recently [21], in compactified string/M theories it is predicted that there will be a single SM Higgs boson with a mass in the range 105 GeV $\le m_H \le$ 129 GeV depending on $\tan\beta$ (the ratio of the Higgs vacuum expectation values in the MSSM). For $\tan\beta > 7$, the prediction is 122 GeV $\le m_H \le$ 129 GeV. From the recent results of the CMS and ATLAS detectors [30,31] the most significant result for the SM Higgs boson mass is about 125 GeV and 126 GeV respectively. A Higgs particle with mass of ≈ 125 GeV would be a triumph for the SM [34]. Recently, by combining electroweak precision data with the results of Higgs boson searches at LEP 2, the Tevatron and the LHC, Erler [35] determine the mass of Higgs boson to $m_H =$ 124.5 $\pm$ 0.8 GeV at the 68% CL. In finite unified theories (FUTs) [36], the mass of Higgs boson is predicted in the range 121–126 GeV. From the above discussion it is clear that the mass of the SM Higgs boson predicted in the string theory, at the LHC, the result obtained by Erler and in FUTs are nearly in the same range. The LHC will continue to collect the data regarding the Higgs boson till the end of 2012. The LHCb experiment is also planning to measure the Higgs boson mass. We expect that these experiments will tell us whether the Higgs boson exists or not. If the discovery of the Higgs boson will be confirmed, two new directions of physics will open up [37]: (i) the detailed investigation of the Higgs will be done to conform to the SM paradigm or to show deviations due to new physics. (ii) there will be investigations for the new physics which complements the Higgs boson, whether supersymmetry or extra dimensions or new strongly-interacting particles or…? Although the Higgs boson belongs to the SM of particle physics its study is a very challenging and fascinating topic which interplays between different branches of physics like particle physics, condensed matter physics and cosmology [38–40].

References

1. P. B. Renton, *Electroweak Interactions*, Cambridge University Press, Cambridge, (1990).
2. D. Griffiths, *Introduction to elementary particles*, John Wiley and Sons, Singapore (1987).
3. P. W. Higgs, *Phys. Lett.* **12**, 132 (1964).
4. P. W. Higgs, *Phys. Rev. Lett.* **13**, 508 (1964); *Phys. Rev*. **145**, 1156 (1966).
5. F. Englert and R. Brout, *Phys. Rev. Lett.* **13**, 321 (1964).
6. G. S. Guralinik, C. R. Hagens and T. W. B. Kibble, *Phys. Rev. Lett.* **13**, 585 (1964).
7. M. Cid and R. Cid, *Physics Education*, **45**(1), 73 (2010) (IOP Publishing).
8. A. Djouadi, *Phys. Rep.* **457,** 1 (2008) [arXiv :hep-ph/0503172].
9. A. Djouadi, *Phys. Rep.* **459,** 1 (2008) [arXiv :hep-ph/0503173].
10. A. Djouadi, in the book *Mass and Motion in General Relativity* edited by L. Blanchet, A. Spallicci and B. Whiting, Springer, pp. 1–23 (2011).
11. S. Adams, *Frontiers Twentieth-Century Physics*, Talor and Francis Ltd, London (2000).
12. G. Kane and M. Shifman, *The Supersymmetric World*, World Scientific Publishing Co. Pte., Singapore (2000).
13. S. K. Soni and Simmi Singh, *Supersymmetry*, Narosa Publishing House, India (2000).
14. S. Sahoo and M. Goswami, *IAPT Bulletin*, **24**(5), 140 (2007).
15. M. E. Peskin, arXiv: hep-ph/0801.1928 (2008).
16. G. Brumfiel, *Nature*, **471,** 13 (2011).
17. M. Duhrssen et al., *Phys. Rev. D* **70**, 113009 (2004).
18. A. Djouadi, J. Lykken, K. Monig, Y. Okada, M. Oreglia and S. Yamashita, arXiv:0709.1893 [hep-ph] (2009).
19. R. Mahbubani; hep-ph/0408096 (2004).
20. S. Sahoo, A. K. Meikap and L. Maharana, *Mod. Phys. Lett. A* **21,** 275 (2006).
21. G. Kane, P. Kumar, R. Lu and B. Zheng, arXiv:1112.1059 [hep-ph] (2011).
22. W.–D. Schlatter and P. M. Zerwas, arXiv:1112.5127 [physics.hist-ph] (2011).
23. The TEVNPH Working Group (The CDF and D∅ Collaboration), arXiv:1203.3774 [hep-ex] (2012).
24. F. Gianotti [on behalf of ATLAS] "Update on the Standard Model Higgs searches in ATLAS" and G. Tonelli [on behalf of CMS], "Update on the Standard Model Higgs searches in CMS", joint CMS/ATLAS seminar at CERN on December 13, 2011, [http://indico.cern.ch/conferenceDisplay.py?confId=164890].
25. S. Akula et al., arXiv:1112.3645 [hep-ph] (2011).
26. E. S. Reich, *Nature*, **480**, 301 (2011).
27. S. Chatrchyan et al. (CMS Collaboration), arXiv :1202.1488 [hep-ex] (2012).
28. G. Aad et al. (ATLAS), *Phys. Lett. B*, **710**, 49 (2012) [arXiv :1202.1408 [hep-ex]].
29. F. Bezrukov et al., arXiv :1205.2893 [hep-ph] (2012).
30. M. Pieri (for the CMS Collaboration), arXiv :1205.2907 [hep-ex] (2012).
31. R. Bernhard (on behalf of the ATLAS Collaboration), arXiv:1205.4629 [hep-ex] (2012).
32. A. Gulov and A. Kozhushko, arXiv: 1105.3025 [hep-ph] (2011).
33. Higgs boson from Wikipedia, May, 2012.
34. A. Djouadi, arXiv: 1203.4199 [hep-ph] (2012).
35. J. Erler, arXiv: 1201.0695 [hep-ph] (2012).

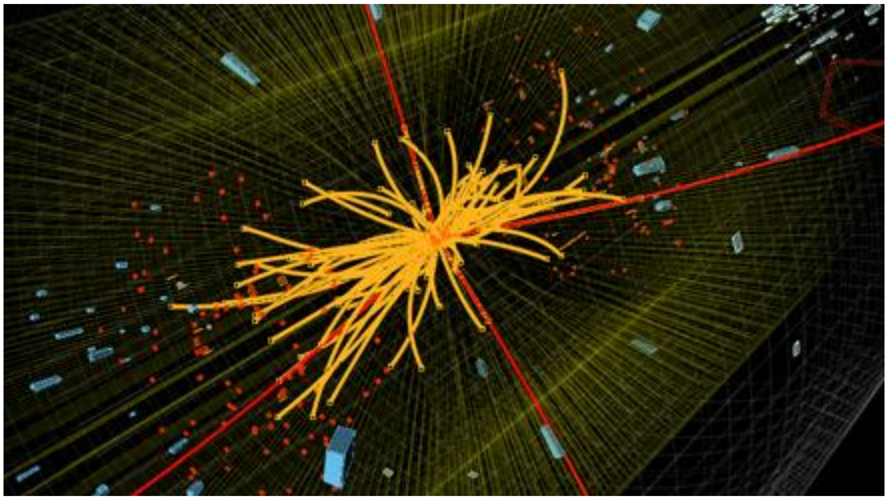
36. S. Heinemeyer, M. Mondragon and G. Zoupanos, arXiv:1201.5171 [hep-ph] (2012).
37. J. Ellis, M. K. Gaillard and D. V. Nanopoulous, arXiv:1201.6045 [hep-ph] (2012).
38. M. S. Chanowitz, arXiv : 0806.0890 [hep-ph].
39. http://www.searchthenetnow.com/graphene-could-help-physicists-probe-the-higgs-bosons-secrets
40. C. B. Jackson, S. Servant, G. Shaughnessy, T. M. P. Tait and M. Taoso, arXiv: 0912.0004 [hep-ph] (2009).

NEWS FLASH

HIGGS WITHIN REACH

The ATLAS and CMS experiments at CERN today (July 4 2012, on the eve of the 36th International Conference on High Energy Physics Conference at Melbourne, Australia) presented their latest results in the search for the long-sought Higgs boson. Both experiments see strong indications for the presence of a new particle, which could be the Higgs boson, in the mass region around 126 (GeV).

Both ATLAS and CMS gave the level of significance of the result as 5 σ sigma level, qualifying to be a discovery.



A Higgs Candidate

A proton-proton collision event in the CMS experiment producing two high-energy photons (red towers). This is what we would expect to see from the decay of a Higgs boson but it is also consistent with background Standard Model

Editorial insert