Vol 29 No. 1 Jan - Mar 2013 ISSN 0970-5953

INDIAN ASSOCIATION OF PHYSICS TEACHERS

IAPT

PHYSICS EDUCATION

www.physedu.in

Volume 29, Number 1

In this Issue

EDITORIAL

(*Submitted* Mar *2013*)

It is a pleasure to publish the present issue of Physics Education, with wide ranging interesting articles. Vinay Venugopal and Piyush Bhagdikar present an analysis of Compton effect using deBroglie wavelength of scattered electrons, a novel idea. Abhijit Potdar discusses teaching of free and forced oscillations using a java applet. Prasad Ravichandran's article on relativistic time dilation is thought provoking. In our regular feature 'Physics Through Teaching Lab' we present an article by D. Hanumeshkumar et al on LABview based weighing machine.

As usual, there is a regular feature on 'Physics Through Problem Solving' by Ahmed Sayeed. The article on Cooper pairs can give some down to earth account of the problem and the article by Elmehdi et al gives some analysis of the students' difficulties with introductory physics courses, particularly in Sharjah.

We have carried a Book review of two recent monographs on Astrophysics the evolution and end-state of Stars by G Srinivasan and I thank Professor J V Narlikar for an expert report of value for readers.

We welcome scholarly discussion and feedback on the material that appears in the journal, which could be carried either as Comments or a full scale Article. Issues of import on pedagogy can be carried as a Letter to Editor.

Pramod S. Joag.
Chief-editor@physedu.in,
pramod@physics.unipune.ac.in
Chief Editor

Teaching of the phenomena of free, damped and forced oscillations in physics through an all-inclusive java applet

Abhijit Poddar[1]

Dept. of Electronic Science,
Surendranath Evening College 24/2 M.G. Road Kolkata 700009 India.

(Submitted 02-11-2011 and resubmitted 23-10-12)

[1] Guest Lecturer. Dept. Of Physics Ramkrishna Mission Residential College (Autonomous) Narendrapur Kolkata 700103 India

Abstract

A java applet has been created for use as an all-inclusive online as well as offline learning object to teach the phenomena of free, damped and forced oscillations in physics in an entirely novel way. Subtle, yet very important features of the above phenomena, which can be difficult to make young learners understand through conventional classroom teaching, can be made easily understandable with the applet's help, through demonstration of animated plots, as well as through the provision for virtual experimentation with appropriate controls. The learner can keep track of the relevant theory including all pertinent equations, which come into focus in synchronization with the dynamic plots in real time. A topical quiz has been embedded inside the applet itself to examine the improvement, if any, in his understanding of the topic. Quantitative as well as interview-based studies to test the effectiveness of the applet have also been carried out. The results indicate that the applet can be most effective when used in conjunction with proper guidance from the teacher and when made part of a well thought-out curriculum.

1. Introduction

The phenomena of free, damped and forced oscillations are amongst the most frequently encountered topics in physics. Students are introduced to the topics when learning the mechanics behind the swinging pendulum. They come across the topics in acoustics as also when studying alternating currents in circuits. Later on they encounter the topics in atomic and nuclear physics as well. The mathematical formulation used to teach the topics however remains more or less the same. It usually involves the solution of second order, homogeneous and inhomogeneous ordinary differential equations. It is imperative that the students, coming out of school and entering college to study physics, have a fair idea of solving differential equations in general. As is often the case, this does not happen and the teacher has to write all the steps of the mathematical formulation on the blackboard, explaining them alongside. He may have to repeat the same when teaching the topics under the different heads. This kills the enjoyment of understanding the physics inherent in the topics. It also stretches the time required to finish the topic in class. In the process, the subject has the potential of becoming boring as well. I recall encountering the yawns and drooping eyes of a few of my students as I frenetically go about performing all of the mathematical deductions on the blackboard. This has led me to think if there could be another way the topics could be taught to make them more interesting and through which the relationships shared by the topics could be made more apparent. With computers and the Internet becoming more and more pervasive and accessible, I thought upon creating an e-learning tool in the form of an all-inclusive educational applet to accomplish this task. The applet would facilitate engaged exploration in the students through the inclusion of interactive and animated simulations, using which they would be encouraged to pose questions to themselves and seek answers from the applet. To that end, I would be guided in some measure by the results of earlier research [1-4] on creating effective educational simulations. In the process, I would also be building upon an applet [5] created by me earlier to teach the topic of resonance, as well as a few other applets [6,7] to teach some other topics as well.

2. Theory of oscillations in lumped systems

I have chosen oscillations in simple, lumped systems like the mass-spring system and its electrical analogue, the series-LCR circuit, as the subjects of this study. In the mass spring-system, the total energy is lumped as kinetic and potential energies in the mass (m) and the spring (represented by the spring-constant k) respectively. In the series LCR circuit, the total energy is lumped as electric and magnetic energies in the coil (of self-inductance L) and the capacitor (of capacitance C) respectively. Let us consider the mass-spring system first. The motion of the mass, displaced initially from its position of rest and constrained only by viscous-damping, is governed by the second-order, homogeneous, ordinary differential equation,

$$m\frac{d^2x}{dt^2}+kx+b\frac{dx}{dt}=0 \quad (1)\ .$$

x is the displacement at time t and b is the damping-constant, the damping force being proportional to the velocity. The solution of Eq.1 subject to the initial conditions,

$$x(t=0)=x_0 \quad (2a)$$

$$\frac{dx}{dt}(t=0)=0 \quad (2b),$$

can be obtained for three different cases:

(i) For small damping: $b < 2\sqrt{km}$,

$$x(t) = x_0\frac{\omega_0}{\omega_1}\exp(-\frac{b}{2m}t)\cos(\omega_1 t-\theta) \ (3)$$

where $\omega_0 = \sqrt{\frac{k}{m}} \quad (4)$

$$\omega_1 = \sqrt{\frac{k}{m}-(\frac{b}{2m})^2} \quad (5)$$

and

$$\theta = \cos^{-1}(\frac{\omega_1}{\omega_0}) \quad (6)\ .$$

The motion is under-damped or oscillatory with an oscillation-frequency ω_1.

(ii) For critical damping: $b = 2\sqrt{km}$,

$$x(t) = x_0(1+\frac{b}{2m}t)\exp(-\frac{b}{2m}t) \quad (7).$$

The motion is said to be critically-damped. The system does not oscillate at all and the initial displacement decays to zero in the least possible time.

(iii) For large damping: $b > 2\sqrt{km}$,

$$x(t) = B_1 \exp\left(\left(-\frac{b}{2m}+\sqrt{(\frac{b}{2m})^2-\frac{k}{m}}\right)t\right)+$$

$$B_2 \exp\left(\left(-\frac{b}{2m}-\sqrt{(\frac{b}{2m})^2-\frac{k}{m}}\right)t\right) \quad (8),$$

where B_1 and B_2 are arbitrary constants that may be determined from the initial conditions. Note that the terms under the square roots are real. The system, in this case too, does not oscillate and the initial displacement decays to zero, albeit slowly. This motion is referred to as over-damped or dead-beat.

The above solutions pertain to the free, natural motion of a damped system not subject to any external force. If the same system is driven by a sinusoidal-external -force of amplitude $F_{\max}$ and frequency ω_2, then the equation of interest would be

$$m\frac{d^2x}{dt^2}+kx+b\frac{dx}{dt} = F_{\max}\sin(\omega_2 t) \quad (9).$$

Its steady state solution would yield

$$x(t) = -\frac{F_{\max}}{D}\cos(\omega_2 t-\phi)$$

$$= \frac{F_{\max}}{D}\sin(\omega_2 t-\chi). \quad (10).$$

$$D = \sqrt{m^2(\omega_2^2-\frac{k}{m})^2+b^2\omega_2^2} \quad (11),$$

and

$$\chi = \phi + 90^0. \quad (13).$$

The steady state velocity can be found to be

$$v(t) = \frac{\omega_2 F_{\max}}{D}\sin(\omega_2 t-\phi) \quad (14),$$

by taking the time-derivative of Eq. 10. From Eqs. 10 and 14, we find that the motion is that of forced oscillation with both the displacement $x(t)$ and the velocity $v(t)$ oscillating with a frequency ω_2 equal to that of the external driving force. $x(t)$ lags the driving force by the phase angle χ while $v(t)$ lags the driving force by the phase angle ϕ. We also note that the amplitude of both $x(t)$ and $v(t)$ vary with ω_2. The amplitude of $x(t)$ becomes maximum when D becomes minimum. This occurs when

$$\omega_2 = \sqrt{\frac{k}{m}-\frac{b^2}{2m^2}} \quad (15)$$

and the situation is referred to *amplitude resonance.* On the other hand, the amplitude of $v(t)$ becomes maximum when $\frac{D}{\omega_2}$ becomes minimum. This occurs when

$$\omega_2 = \sqrt{\frac{k}{m}} = \omega_0 \quad (16)$$

and the situation is referred to *velocity resonance.* In this case, the phase angle ϕ, at resonance, becomes zero and consequently the velocity oscillates in phase with the driving force.

It is important to bear in mind that

solution for $x(t)$ given in Eq. 10 (and also for $v(t)$ given in Eq. 14 for that matter) correspond to the state of the system after it has reached a steady state. In the intervening period between $t = 0$ and this time, the actual solution for $x(t)$ would be the sum of this steady state solution and any one of the transient solutions for $x(t)$ given in Eqs. 3, 7 and 8. However the transient solutions, true to their name, die out quickly, enabling the system to be in the true steady state described only by Eqs. 10 and 14.

Next we dwell on the series LCR circuit. The corresponding equation of motion for the system driven by an external sinusoidal voltage source of amplitude $V_{\max}$ and frequency ω_2 is given by

$$L\frac{d^2q}{dt^2} + \frac{q}{C} + R\frac{dq}{dt} = V_{\max}\sin(\omega_2 t) \quad (17).$$

With the voltage source removed and the system left to itself, the equation of motion becomes

$$L\frac{d^2q}{dt^2} + \frac{q}{C} + R\frac{dq}{dt} = 0 \quad (18).$$

If we compare Eq. 17 with Eq. 9 and Eq. 18 with Eq. 1, we find that the equations are exactly analogous if the self-inductance L corresponds to the mass m, the capacitance C corresponds to inverse of the spring-constant k, the resistance R corresponds to the damping constant b and the charge q corresponds to the displacement x. The velocity $v = \frac{dx}{dt}$ would now correspond to the current $i = \frac{dq}{dt}$.

The voltage across R ($V_R = iR$) would now correspond to the damping force (bv), the voltage across C ($V_C = \frac{q}{C}$) would now correspond to the restoring force provided by the spring (kx) and the voltage across L ($V_L = L\frac{di}{dt}$) would now correspond to the net force on the mass ($m\frac{dv}{dt}$). What this implies, is, that we need not strain ourselves trying to solve the equations all over again for the electrical system as we have already found the solutions for the mechanical one, evidenced through Eqs. 3,7,8, 10 and 14. We only need to replace the values of the lumped parameters in the mechanical system with those of their electrical counterparts. We would then have found that velocity resonance corresponds to current resonance wherein the amplitude I_0 of the alternating current i becomes a maximum when the frequency of the external alternating voltage source ω_2, equals

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (19),$$

the natural frequency of the resistance-less motion. The current is then in phase with the source voltage, and the corresponding impedance Z is equal to R. The circuit then behaves as if it is a purely resistive one. We may also observe voltage resonance in the LCR series circuit, corresponding to amplitude resonance in the mechanical case. The amplitude of the oscillating charge q on the capacitor and equivalently that of the oscillating voltage across the capacitor $V_C = \frac{q}{C}$ would then be a maximum at a value of the source frequency

$$\omega_3 = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} \quad (20),$$

which, importantly, is different from both ω_0 and ω_1. In conclusion, all features of the oscillations in the mechanical system, which may or may not have been discussed above, would also have their exact analogues in the electrical system.

3. Educational applets: What are they and how they may be improved upon?

Virtual simulations of theoretical concepts and laboratory experiments have long been used as supplements to conventional classroom teaching. Many an initiative [8-13] has been taken in this regard, especially in the last few years. Most of these initiatives take the help of Flash or java applets, which are small programs that are temporarily downloaded onto the user's computer and executed. These applets, through which the simulations may be carried out, are embedded inside a parent html page. In some cases the applets may be downloaded and saved onto the local computer and executed offline. It is important to note that more often than not, the applets are designed more as tools to supplement the theoretical understanding. As a consequence, very little theory, which may include the working formula, is provided alongside the applet. This little bit of theory, along with the instructions on how to use the applet, are incorporated in the html page housing the applet or in separate files, which may be downloaded for offline use. Hyper-links, which enable the user looking for more information to navigate to other websites, web-forms through which the teacher may collect feedback and queries from the students, sets of questions and answers as well as other features, may be found inside the parent html page. However, with the rapid rise in both the speed of the internet and its affordability, there is no reason why we cannot create and make use of larger sized applets with bigger file-sizes, housing all of the features stated above in neatly arranged panels and dynamically link the contents of the different panels. The benefit that would accrue from such a design is manifold. Since the applet would have a size not greater than that of the display-screen-area, all the main components of the applet stacked in the different panels, would be clearly visible to the learner at all times. The learner would then be able to watch a simulation of an experiment in one panel and browse through the related theory, which crops up simultaneously in another panel, all at the same time and without losing focus.

This would also help the learner to relate the experiment to the theory, better and faster. The notion of a subject-topic being segregated into a theoretical part and an experimental part would also cease to haunt the young minds. This, in turn would lead to a better and a more complete understanding of the topic. The all-encompassing design of the applet would also help foster intuitive links between different aspects of a given topic.

4. Existing applets on damped and forced oscillations

One of the simplest applets on damped oscillatory motion is that of Eric Woolgar [14], wherein the values of the parameters pertaining to mass, spring-constant and damping factor may be inserted in the corresponding text-boxes and the resulting plots of different types of motion may be visualized. One also finds a set of questions related to a few interesting situations pertaining to the solution of Eq. 1, a fact not surprising, given the author happens to be a mathematician. With Walter Fendt's applet [15] on forced oscillation, one may change the frequency ω_2 of the driving force and view, in addition to the displacement versus time plots, also the plots pertaining to displacement's amplitude versus ω_2 and displacement's phase versus ω_2. Michael Bergdorf and Stephan Kaufmann's applet [16] is functionally and feature-wise similar to Fendt's applet, but has an additional provision for letting the user change the value of the initial displacement. Thanks to MIT's open courseware project, we have another applet [17] on forced and damped vibration. In the applet, apart from being able to observe both the transient and steady-state temporal plots for the displacement (x) for different values for the parameters chosen by the user, one can also observe how the corresponding phase-space trajectories (x versus velocity v) evolve with time. Wee Loo Kang's applet [18], hosted under NTNUJAVA Virtual Physics Laboratory [7], has many extra controls and the ability to display the values of many more variables related to the simulation. However, it takes some time and effort getting to learn how to use it. Last but not the least, the Physics Education and Technology Project (PhET) team at the University of Colorado, Boulder, have designed a resonance applet [19], which, in spite of not having been designed to produce plots like those found in the other applets discussed above, would appeal to first-time learners of the topic. The use of Flash in addition to java makes the applet closely resemble a real-world mass-spring resonating system and makes it easier for the student to relate to the phenomenon.

5. Research related to the design of effective simulations and studies to test their efficacy

In recent years, a lot of insightful research [1-4] has been carried out, primarily by the PhET team at Colorado, to find out the most effective ways to design and use educational simulations that may be carried out through applets.Wieman et al.[1] have discussed strategies like using the simulations in the form of simple animated illustrations, in concept tests or in the form of interactive classroom demonstrations. They have also suggested using the simulations in different settings: as group activities in the class where students may be put through question-answer sessions; as simulation-based homework assignments as tools to conduct virtual experiments in situations where performing actual experiments is not possible and also as a pre-lab or post-lab supplement to the conventional laboratory class. Adams et al.[2,3] have conducted over two-hundred student interviews and thereafter come out with recommendations on both the design as well as qualitative aspects related to creating effective simulations. They have opined that the simulations should be research-based, engaging, interactive, animated, encourage exploration, promote learning and should relate to the real world. Podolefsky et al.[4], on the basis of interviews with students exposed to a wave-interference simulation, have analyzed the factors that promote engaged exploration.

McKagan et al. [20] have developed a curriculum pertaining to the photo-electric effect which includes, apart from an interactive computer simulation, interactive lectures, peer instruction and home assignments. They have then designed two exam questions to test the efficacy of the curriculum and found that it indeed helps the students to correctly predict the results of photoelectric experiments. In another work, McKagan et al.[21] have used student interviews to gain new insights into student thinking related to the PHET–simulations [10] on quantum theory and quantum mechanics and made use of the results to help refine the simulations as well. Frenkestein et al. [22] and later Keller et al. [23] have studied the effectiveness of another computer simulation: the 'Circuit Construction Kit', as a substitute for real laboratory equipment. Based on statistical analysis, they have found that in situations where students were exposed to the simulation, their performance improved remarkably.

Statistical studies to test the effectiveness of simulations have also been carried out by the likes of Tambade and Wagh [24], who have used the methodology of pre-test and post-test to study the positive effect of computer simulations and animations in improving the understanding of the concepts of force, field and potential in electrostatics. Tadesse [25] has also used student pre-test and post-test results to show how students' performance was enhanced in theoretical as well as practical classes through the use of computer simulations.

6. Design and working of the all-inclusive oscillations applet

The creation of my java applet on free, damped and forced oscillations has been motivated by my ideas, discussed in section 3, related to teaching the topic in a novel and useful way. The recommendations emanating from research on designing physics simulations, discussed in the previous section, have also been very helpful in putting my ideas to work. As it has turned out, my applet is fundamentally different from the other applets on the same topic discussed in section 4. It has an innovative and visually appealing layout. It is more feature-rich and is more fun to use. As we shall see, it can also prove to be much more useful.

The applet has been created using the Java Standard Edition Development Kit (JDK) version 5.0. Since the applet would be executed inside the browser of the host computer, Java Runtime Environment (JRE version 5.0 or later) must be installed in the host computer. One may note that both the JDK and the JRE are available as free downloads [26]. Fig. 1 is a screenshot of the applet when it is run for the first time. As can be seen, it is divided into four panels, one containing an option-box to select different tasks, another to house the controls to perform the simulations, and the remaining two to primarily display the results of the simulations. Clicking on the 'Read complete theory' option in the option-box in the top-left panel, opens up a text-pane in java, containing all the pertinent theory, including all the equations. The pane has two scrollbars in the horizontal and vertical directions, which make it possible to pack in as much theoretical material as required. The user can smoothly navigate through the theory with the scrollbars without losing focus on the simulations. This is made possible because of the fact that the box can cover only an area, which is no greater than that of the top-left panel. All the other panels continue to be displayed. The bottom-left panel contains the slider controls using which one may change the values of different variables governing free and damped oscillation. One can use the sliders corresponding to the parameters m (or L) and k (or $1/C$) to fix the value of the angular frequency ω_0 of free un-damped oscillation and in addition use the slider for the parameter b (or R) to fix the angular frequency ω_1 of damped

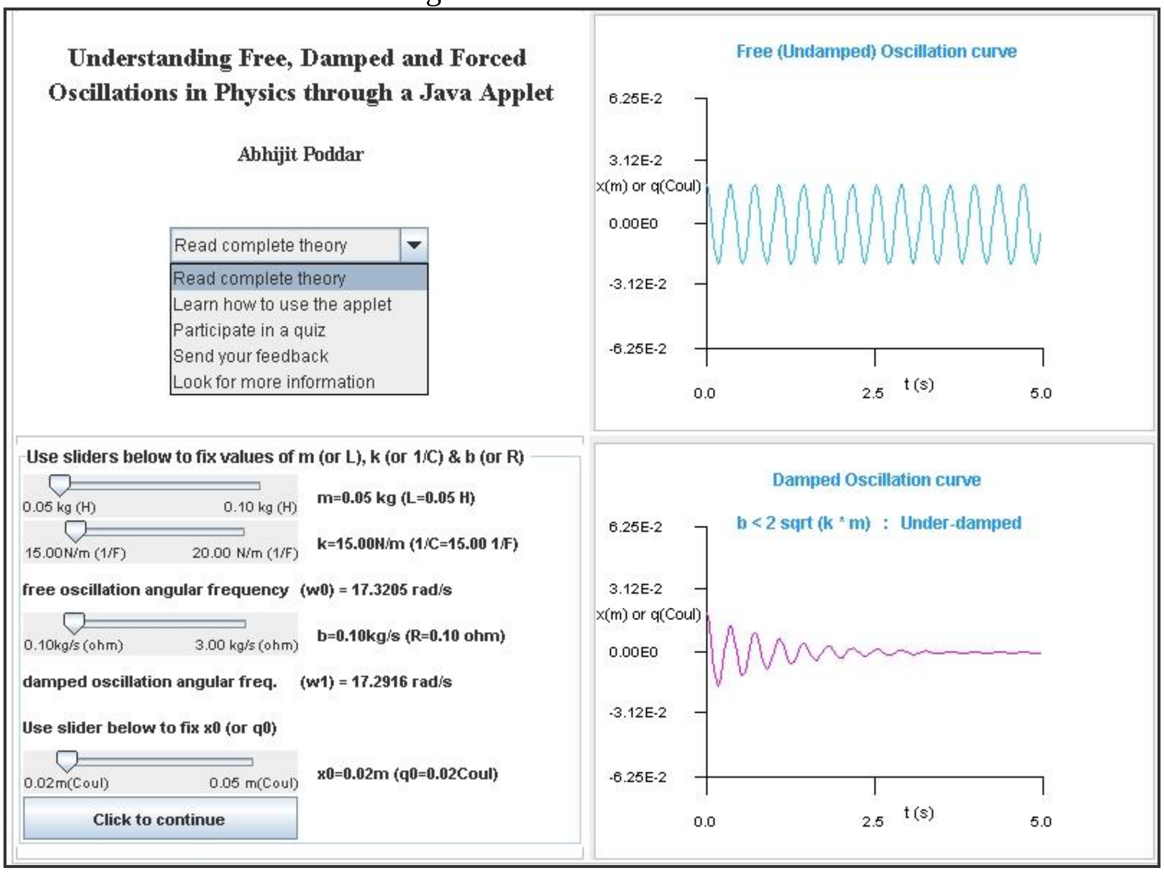


Figure 1: A view of the applet as it is run for the first time.

oscillation. An additional slider has been introduced to fix the value of the initial displacement x_0 (or the initial charge q_0). For each set of slider positions in the bottom-left panel, the corresponding waveform for free oscillations of the displacement x (or charge q) and also that for the damped oscillations of the same x (or q) can be visualized in the top-right and in the bottom-right panels respectively, as is shown in Fig. 1. The student or learner can read the relevant theory on free and damped oscillations, which comes into focus in the top-left panel, the moment he starts to drag any of the four sliders. At the same time he can explore around by playing with the slider controls, observe the resultant visualizations and find out, just as predicted by theory,

(a) how ω_0 changes with *m* and *k* in the mechanical case and equivalently, with *L* and *1/C* in the electrical case ;

(b) how ω_1 changes with *m*, *k* and also *b* in the mechanical case and equivalently, with *L*, *1/C* and also *R* in the electrical case;

(c) how ω_0 and ω_1 are not affected by changes in x_0 ;

(d) how, slowly increasing the value of *b*, results in under-damped oscillatory motion (for which $b < 2\sqrt{km}$ or equivalently $R < 2\sqrt{L/C}$) giving way to critically damped motion (for which $b = 2\sqrt{km}$ or equivalently $R = 2\sqrt{L/C}$) which again gives way to over-damped

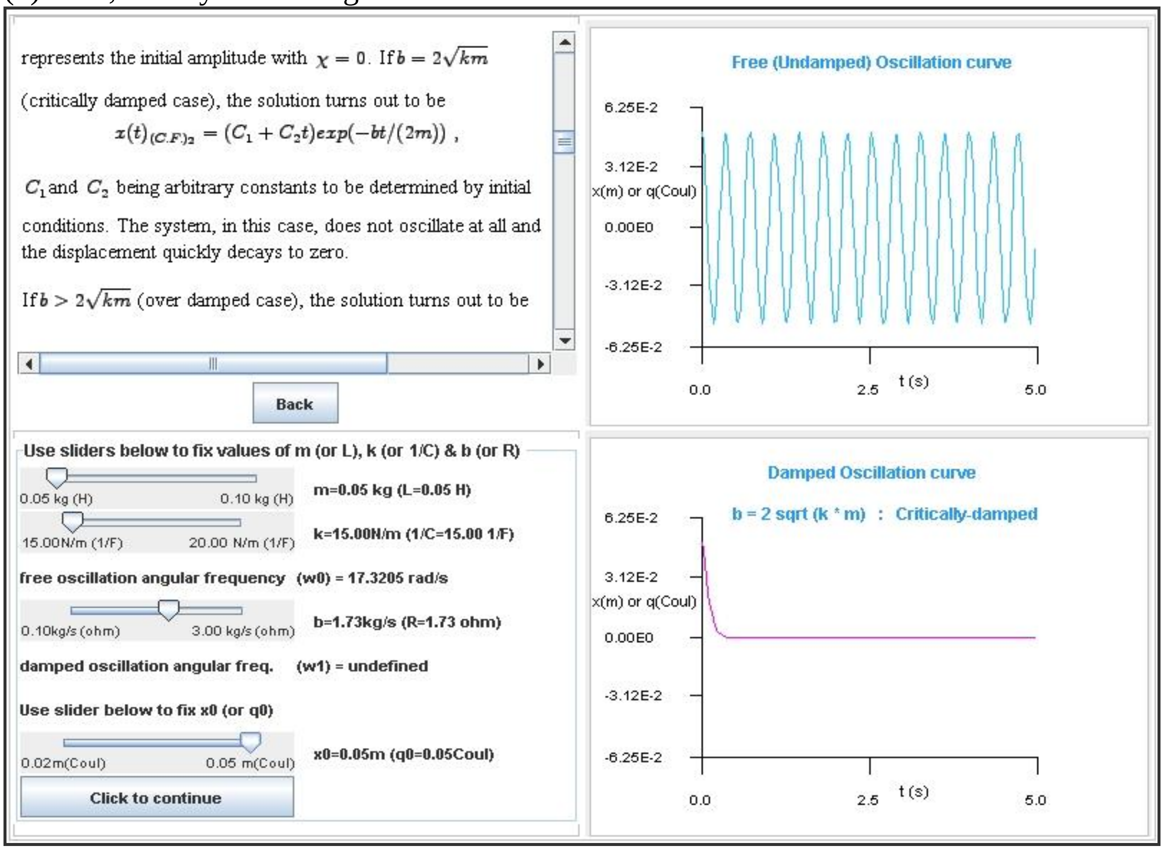


Figure 2: A screenshot of the applet showing, how, one may read the pertinent theory, simulate the case of critical damping with the slider-controls and observe the resulting plot at one and the same time.

motion (for which $b > 2\sqrt{km}$ or equivalently $R > 2\sqrt{L/C}$). If the student so desires, he may at first play around with the relevant controls and then look for explanations for the changes in the observed phenomena by going through the relevant theory, without losing focus. This feature of engaged exploration possible through the applet, is in consonance with the conclusions drawn from earlier research on the requirements of creating an effective simulation [2,4]. A screenshot of the applet showing critical damping is presented in Fig. 2.

One may have observed the 'Click to continue' button at the bottom of the bottom-left panel in Figs. 1 and 2. Pressing this button opens up a set of new controls in the same panel, using which, one may simulate the phenomenon of forced oscillation and resonance for the same system. Both types of resonance, amplitude (or voltage) resonance and velocity (or current) resonance, talked about earlier in section 2, can be studied in detail at the same time. This is made possible through the use of multiple-representations of the same or related phenomena in the form of graphs, animations and pertinent text, in

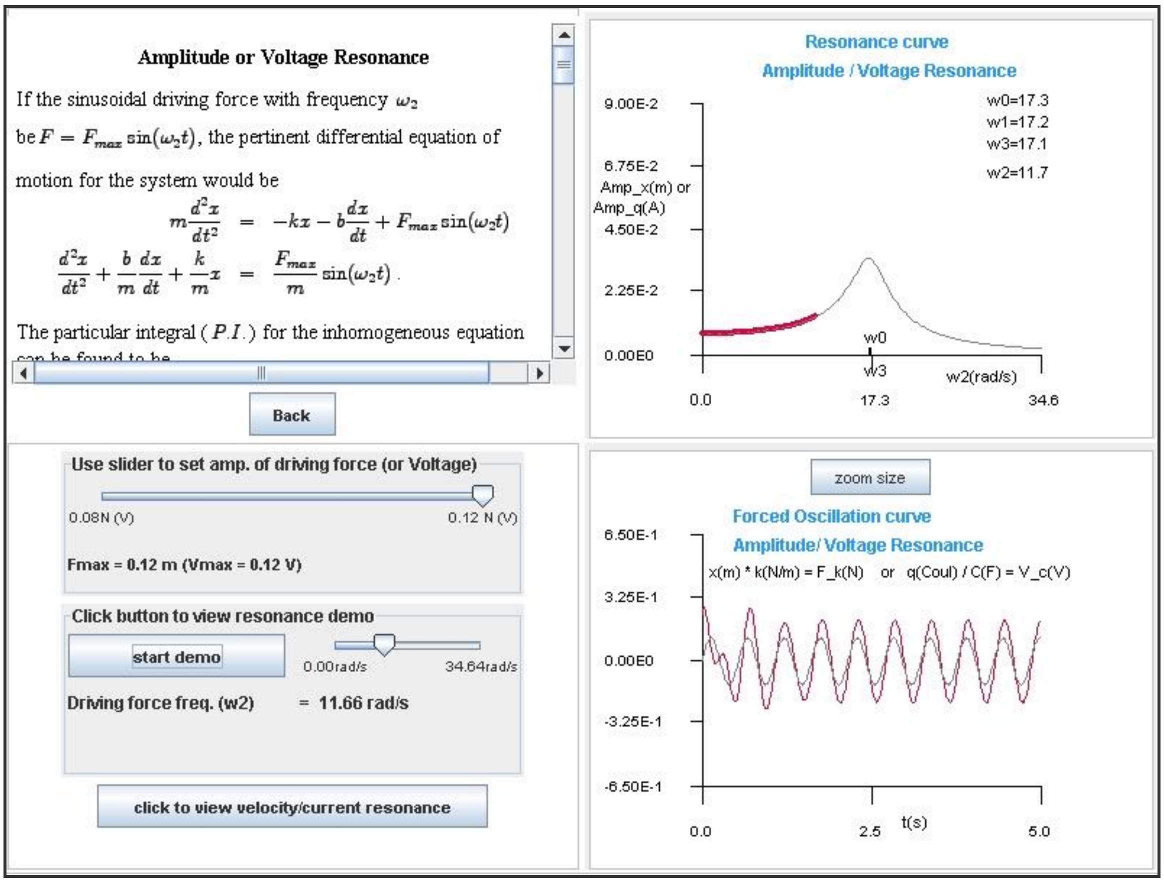


Figure 3: A screenshot of the applet showing, how, one may simultaneously refer to the theory and perform a virtual experiment to study the phenomenon of amplitude or voltage resonance.

agreement with the conclusions drawn from earlier research [1,2]. At the very instant the student clicks the 'click to continue' button, the relevant theory on amplitude or voltage resonance comes into focus in the top-left panel as shown in Fig. 3. The corresponding simulation can now be started by fixing the value of the amplitude of the driving force (or voltage) using the relevant slider and clicking the 'start demo' button in the bottom-left panel shown in Fig. 3. The angular frequency of the driver ω_2 gets automatically incremented and for each value of ω_2, one may observe the displacement x (more specifically the restoring force kx) and equivalently the charge q (more specifically the voltage across the capacitor q/C) in the bottom right panel, as they evolve with time. The effect of the transient response dominates at the start but soon gives way to a fixed-amplitude steady-state oscillatory response with a frequency equalling the driving frequency ω_2. At each value of ω_2, the steady-state amplitude of x (or q) is plotted versus ω_2 in the top-right panel and this results in the so-called amplitude (or voltage)

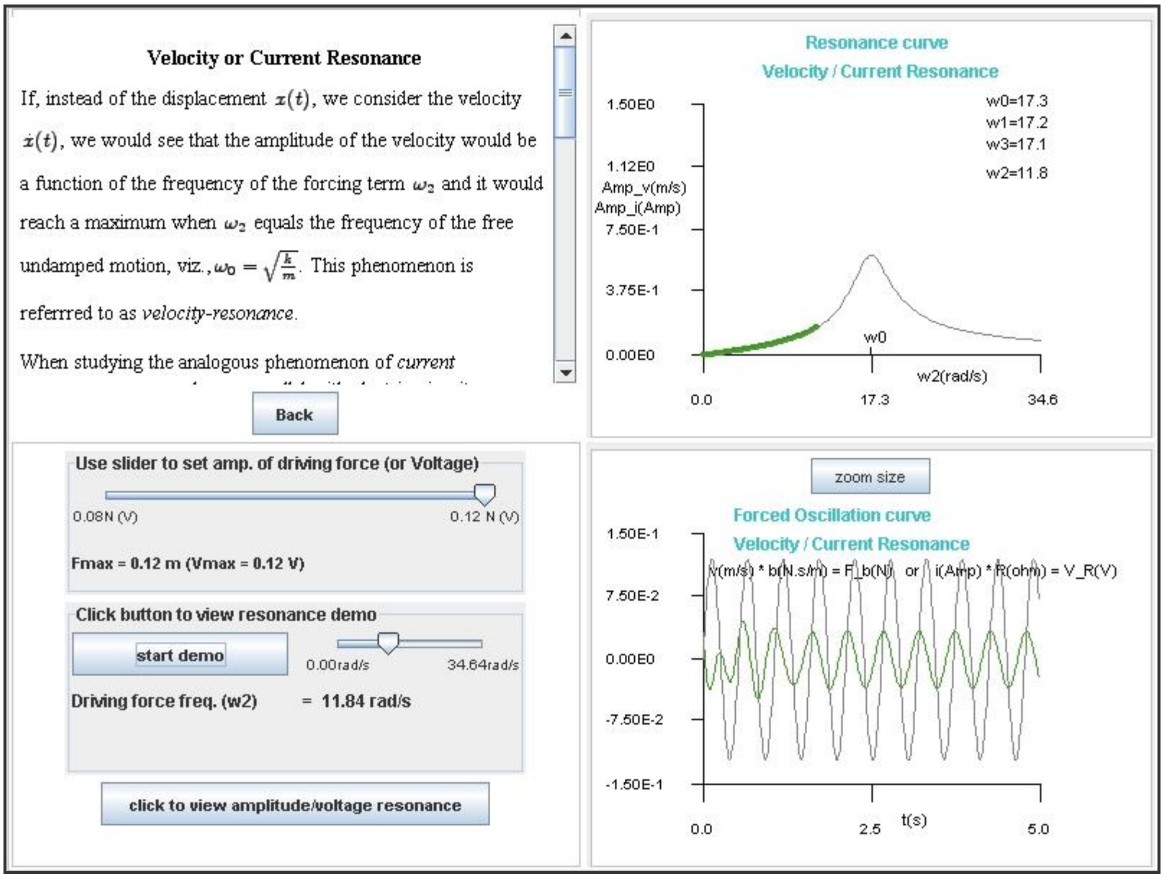


Figure 4: A screenshot of the applet showing the unfolding of the phenomenon of velocity or current resonance.

resonance curve, whose maximum is reached when ω_2 equals a frequency ω_3, as can be observed by following their values on screen. By playing with the respective controls, one may further explore

(a) how the amplitude of x (or q) changes with changes in m (or L), b (or R) and k (or C).

(b) how the amplitude of x (or q) changes with changes in the amplitude and frequency of the driving force (or voltage) .

(c) how the amplitude of x (or q) becomes maximum at a frequency ω_3, which is not only less than ω_0 but also less than ω_1 when the corresponding transient motion is oscillatory with frequency of oscillation ω_1. One may simultaneously look for explanations for the results of the above exploration by going through the relevant theory in the top-left panel, as can be seen in Fig. 3.

When one clicks the 'click to view velocity/current resonance' button at the bottom of the bottom-left panel shown in Fig. 3., he may now follow how 'velocity resonance' in the same mechanical system and its electrical counterpart: 'current resonance' in the same electrical system, occur. These correspond respectively to the amplitude of the

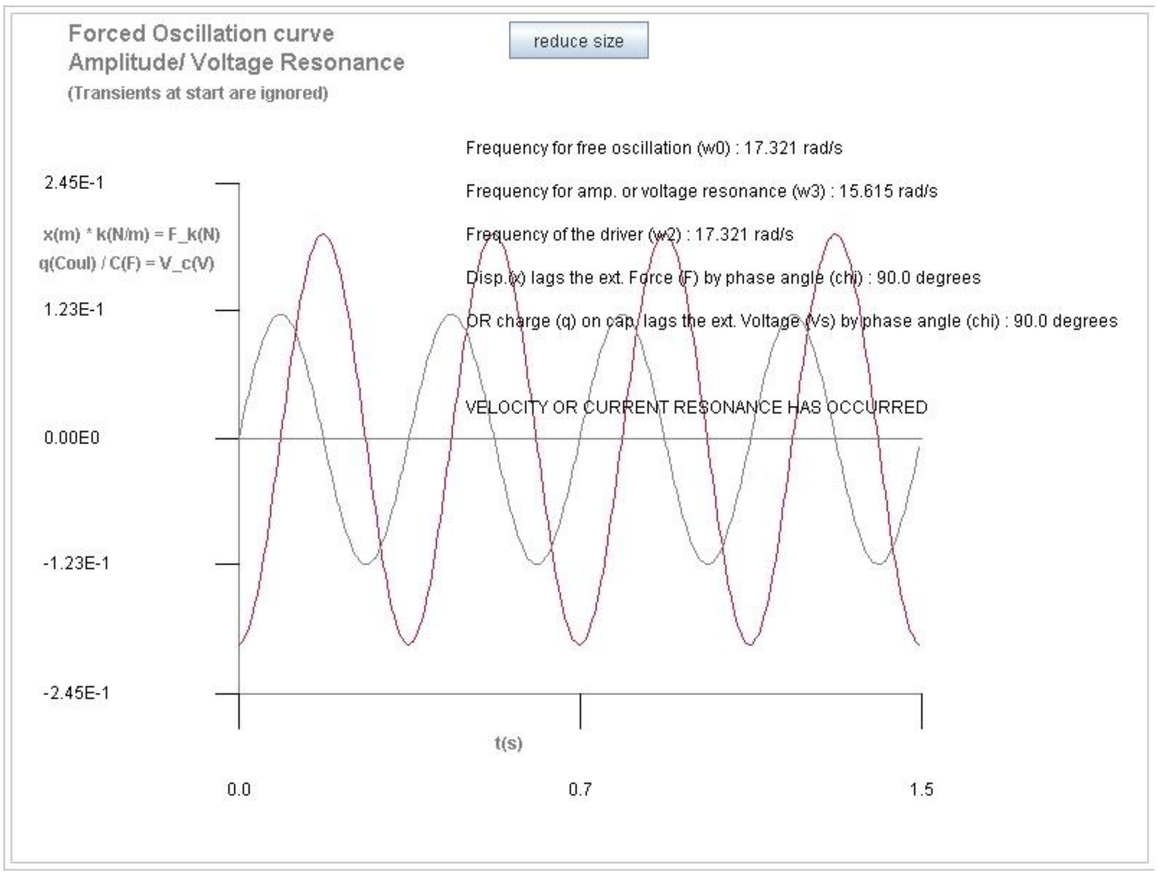


Figure 5: A screenshot of the zoomed bottom-right panel, showing the phase relationship between the driving force or voltage and the displacement or charge (more specifically the restoring force of the spring or the voltage across *C*) at the frequency ω_0.

velocity (v) of the mass or the amplitude of the current (i), reaching a maximum when the driving frequency ω_3 equals ω_0, giving rise to the so-called velocity (or current) resonance curve. The phenomenon can be best understood by following the corresponding animation in the top-right panel, as can be seen in Fig. 4. The variation of v (more specifically the damping force bv) and equivalently of i (more specifically the voltage across the resistor Ri) with time can also be followed in the bottom right panel. As in the amplitude-resonance case, one may change the values of the different parameters, explore their effect on v (or i), and seek explanations from the related theory at the same time.

One may have noticed the 'zoom size' button on the bottom-right panel of Fig. 3. Clicking the button zooms the panel-size, enabling us to follow more closely, the phase relationship between the driving force, whose time-variation is shown in grey, and the displacement x (more specifically the restoring force kx) whose time variation is shown in crimson. In the electrical case, this would correspond to following the phase relationship between the driving voltage

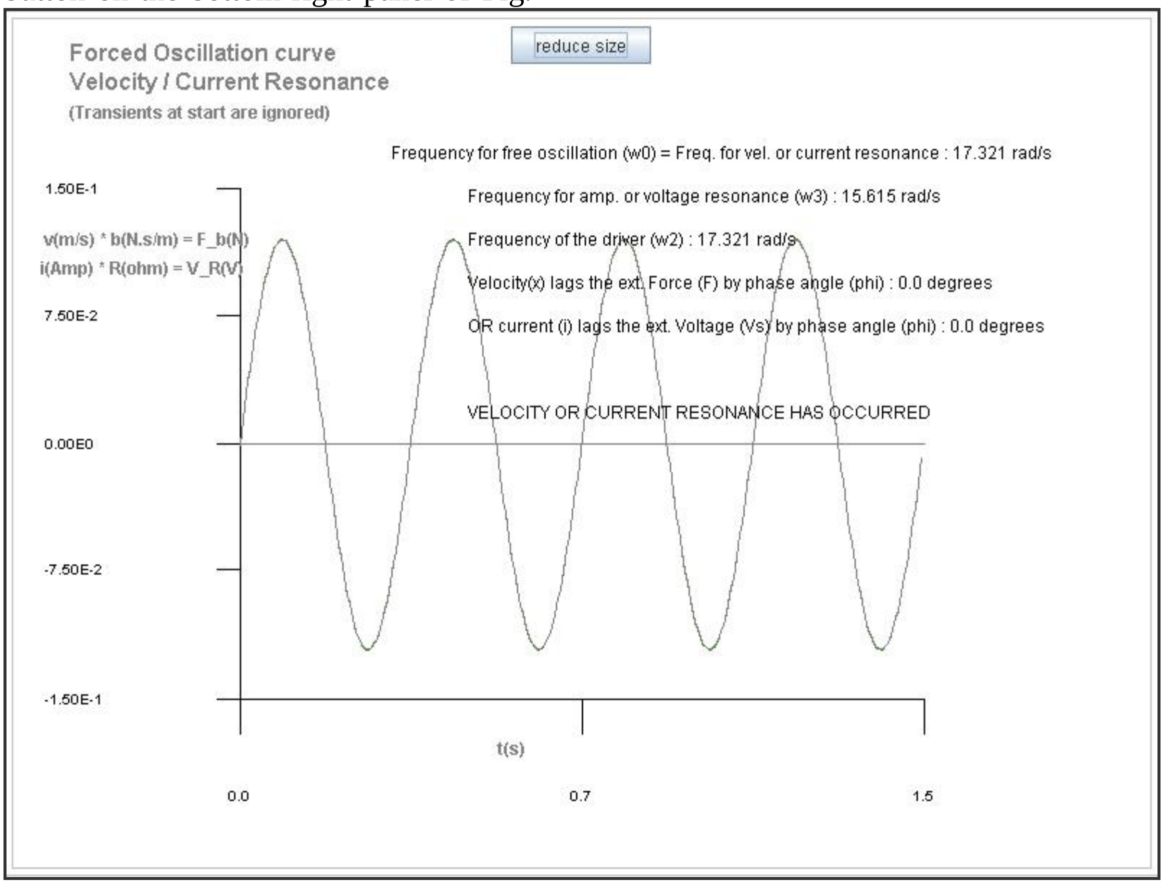


Figure 6: Another screenshot of the zoomed bottom-right panel, showing the in-phase relationship between the driving force or voltage and the velocity or current (more specifically the damping force or the voltage across *R*) at the frequency ω_0.

and the charge q (more specifically the voltage across the capacitor q/C). Of particular interest is the situation at the frequency of free oscillation ω_0, when x (or q) lags the driving force (or voltage) by 90 degrees as shown in Fig. 5. Similarly, clicking on the 'zoom size' button on the bottom-right panel shown in Fig. 4, helps us to visualize more closely the phase relationship between the driving force and the velocity v (more specifically the damping force bv). In the electrical case, this would correspond to following the phase relationship between the driving voltage and the current i (more specifically the voltage across the resistor Ri). Of particular interest is the situation at the frequency of free oscillation ω_0, when v (or i) is exactly in phase with the driving force (or voltage) as can be seen in Fig. 6.

For a mechanical system, the large displacement amplitudes that result at resonance could potentially damage the mechanical structure itself. For an electrical system, the large currents that result at resonance could burn the electrical circuit. It makes sense, therefore, to include some sort of sound to drive home this point and that is what

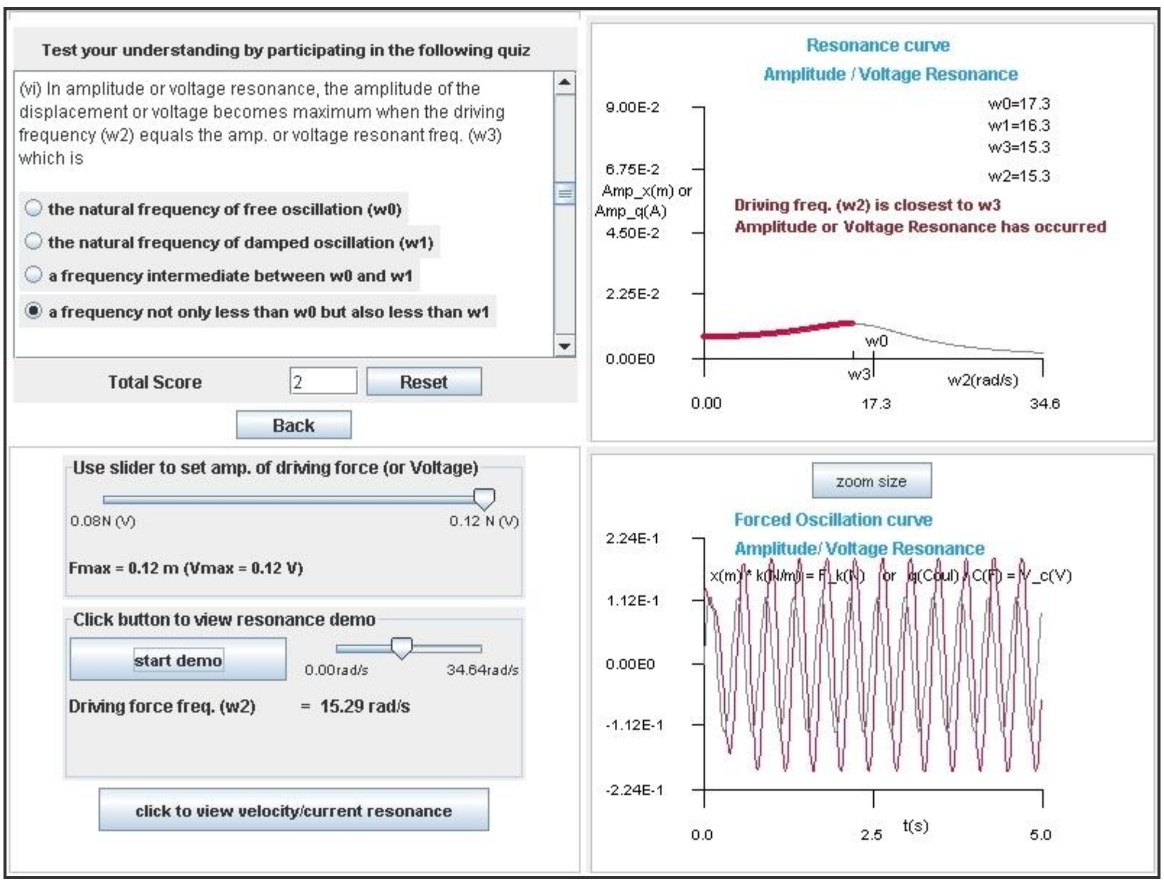


Figure 7: A screenshot of the applet showing how one may participate in a topical quiz and seek answers from within the applet itself, if required.

has been done. Calandra and co-workers [27] have shown that with increasing bandwidth of the Internet, digital audio is becoming more and more common in online courses and suggested ways how audio can be included in e-learning courseware. Taking a cue from their work, I have incorporated the sound of shattering glass and that of a fire alarm at the appropriate instant. Voice recordings have also been introduced at appropriate moments of the simulation to give it a real-world classroom-touch.

Another nifty feature of the applet is the inclusion of a set of quiz questions inside the applet itself, to facilitate the testing of ones understanding of the topic, at any time before, during or after its use. There is also a provision for keeping a score of the marks obtained for choosing the right answers. Questions seeking answers to subtle aspects on the topic have been included in the quiz. The student has the option of looking for the answer in the applet itself, at the same time. As an illustration, in order to seek the answer to a question, 'What would the value of the frequency for amplitude or voltage resonance be vis-à-vis the frequencies for free oscillation ω_0 and damped oscillation ω_1?', one can play around with the controls, watch the unfolding of amplitude-resonance in the top-right panel and note the values of ω_0, ω_1 and the frequency ω_3 at which amplitude resonance occurs. This is depicted in Fig. 7.

Earlier it had been pointed out that the goal of this work was primarily to create a 'self-contained' applet, which would depend very little on the web page to which it belonged. To this end, gathering feedback and being able to answer the queries of the users of the applet has been made possible from within the applet itself through the use of a PHP form. One may have noticed the 'Send your feedback' option in the drop-down list in the top-left panel shown in Fig. 1. Clicking this option would display a PHP form, asking the student to provide feedback regarding the applet and to send any question he may have regarding the topic. The student may view the response to his queries by clicking the link at the bottom of a confirmation page. Of course he has to wait till the teacher has noted down his response in a text file and uploaded it to the server hosting the applet. Alternatively, the teacher may send his response directly to the student at the email address provided by him in the PHP form itself.

Last but not the least, the student or learner should always be encouraged to look for more information, assimilating more knowledge in the process. To this end, a provision for linking to other web sites containing additional resources on the pertinent topic has also been made. This can be achieved by clicking the 'Look for more information' option in the drop-down list in the top-left panel shown in Fig. 1.

It bears mentioning that it would not be difficult to accommodate more features within the applet. The design of the applet has been deliberately made an open-ended one.

7. Assessment of the efficacy of the applet

To test the efficacy of my applet, I have made use of a simple pre-test – post-test study on ten of my students to study the effect of my applet in improving their understanding of the topic. The students were first lectured on the topic by myself for a couple of hours and then each of them were handed over a sheet of paper having fifteen multiple choice questions, each carrying two marks, which they were to answer in fifteen minutes. They were allowed to consult only their class notes if needed. This constituted the pre-test in my study. Next they were exposed to the applet by themselves, allowed to play around with it for an hour, and made to participate in the quiz embedded inside the applet for fifteen minutes. The quiz comprised the same set of fifteen questions and the students were at liberty to take the help of the simulations inside the applet in seeking the answers. This constituted the first post-test in my study. Subsequently the students were made to use the applet again for half an hour, but this time with my help and guidance, before participating in the same quiz again for fifteen minutes. This constituted our second and last post-test. As during the first post-test, the students were allowed to take the help of the simulations inside the applet, if required.
The use of the applet entailed very little expertise in the use of a computer, and all the students who were tested, did have that level of expertise. I therefore did not feel the need to factor in the effect of each student's computer proficiency in the study.

Statistical tests were carried out on the data (scores) by using the some of the statistical functions included in the spreadsheet program Gnumeric [28] and the results are shown below:

Table 1. M = Mean, σ = Standard Deviation

Test	M	σ
Pre-test	14.20	6.76
First post-test	16.00	6.86
Second post-test	21.6	6.10

Table 2. Paired (type-1) 2-tailed t-test probability (*P)* and Mean of Normalized Gain (<*G*>)

Comparing groups	*P*	<*G*>
First post-test with pre-test	0.000725	0.15
Second post-test with pre-test	6.3E-06	0.52
First post-test with second post-test	0.000202	0.42

G = Normalized Gain [29]

$$= \frac{post\text{–}test\ score - pre\text{–}test\ score}{Max\text{–}score - pre\text{–}test\ score}$$

Post-test - pre-test combinations are as depicted in 1st column of Table 2 (When being compared to the 2nd post-test results, the 1st post-test plays the role of pre-test in the above formula)
Max-score = 30.

P gives the probability of the difference in the means being more due to chance than due to the effect of the applet (value < 0.05 is considered to be statistically significant). Each of the data sets was successfully tested for normality before applying the t-tests to the data-set pairs.

From the data presented in Table 1 we observe that the mean score from the pre-test increases by a meagre 12.7% after the first post-test and by a whopping 52.1% after the second post-test. Interestingly there is also a 35.0% increase from the first post-test mean score to the to the second post-test mean score. The standard-deviation values remain nearly equal. The results of the paired 2-tailed t-test shown in second column of Table 2 tells us that the increase in scores from one test to another is statistically significant in all three cases. However, the most significant increase in the scores occurs from the pre-test to the second post-test. The mean normalized gain <G> in the scores is also greatest after the second post-test when compared to the scores after the pre-test. From the above, we may infer that the use of the applet and its embedded simulations must have helped the students to have a better understanding of the topic. This is in tune with the outcome of similar studies on the positive effect of using computers [30,31] and computer simulations [20-25] on the teaching of different topics in physics. However, instructor guidance also seems to have played a major role in the process, corroborating the findings of earlier research [1].

The quantitative study of my applet's efficacy was somewhat constrained by my inability to test a larger number of students, at least for the time being, although I do believe that a larger statistical sample size would not have had a different effect on the overall conclusion. Nevertheless, I decided to also undertake a qualitative study, in tune with similar work [21] reviewed in section 5, on testing the effectiveness of simulations on the teaching of quantum mechanics. In the process I expected to gain deeper insights into students' questions on the topic and misconceptions, if any. I chose to have direct one-to-one interactions with a couple of students, one being the weakest and the other being the brightest boy in my class. Incidentally, but not surprisingly, they had had the lowest and the highest score respectively consistently in all the three tests, I had given them. A couple of questions I put to them were as follows:

The source voltage in a series LCR circuit equals the sum of the voltages across L, C and R.

(a) At the current resonant frequency ω_0, the voltage across R becomes a maximum and equals the voltage of the source. That would seem to suggest that the voltages across C and L would be zero at the resonant frequency. Is it correct? Explain your answer, both qualitatively and quantitatively.

(b) At the voltage resonant frequency ω_1, the voltage across the capacitor becomes a maximum. Explain intuitively, why, the voltage across the inductor cannot be a maximum at the same frequency ω_1, given that the maximum voltage across the inductor must equal the maximum voltage across the capacitor.

Both the students were allowed to take the applet's help in answering the questions. However, the weaker student failed to answer both the questions while the brighter student, in spite of being able to correctly answer only the first question after taking help from the applet, was yet to develop a complete understanding of the phase-relationships between the different waveforms, which is so vital in relation to the question. It was then that I intervened and spent some time explaining to the students with the help of the relevant animated plot in the applet, a screenshot of which is shown in Fig. 6, how, at the resonant frequency ω_0, the instantaneous waveforms for the voltages across C and L, not only are out of phase as always but also have equal magnitudes and therefore cancel out, as a result of which, the voltage across the resistor attains its maximum value which is equal to that of the source voltage. In the process I also used the applet to explain to them the concept of phase and phase difference. Thereafter, the students were able to comprehend why the answer to the first question was a no and seemed pretty relieved at that. Subsequently, they could use the relevant equations with greater confidence to verify the answer quantitatively.

As for the second question, I took the help of the animated plots in the applet once again to reason out and explain to both the students why voltages across C and L cannot be same at ω_1 as they would then cancel out (their phases being opposite at all times) resulting in the voltage across R becoming a maximum and equal to the source voltage at ω_1 in addition to the same happening at ω_0, which cannot be. Thus the maximum voltage across the inductance, which equals the maximum voltage across the capacitance, must occur at a frequency different from ω_1. The students were then instructed to apply the relevant theory to verify that this new frequency would be symmetrically placed on the other side of ω_0. To my satisfaction, it took little time thereafter, for the students to establish the theoretical correctness of the predictions by themselves.

The above interaction once again highlighted the importance of instructor guidance in the use of the applet to make it more effective. What it also showed was the necessity of integrating the applet with a well-designed curriculum that could include lectures, question–answer sessions and home assignments [1,20,21] and that is precisely what I did as a follow up exercise: designed an applet-centric curriculum to be used henceforth to teach the topic in question. Importantly, a few aspects related to the topic, which did not find place in the applet, were included in the lectures. Moreover, laboratory experiments, like that of the study of the resonance phenomena in a series-LCR circuit with the help of an oscilloscope, which were till now conducted independently, were now made part of the curriculum. In the process the students could relate the theory, simulations through the applet as well the results of real experimentation in the laboratory class, much better. In the near future, I also plan to include in the curriculum, actual experiments to study transient phenomena with the help of a PC and a data acquisition interface [32].

From the interactions I had with the students, I also realized how the applet could make seemingly queer phenomena like that of voltage magnification understandable. That the voltage across the capacitor (and also across the inductor, the two being equal and opposite at ω_0) could exceed the voltage of the source itself seemed intriguing, more so after they had collected the relevant data in a laboratory experiment to study resonance in a series LCR circuit a few days earlier. The animated visualization of the voltage waveforms for the source and the capacitor, a screenshot of which is shown in Fig. 6, helped remove their doubts. Through the applet, they could now also intuitively link the phenomena of voltage magnification in the electrical circuit to that of the analogous force magnification in the mechanical mass-spring system.

The interactions also brought out the fact that students suffered from a few misconceptions. One of the misconceptions was that the amplitude resonance and velocity resonance occur in different systems, while in actuality they refer to the same system at the two different values of the driving frequency, one equalling ω_3 and the other ω_0 respectively. By designing the applet in such a manner that one could toggle between observing either of the two resonance curves forming at any point while the animation was running, helped remove the misconception.

8. Summary and conclusion

To summarize, a self-contained java applet has been created to teach the phenomena of free, damped and forced oscillations in both mechanical and electrical systems at the same time and in a novel and interesting way. The applet contains all the relevant theory, simulated experiments and animations, an interactive quiz, provision for gathering feedback, posting questions and receiving answers, as well as linking to additional resources, neatly packed inside itself. The theory and experiment go hand-in-hand, with the relevant theory appearing in a panel in sync with the virtual experiment and visualization, taking place in another panel. While designing the applet, the recommendations coming out of earlier research on designing effective simulations have been followed as far as practicable. The efficacy of the applet has been studied quantitatively as well as through student interviews. It has been found that subtle yet important features of the topic can be more easily understood and the overall understanding of the topic can be remarkably improved through the applet, helped in no small measure by the timely intervention and proper guidance of the teacher. The applet must also be made part of a well-thought out curriculum to be most effective. The applet may be used online as well as offline at any time. In situations where laboratory experimentation is not feasible, the applet can help in virtual experimentation as well.

The eager look in the faces of the students in the class being taught with the applet's help betrays the interest and sense of participation generated in them. Teaching becomes all the more satisfying too. With the cost of computers and internet accessibility coming down fast, coupled with the increasing speed of the internet, applets such as these, as part of a well designed curriculum, have great potential in spreading the reach and scope of online-teaching, specifically of experiment based science topics.

Acknowledgements

The author wishes to thank the referee for his insightful suggestions, which have helped him place the work in the proper context.

References

1. C. Wieman, W. Adams, P. Loeblein, and K. Perkins, *'Teaching physics using PhET simulations'*, The Physics Teacher, vol. 48 (4), 225 (2010).

2. W. K. Adams, S. Reid, R. LeMaster, S. B. McKagan, K. K. Perkins, M. Dubson and C. E. Wieman, '*A Study of Educational Simulations Part I - Engagement and Learning' ,* J. of Interactive Learning Research, vol. 19 (3), 397 (2008).

3. W. K. Adams, S. Reid, R. LeMaster, S. B. McKagan, K. K. Perkins, M. Dubson and C. E. Wieman, '*A Study of Educational Simulations Part II-Interface Design',* J. of Interactive Learning Research, vol. 19 (4), 551 (2008).

4. N.S.Podolefsky, K. K. Perkins and W. K. Adams '*Factors promoting engaged exploration with computer simulations'* Phys. Rev. ST Phys. Educ. Research, vol. 6, 020117 (2010).

5. A. Poddar, '*Teaching Resonance out of an E-Briefcase*', Phys. Edu. 27 (1), vol. 19 (2010).

6. A. Poddar, *Topic-specific Learning Management Systems through self-contained Applets*, International Conference on E-learning in the Workplace, Columbia University, New York, June 9 to 11, 2010, edited by Dr. David Guralnick, ISBN number (CD): 978-0-9827670-0-9.

7. A. Poddar, *Applet based Learning Management Systems to teach Electronics*, International Conference on E-resources in Higher Education, Bharathidasan University, Tiruchirapalli, India, February 19 and 20 2010, edited by Dr. S. Srinivasaragavan and Dr. E. Ramganesh, pp. 111-115, ISBN number: 978-81-908078-9-0.

8. W. Christian, G. Novak, *Java Physics Applets*, Davidson College: http://webphysics.davidson.edu/Applets/Applets.html.

9. *MichiganTech Physics Academic Resources:*http://www.phy.mtu.edu/links/Interactive_Physics.html.

10. *Interactive Science Simulations*, University of Colorado Boulder: http://phet.colorado.edu/index.php.

11. NTNUJAVA *Virtual Physics Laboratory:*http://www.phy.ntnu.edu.tw/ntnujava.

12. P. Falstad, *Math and Physics Applets:*http://www.falstad.com/mathphysics.html.

13. W. Fendt, *Java Applets on Physics:* http://www.walter-fendt.de/ph14e.

14. E. Woolgar's damped vibration applet, http://www.math.ualberta.ca/~ewoolgar/java/Hooke/Hooke.html

15. Walter Fendt's resonance applet,

http://www.walter-fendt.de/ph14e/resonance.htm

16. M. Bergdorf and S. Kauffmann's forced oscillation applet, http://www.zfm.ethz.ch/meca/applets/ems/ems.htm

17. MIT's open courseware applet on forced and damped vibration, http://math.mit.edu/daimp/ForcedDampedVib.html

18. Wee Loo Kang's oscillation applet, http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=494

19. PhET resonance applet, http://phet.colorado.edu/en/simulation/resonance

20. S. B. McKagan, W. Handley, K. K. Perkins and C. E. Wieman, '*A Research-Based Curriculum for Teaching the Photoelectric Effect',* Am J. of Physics, vol. 77 (1), 87 (2009).

21. S. B. McKagan, K. K. Perkins, M. Dubson, C. Malley, S. Reid, R. LeMaster, and C. E. Wieman, *'Developing and Researching PhET simulations for Teaching Quantum Mechanics'*, Am. J. of Physics, vol. 76 (4), 406 (2008).

22. N.D. Finkelstein,, W.K. Adams, C.J.Keller, P.B. Kohl, K.K. Perkins, N.F. Podolefsky and S. Reid, '*When learning about real world is better done virtually: A study of substituting computer simulations for laboratory equipment',* Phys. Rev. ST Phys. Educ. Research, vol. 1, 010103 (2005).

23. C. J. Keller, N. D. Finkelstein, K. K. Perkins and S. J. Pollok '*Assessing the effectiveness of a computer simulation in introductory undergraduate environments',* Physics Education Research Conference PER Conference series, vol. 883, 121 (2006).

24. P. S. Tambade1 and B. G. Wagh, *'Assessing the Effectiveness of Computer Assisted Instructions in Physics at Undergraduate Level'* Eurasian J. Phys. Chem. Educ., vol. 3 (2), 127 (2011).

25. A. T. Tadesse '*Effectiveness of computer simulations in physics teaching/learning: The effects of computer simulations on undergraduate students' physics achievement',* LAP Lambert Academic Publishing (2012).

26. Java: http://www.java.com.

27. B. Calandra, A.E Barron and I Thompson-Sellers) '*Audio Use in E-Learning: What, Why, When and How?',* Int. J. of E-Learning, vol. 7 (4), 589 (2008).

28. Gnumeric spreadsheet program: http://projects.gnome.org/gnumeric/

29. R. R. Hake, '*Interactive engagement vs. traditional methods: A six –thousand-student data of mechanics test data for introductory courses*', Am. J. of Physics, vol 66 (1), 64 (1998).

30. S. G. Thube And A.D. Shaligram, '*Effectiveness of Computer Assisted Teaching of Geometrical Optics at Undergraduate Level'*, Phys. Edu, January − March issue, 263 (2007).

31. G. P. Pimpale1 and R. V. Vadnera, '*Investigating the Effectiveness of Digital Interactive Multimedia Package in Astronomy to Promote Scientific Temper*', Phys. Edu., 28 (2), article no. 3 (2012).

32. Phoenix: Computer Interfaced Science Experiments, http://www.iuac.res.in/~elab/phoenix/

Relativistic time dilation: theoretically a reality associated only With the light clocks

Prasad Ravichandran

Undergraduate, Department of Physics
Loyola College Chennai, Madras University

(Submitted Nov 2012)

Abstract

In this article, a new version of Einstein's thought experiment reveals something which was never pondered about. Light clock used in thought experiment by Einstein which runs on light pulse is not an ideal clock to measure time in moving situations. A light motion analysis presented in article brings out a special peculiarity of light. "The speed of light is a constant, independent of the motion of the light source and all observers. Whereas the direction of light is dependent upon the motion of source and the observers" is the peculiar trait of light which gives time dilation effect in light clock used in thought experiment. Other clocks which don't run in light pule don't show this effect. There is no such peculiarity in machinery of the other clock that might cause the time dilation effect in thought experiments. Practical experiments are in good agreement with relativistic time dilation effect. The light clock thought experiment which gave time dilation concept itself is revealing something different in the new version of thought experiment followed by the light motion analysis.

1. Introduction

Newtonian mechanics was incapable of explaining the peculiar aspects of light and its motion. Albert Einstein framed STR to account for peculiar behavior of light. Einstein used one of the laws of Maxwell's electromagnetism which is

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

as a base for devising special theory of relativity. This law of electromagnetism was given representation as a postulate by Einstein in STR, which states that-

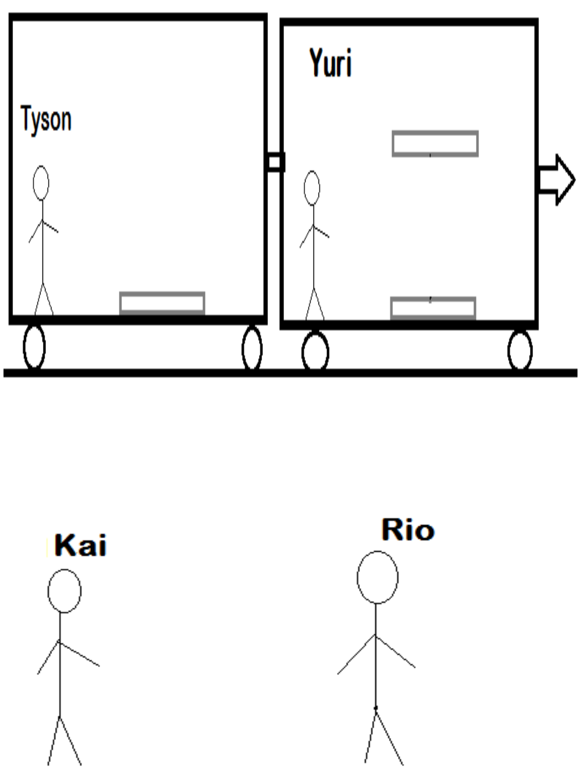
"The speed of light is a constant, independent of the motion of the light source and all observers"

Though this postulate seems silent it gave out breaking results with Einstein's thought experiment .In thought experiment Einstein considered a light clock in a moving train .A person in the train measures time Δt_0 for one tick. A stationary observer measures time Δt for one tick. The relation between Δt_0 and Δt came out as –

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

This is the famous time dilation equation. This time dilation equation and its further application brought together gave many results. If Einstein's thought experiment is seen in a different point of view it reveals something else. It reveals something which was never pondered about.

2. New version of Einstein's thought experiment

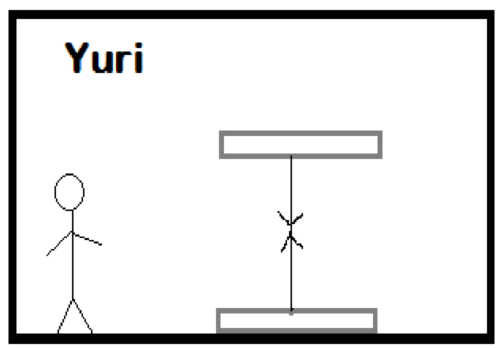


Consider a thought experiment. A train is moving in a track with velocity *v*.

It has two wagons and an engine. Yuri is standing in a wagon with light clock. (Light clock consists of two mirrors separated by a distance *l* between which a light pulse moves. Time for light to move from bottom mirror to top mirror and then back is one tick).Rio is a stationary observer who is watching this wagon. Tyson is standing on another wagon with ball clock. (Ball clock consists of machinery which ejects a ball upward with a velocity say α. Time taken by ball to go up and return to the machinery is one tick).Kai is a stationary observer who is watching this wagon.

<u>Yuri-Rio Observation</u>

1. From point of view of Yuri



Total distance travelled by light pulse= *l*+*l*

Speed of light pulse=*c*

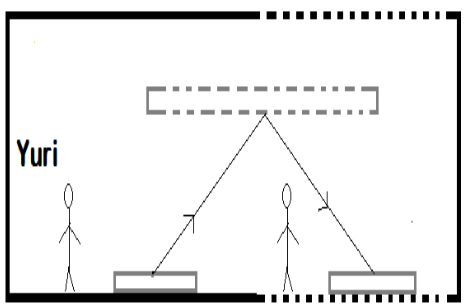
Time for one tick,

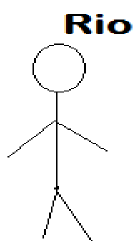
$$\Delta t_0 = \frac{total\ distance\ covered}{speed}$$

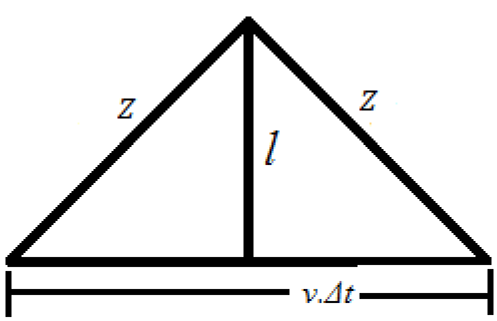
$$\Delta t_0 = \frac{l+l}{c}$$

$$= \frac{2l}{c} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

2. From point of view of Rio







Δt→time for one tick i.e., time taken to go from bottom mirror to top mirror and then back

$$Z^2 = l^2 + \left(\frac{v.\Delta t}{2}\right)^2 \ldots\ldots\ldots\ldots (2)$$

Light pulse travels 2Z at speed c in time Δt, it implies that

$$2Z = c.\Delta t$$
$$Z = c.\Delta t/2$$

Substituting this value in (2)

$$\left(\frac{C.\Delta t}{2}\right)^2 = l^2 + \left(\frac{v.\Delta t}{2}\right)^2$$

$$\frac{c^2.\Delta t^2}{4} = l^2 + \left(\frac{v^2.\Delta t^2}{4}\right)$$

$$\Delta t^2 = \frac{4l^2}{c^2} + \frac{v^2\Delta t^2}{c^2}$$

$$\Delta t^2 - \frac{v^2.\Delta t^2}{c^2} = \frac{4c^2}{c^2}$$

$$\Delta t^2 \left(1 - \frac{v^2}{c^2}\right) = \frac{4l^2}{c^2}$$

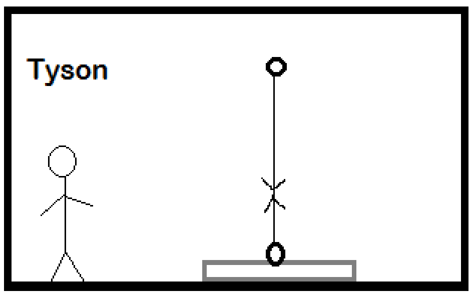
Substituting equation (1) in above

$$\Delta t^2 \left(1 - \frac{v^2}{c^2}\right) = (\Delta t_0)^2$$

$$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

<u>Tyson –Kai observation</u>

1. From point of view of Tyson



Initial velocity with which ball is projected by machinery=α

For half motion,
Initial velocity =α
Acceleration=-g
Final velocity=0

Using $v = u + at$

$$0 = \alpha + (-g)(t_{1/2})$$

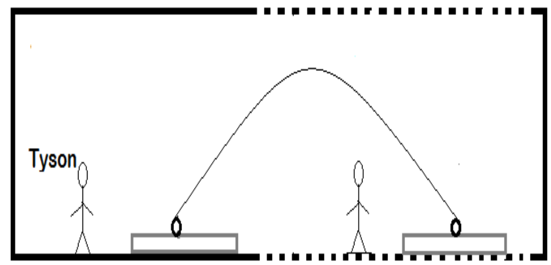
$$-\alpha = (-g)(t_{1/2})$$

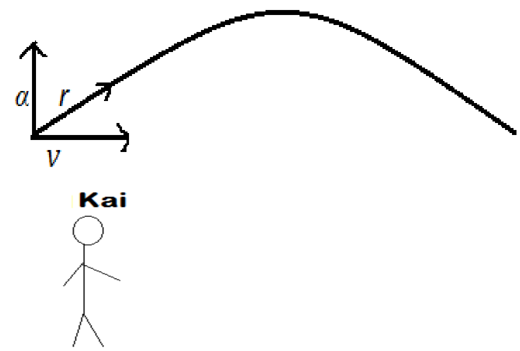
$$t_{1/2} = \frac{\alpha}{g}$$

Therefore, the time for one tick i.e. time to go up and then come back is

$$T_0 = \frac{2\alpha}{g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

2. From point of view of Kai





For Kai the ball moves in combined effect the of train's horizontal velocity and vertical velocity of ejection given by the machinery. $r = \sqrt{\alpha^2 + v^2}$ is the resultant velocity of ball in resultant direction. It is well known fact that whatever be the horizontal velocity of train it will only affect the horizontal displacement of ball whereas vertical velocity and thus the vertical height will remain unaffected. (Vertical velocity is the velocity, with which machinery projects the ball)

For half motion,
Initial vertical velocity of ball=α
(Because the horizontal velocity of train will affect only horizontal component not the vertical velocity of ball)

Acceleration=-g

Final vertical velocity of ball=0

Using $v = u + at$

$$0 = \alpha + (-g)(t_{1/2})$$

$$-\alpha = (-g)(t_{1/2})$$

$$t_{1/2} = \frac{\alpha}{g}$$

Therefore time for one tick i.e., time taken to attain height and come back is $T = \frac{2\alpha}{g}$………….. (4)

From (3) and (4) it is clear that,

$$T = \frac{2\alpha}{g} = T_0$$

$$T = T_0$$

Rio's time measurement and Yuri's time measurement for light clock are not the same. Yuri measured a smaller time of tick than Rio. It can be concluded that moving clock ticks slower. This is what Einstein proposed in his thought experiment using light clock.

Kai's time measurement and Tyson's time measurement for ball clock are the same. It can be concluded that moving clocks doesn't run slower.

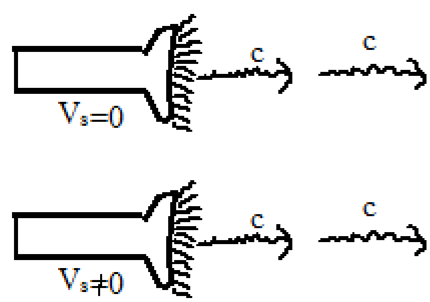
Which perception is right? Which perception is a reality? Was Einstein's thought experiment a failure?

Einstein's thought experiment is not a failure. Moving light clocks indeed get slower. Einstein was right. But the conclusion that since the light clocks get slower hence other clocks, physical clocks, biological clocks will also tick slower was a misconception. Actually time doesn't run slower with increasing speed, it is the light clock that ticks slower. Light is much more peculiar than what it is thought about. Light clock which runs on light pulse is not an ideal clock for measuring time in moving situations. The coming Light Motion Analysis will reveal the fact.

3 Light Motion Analysis

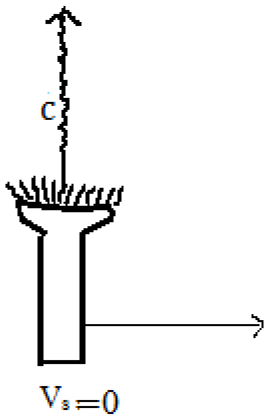
Postulate of STR states that-
"The speed of light is a constant, independent of the motion of the light source and all observers"

Whether the direction of motion of light depends upon the motion of source? Whether the direction of motion of light is affected by the motion of source? There is no description about the direction of light in the postulate. It can be claimed that direction of motion of light has no role to play. But that is not the case. When direction of motion of source and direction of ejection of light are same then obviously light will move in the same direction of ejection with speed c. The following illustration depicts that→

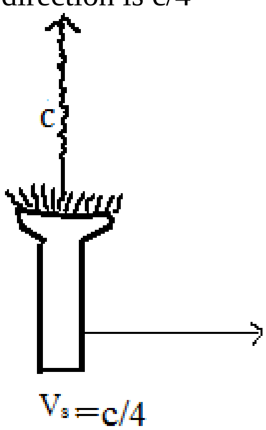


When the direction of motion of source is perpendicular to direction of ejection of light and if the direction of motion of light is not affected by source then whatever be the velocity of motion of source, light will always move in its direction of ejection. The following illustration depicts this→

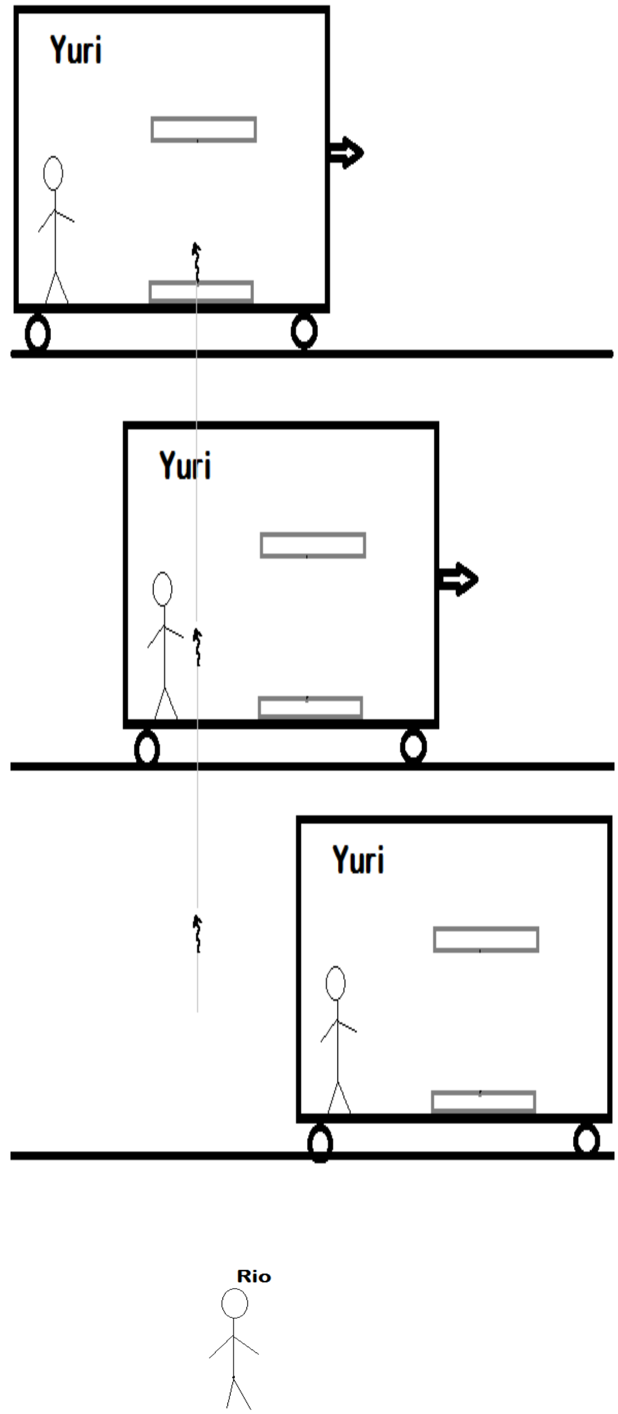
(a) Velocity of source in horizontal direction is zero



(b)Velocity of motion of source in horizontal direction is c/4



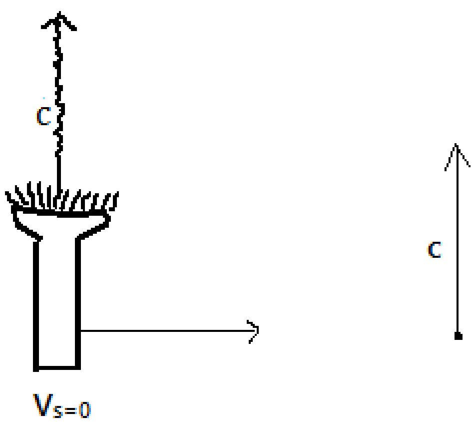
If the above consideration and illustration is right then its analogous condition where the light clock moves in horizontal direction with light pulse ejected in vertical direction will give the following scene to Rio (the stationary observer watching the wagon).Light irrespective of the motion of source will trace the ejected direction. The following illustration depicts such scenario→



But this scenario is not the truth. Rio observed something else. Rio didn't observe the light travelling in a vertical straight line path; instead he observed the light pulse travelling in an inclined straight line path. Hence it can be concluded that direction of motion of light is affected by the direction of motion of source.

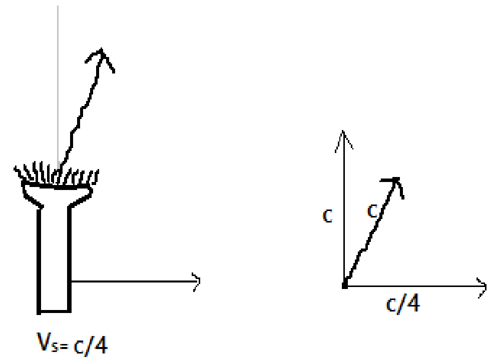
When the direction of motion of source is perpendicular to the direction of ejection of light, then light traces a direction which is resultant of the lights velocity(c) and velocity of source (V_s) but speed of light still remains c in this resultant direction. It's only the direction of light which gets affected by the motion of source but not the speed. The following illustration depicts this→

(a)Velocity of motion of source in horizontal direction is zero



Velocity of source is zero. Therefore the resultant direction of light pulse lies in direction of ejection of itself. Light pulse moves along the resultant direction with its usual speed.

(b) Velocity of motion of source is C/4 in horizontal direction



Source has velocity c/4 in horizontal direction; light has velocity of ejection c in vertical direction. The light pulse will trace a direction which is combined effect of c and c/4.It will move in resultant direction with its usual velocity c.

Direction of motion of light

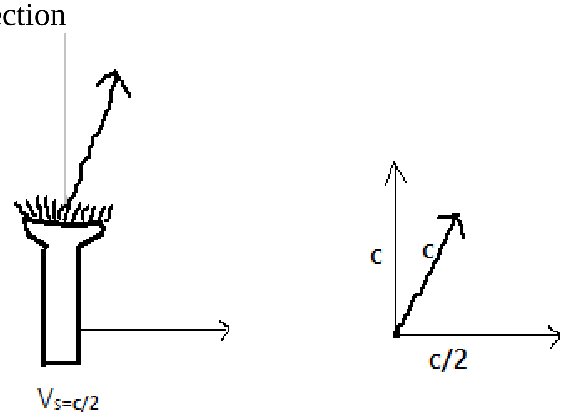
$$\theta = tan^{-1}\left(\frac{c}{c/4}\right)$$

$$\theta = tan^{-1}(4)$$

$$\theta = 75.9°$$

Velocity of motion of light=c

(c)Velocity of source is c/2 in horizontal direction



Light moves in the resultant direction with its usual speed. Direction of motion of light

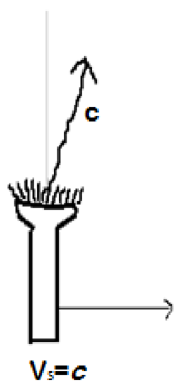
$$\theta = tan^{-1}\left(\frac{c}{c/2}\right)$$

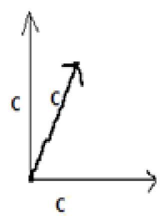
$$\theta = tan^{-1}(2\ )$$

$$\theta = 63.4°$$

Velocity of motion of light = *c*

(d)Velocity of motion of source is c in horizontal direction





Light travels in the resultant direction with its usual speed.

Direction of motion of light

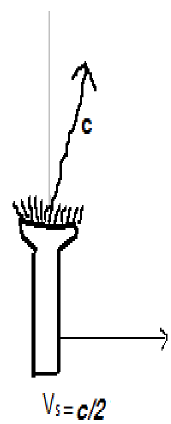
$$\theta = tan^{-1}\left(\frac{c}{c}\right) = 45°$$

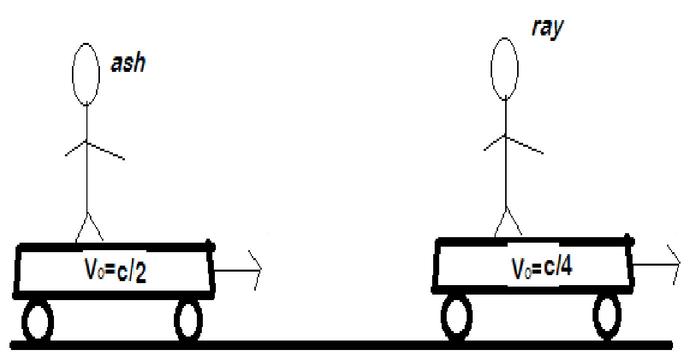
Velocity of motion of light =c
The direction of light is affected by the motion of light source. Hence direction of light is dependent upon the motion of light source.

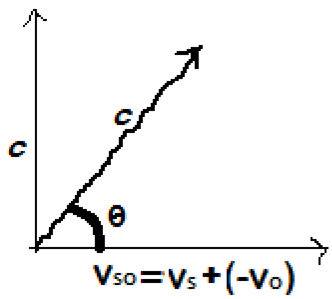
Whether the direction of light is dependent upon the motion of observers?

Yes, direction of light is dependent upon the motion of observers. Every observer will have different perception of lights direction depending upon their velocity with respect to the source. The following illustration depicts this→

Source is moving with velocity *c*/2 in horizontal direction and the observer Ray is moving at velocity *c/4* and observer Ash is moving at velocity c/2







(a)For Ray

Velocity of source with respect to Ray $= V_s - V_o = \frac{c}{2} - \frac{c}{4} = \frac{c}{4}$

Direction of motion of light,

$$\theta = tan^{-1}\left(\frac{c}{V_{SO}}\right)$$

$$\theta = tan^{-1}\left(\frac{c}{c/4}\right)$$

$$\theta = 75.96°$$

Velocity of motion of light=c

In Ray's frame light traces direction $\theta = 75.96°$ with its usual speed c

(b) For Ash

Velocity of source with respect to Ash

$$= V_s - V_o = \frac{c}{2} - \frac{c}{2} = 0$$

Direction of motion of light,

$$\theta = tan^{-1}\left(\frac{c}{V_{SO}}\right)$$

$$\theta = tan^{-1}\left(\frac{c}{0}\right)$$

$$\theta = 90°$$

,

Velocity of motion of light=c

In Ash's frame of reference light traces direction $\theta = 90°$ with its usual speed

The result of Light Motion Analysis is→

> "The speed of light is a constant, independent of the motion of the light source and all observers. Whereas the direction of light is dependent upon the motion of source and the observers."

In other words,

> "Every observer will have same speed of light irrespective of their velocity with respect to the light source. Whereas each one of them will have different perceptions of direction of light depending upon their velocity with respect to the light source."

4. Interpretation of thought experiment with the Light Motion Analysis Result

Light clock possess velocity in horizontal direction along with the train whereas light is ejected in vertical direction. The velocity of motion of source (light clock) along with train brings a change in the direction of light pulse. This change in direction of light pulse is observed by Rio (in his frame of reference light clock is in relative motion).The change in direction of light pulse cannot be observed by Yuri because in Yuri's frame of reference light clock is at relative rest. In Rio's frame of reference light travels an inclined straight line path with speed c. In Yuri's frame of reference light travels a vertical straight line path with speed c. Hence Rio observes longer time for light pulse to go from bottom mirror to top mirror and come back. Whereas Yuri observes smaller time for light pulse to go from bottom mirror to top mirror and come back. Overall, it appears that time runs slower to Yuri. Light moves in resultant direction with its usual speed.

Observers in every frame of reference will see different trajectory (direction) but same speed for light. Hence observers in every reference frame will have different time of tick than others.

Ball clock moves in horizontal direction along with train whereas the ball is of ejected in vertical direction. Velocity of motion of ball clock along with train changes the direction of motion of ball and also the resultant velocity of ejection of ball. This change is observed by Kai (in his reference frame ball clock is in relative motion).The change in direction of motion of ball and the change in velocity of ejection is not observed by Tyson because in his reference frame ball clock is at relative rest. Kai observes a projectile path of ball with velocity of ejection $\sqrt{\alpha^2 + v^2}$ .Tyson observes a vertical path of ball with velocity of ejection α.Kai observes a larger trajectory and larger velocity whereas Tyson observes a smaller trajectory and smaller velocity. Overall, time of tick for Kai and Tyson comes out as same. Ball moves in resultant direction with its resultant velocity. Observers in every frame of reference will see different trajectory and different velocity of ejection. Hence observers in every reference frame will have the same time of tick.

Hence it is clear that time dilation is just a peculiarity associated with light clocks which runs on light. "Motion of source or observer affects the direction of light but not the speed" is the peculiar trait of light which gives time dilation affect. In thought experiment, light clock which runs on light pulse is not an ideal clock to measure time in moving situations; relative motion does not affect any other clock (ball clock, physical clock) because they don't run on light pulse.

5. Conclusion

Light clock used in the thought experiment is not an ideal clock to measure time in moving situtations."The speed of light is a constant, independent of the motion of the light source and all observers. Whereas the direction of light is dependent upon the motion of source and the observers" is the peculiar trait of light which causes the moving light clocks to get slowed down. Other clocks which don't run in light pule don't show this effect. There is no such peculiarity in machinery of the other clock that might cause the time dilation effect in theoretical thought experiments, we simply cannot consider without any reason that other clocks will appear to run slow just like the light clock. Experiments like comparison of muon lifetimes at different speeds, measurement of the doppler shift from a source moving at right angles to the line of sight, flying atomic clocks in east and west direction etc. are in agreement with relativistic time dilation. The light clock thought experiment which gave time dilation concept itself is revealing something different in the new version of thought experiment followed by the light motion analysis.Theoretically it comes out that relativistic time dilation is mere a reality associated only with the light clocks though practical experiments reveal something else. Maybe the actual picture of relative reality is different from what it was thought.

References

[1] Nicholas J. Giordano, *College Physics: reasoning and relationships*, Brooks/Cole,Cengage Learning(USA)(2010),Page 919-920

[2] Halliday, Resnick and Walker, *Fundamentals of physics*,6th Edition,Wiley Publications, Page 920-945

de Broglie Wavelength and Frequency of Scattered Electrons in the Compton Effect

Vinay Venugopal* and Piyush S Bhagdikar

Division of Physics, School of Advanced Sciences
VIT University, Chennai Campus, Vandalur Kelambakkam Road
Chennai 600127, India
*Corresponding author. Email:vinayvenugopal@vit.ac.in

(*Submitted* 01-02-2013)

Abstract

The undergraduate courses on modern physics generally consider the particle interpretation of Compton Effect. Motivated by a student's solution in an undergraduate examination on modern physics here we consider the wave characteristics of recoiled electrons in the Compton Effect. The de Broglie wavelength, wave and clock frequency of the scattered electrons are expressed in terms of the wavelength and the frequency of the incident and the scattered photons respectively using the familiar particle interpretation of the Compton effect, where initially the electron is at rest and its spin is ignored. Both non-relativistic and relativistic cases are considered. Numerical values of de Broglie wavelength, wave and clock frequency of the scattered electron are calculated for an incident photon energy that was used in the original experiment of Compton as a function of the scattering angle of the recoiled electron. Considering the relativistic effects which are however insignificant for the de Broglie wavelength of the recoiled electron under these conditions, the minimum value obtained is in the range of X-rays. The non-relativistic de Broglie wave frequency obtained by neglecting the rest mass of the electron leads to an underestimation of its value. The implications of de Broglie wavelength and clock frequency for Compton scattering experiments are briefly discussed and possible extensions of the obtained mathematical formulations are indicated. The results are useful for understanding the wave-particle duality of the recoiled electron in the context of the Compton Effect.

1. Introduction

In several undergraduate texts on modern physics [1,2], the concept of the particle nature of wave (light) is introduced prior to that of wave nature of the particle (electron). The latter is demonstrated by the Davisson-Germer experiment. The former is illustrated using the Compton Effect where a photon collides with a stationary electron which is treated as a particle using the laboratory frame of reference and its spin is ignored. Rarely the concept of de Broglie waves are discussed in the context of Compton scattering. Several attempts have been made to obtain the expression for Compton shift by considering the interaction of electromagnetic waves with electron [3-5]. Schrödinger [4] considered the interaction classical electromagnetic waves with the de Broglie waves of the electron. Pedagogical exposition of Schrödinger's treatment was considered by Strand [6] and an approach similar to that of Schrödinger has been used by Su [7]. Compton Shift has also been explained as a double Doppler shift considering the interaction of electromagnetic wave train with electron [8]. But generally these are not discussed in the undergraduate courses on modern physics.

de Broglie when attempting to obtain the relativistic transformation of Planck-Einstein's equation [9] proposed three different frequencies to a particle with rest mass m_e (i) frequency of the internal energy of the particle at rest $\nu_C = m_e c^2 / h$ (ii) frequency of the total energy of the particle as measured by a 'fixed' observer: $\nu_{dB} = \gamma m_e c^2 / h$, where γ is the Lorentz factor (iii) 'internal periodic phenomenon' /clock frequency as measured by a 'fixed' observer: $\nu_{cl} = \nu_C / \gamma$. Recently there is a renewed interest [10-13] to understand the internal clock frequency of the electron. The experiments of Catillon *et al.* [11], aimed at detecting the clock frequency of the electron in terms of interactions electrons with atoms indicate that one needs to consider the internal motion of the centre of charge of the electron around its centre of mass [10] with a frequency twice that of ν_C corresponding to the frequency of internal motion of the electron (zitterbewegung proposed by Schrödinger) used by Dirac.

This paper was motivated by a student's (the second author) solution to the problem posed in an undergraduate modern physics examination at VIT University, Chennai Campus, Chennai (August, 2011) on calculating the de Broglie wavelength of recoiled electrons in the Compton effect. The de Broglie wavelength and the de Broglie wave and clock frequency of the scattered electrons are generally expressed in terms of relativistic velocity of the recoiled electrons. Here we obtain these parameters in terms of the wavelength and frequency of the incident and the scattered photons. The familiar expressions for the conservation of energy and momentum that describe the Compton scattering by considering the particle nature of photon and electron (initially at rest and ignoring its spin) are used. Both non-relativistic and relativistic cases are considered. Considering an incident photon energy that was used in the original experiment of Compton [13], numerical values of the de Broglie wavelength, wave and clock frequency are obtained. The implications of de Broglie wavelength and clock frequency for Compton scattering experiments are briefly discussed and possible extensions of the obtained mathematical formulations are indicated. Hence these results can be effective for understanding the wave-particle duality of the recoiled electrons in the context of the Compton effect.

2. Results and Discussion

We obtain the wave characteristics of the scattered electrons from the familiar Compton scattering mechanics where the photon and the electron are treated as particles in the frame work of special theory of relativity. The electron before the collision is considered to be free (valid when the binding energy of the electron is negligible compared to the energy of the incident photon) and at rest in the laboratory frame of reference, and its spin is ignored. Figure 1 shows the geometry of Compton scattering in terms of the angles of the scattered photon (ϕ) and the electron (θ) respectively with respect to the direction of the incident photon.

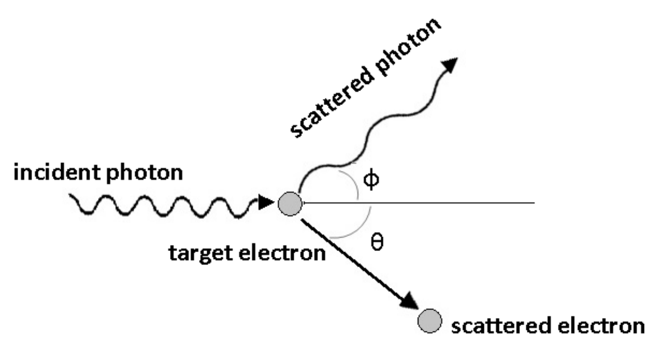


FIG. 1: The geometry of Compton scattering showing the directions of the scattered photon and electron with respect to the direction of the incident photon.

The relativistic de Broglie wavelength of the scattered electron can be obtained by applying the conservation of energy (Fig. 1),

$$p_{ph}c + m_e c^2 = \left(m_e{}^2 c^4 + p_e'^2 c^2\right)^{1/2} + p_{ph}{}' c \qquad (1)$$

where p_{ph} and $p_{ph}{}'$ are the momenta of the incident and the scattered photon respectively, $p_e{}'$ is the relativistic momentum of the scattered electron, m_e is the rest mass of the electron and c is the velocity of light. Substituting $p_{ph} = h/\lambda$, $p_{ph}{}' = h/\lambda'$ and $p_e{}' = h/\lambda_{dB}^R$ (where h is the Planck's constant) respectively in Eq. (1),

$$\lambda_{dB}^R = \left(\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)^2 + \frac{2}{\lambda_C}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)\right)^{-1/2} \qquad (2)$$

where λ_{dB}^R is the relativistic de Broglie wavelength of the scattered electron, λ and λ' are the wavelength of the incident and scattered photon respectively, $\lambda_C = h/m_e c$ is the Compton wavelength. A primitive form of this equation was derived by the student in the examination. A similar expression for the relativistic de Broglie wavelength of the scattered electron can be obtained using the conservation of momentum (Fig. 1)

$$\vec{p}_{ph} - \vec{p}_{ph}{}' = \vec{p}_e{}' \qquad (3)$$

Taking the self vector dot product and substituting $p_{ph} = h/\lambda$, $p_{ph}{}' = h/\lambda'$ and $p_e{}' = h/\lambda_{dB}^R$,

$$\lambda_{dB}^R = \left(\frac{1}{\lambda^2} + \frac{1}{\lambda'^2} - \frac{2\cos\phi}{\lambda\lambda'}\right)^{-1/2} \qquad (4)$$

where ϕ is the angle between the incident and the scattered photon. The de Broglie wavelength in Eq. (4) can be expressed in terms of the angle between the directions of the scattered electron with respect to the incident photon (θ) using the well known relation,

$$\phi = 2\cot^{-1}\left[\left(1 + \frac{\nu}{\nu_C}\right)\tan\theta\right] \qquad (5)$$

where $\nu_C = m_e c^2/h$.

Under the non-relativistic kinematics,

$$h(\nu - \nu') = \frac{1}{2}mV^2 \qquad (6)$$

where ν and ν' are the frequencies of incident and scattered photons respectively, and V is the velocity of the scattered electron. Hence the non-relativistic de Broglie wavelength of the scattered electron is

$$\lambda_{dB}^{NR} = \left(\frac{2}{\lambda_C}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)\right)^{-1/2} \qquad (7)$$

The de Broglie wave frequency, associated with an electron is

$$\nu_{dB}^R = \frac{\gamma m_e c^2}{h} = \gamma\nu_C \qquad (8)$$

where γ is the Lorentz factor. Hence $\nu_C = m_e c^2/h$ with a value of 1.2356×10^{20} Hz is the rest frequency that was identified by de Broglie with the internal clock of the electron. A non-relativistic expression for the de Broglie wave frequency is obtained by neglecting the rest mass of the electron and treating the scattered electron as a free particle. Hence mc^2 in Eq. (8) is replaced by the kinetic energy $E_k = p^2/2m$, where p is the non-relativistic momentum of the scattered electron. Thus Eq. (8) becomes

$$\nu_{dB}^{NR} = \frac{p^2}{2mh} \qquad (9)$$

Using $p = h/\lambda_{dB}^{NR}$ and substituting for λ_{dB}^{NR} from Eq. (7) in Eq. (9),

$$\nu_{dB}^{NR} = \nu - \nu' \qquad (10)$$

The expression for the relativistic de Broglie wave frequency in terms of the frequency of the incident and scattered photon is obtained starting from

$$\nu_{dB} = \frac{V_p}{\lambda_{dB}} \qquad (11)$$

where V_p is the phase velocity of the de Broglie waves associated with the scattered electron. Substituting for λ_{dB} from Eq. (2), using $V_p = c^2/V$ and expressing V in terms of ν and ν' using the relativistic kinetic energy of the scattered electron ($K = h(\nu - \nu')$), Eq. (11) becomes

$$\nu_{dB}^{R} = c\left(1 - \left(\frac{1}{\left(1 + \frac{(\nu - \nu')}{\nu_C}\right)^2}\right)\right)^{-1/2} \left(\frac{1}{c^2}(\nu - \nu')^2 + \frac{2m_e}{h}(\nu - \nu')\right)^{1/2} \qquad (12)$$

Similarly using Eq. (4),

$$\nu_{dB}^{R} = \left(1 - \left(\frac{1}{\left(1 + \frac{(\nu - \nu')}{\nu_C}\right)^2}\right)\right)^{-1/2} \left(\nu^2 + \nu'^2 - 2\nu\nu'\cos\phi\right)^{1/2} \qquad (13)$$

The internal clock frequency of the scattered electron as measured by an observer in the laboratory frame is

$$\nu_{cl} = \frac{\nu_{dB}^{R}}{\gamma^2} = \frac{\nu_{dB}^{R}}{\left(1 + \frac{(\nu - \nu')}{\nu_C}\right)^2} \qquad (14)$$

where one should substitute for ν_{dB}^{R} using Eq. (12) or (13).Note that λ' in Eq. (2), (4) and (7) can be expressed in terms of λ and ϕ (or θ) while ν' in Eq. (12), (13) and (14) can be expressed in terms of ν and ϕ (or θ). Table I shows the calculated de Broglie wavelength (from Eq. (7) and (4)), de Broglie wave frequency (from Eq. (10) and (12)), and de Broglie clock frequency (from Eq. (14)) for an incident photon of wavelength 0.707831 Å using MATLAB R2008b. This wavelength was chosen for the purpose of illustration, since Compton in his original experiment [14] used MoKα_1 X-ray source. For comparison, the value of ν_C is shown in the last column. The variation of λ_{dB}^{R}, ν_{dB}^{R}, and ν_{cl} as a function of θ for an incident photon of wavelength $\lambda = 0.707831$ Å are shown in Fig. 2 obtained from Eq. (4), (12) and (14) respectively plotted using MATLAB R2008b.

TABLE I. The calculated de Broglie wavelength, wave frequency and clock frequency of the scattered electron for an incident photon of wavelength $\lambda = 0.707831$ Å at different angles of the scattered photon ($\phi = 45°, 90°, 135°, 180°$). The values of the constants c, h, m_e, λ_C , and the wavelength of $\mathrm{MoK}\alpha_1$ radiation were obtained from the NIST website.[15]

θ (°)	λ_{dB}^{NR} (Å)[a]	λ_{dB}^{R} (Å)[b]	ν_{dB}^{NR} ($\times 10^{17}$ Hz)[c]	ν_{dB}^{R} ($\times 10^{20}$ Hz)[d]	ν_{cl} ($\times 10^{20}$ Hz)[e]	ν_{C} ($\times 10^{20}$ Hz)[f]
0	0.3658	0.3656	2.7173	1.2383073	1. 2328786	1.2355899
21.82	0.3941	0.3939	2.3414	1.2379313	1. 2332530	1.2355899
44.03	0.5090	0.5089	1.4037	1.2369936	1.2341878	1.2355899
66.81	0.9295	0.9294	0.4210	1.2360110	1.2351692	1.2355899

[a]Using (7), [b]Using (4), [c]Using (10), [d]Using (12), [e]Using (14), [f]m_0c^2/h

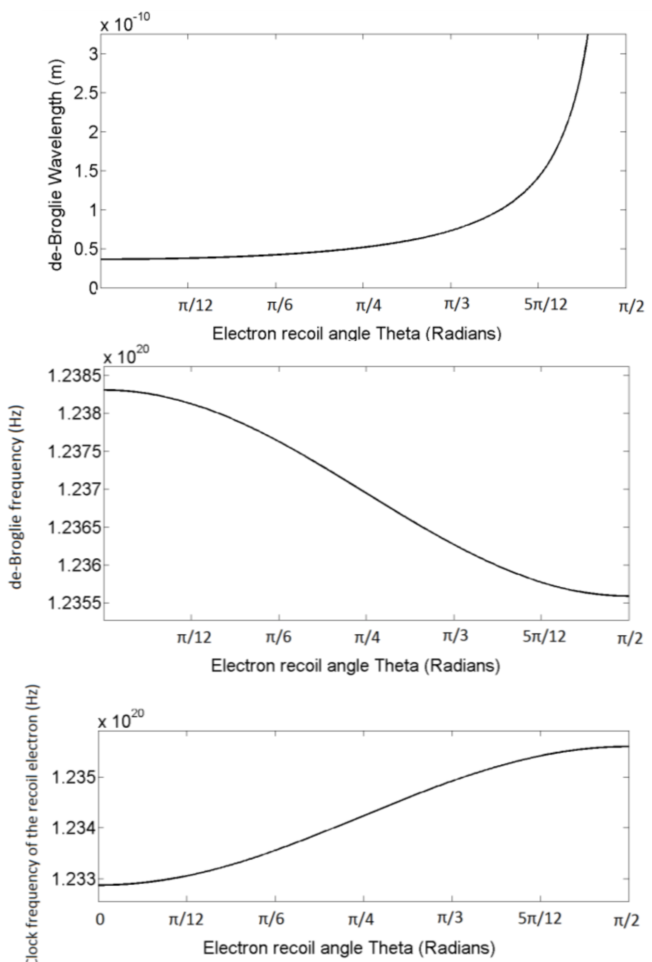


FIG. 2. The variation of (a) de Broglie wavelength (b) de Broglie wave frequency (c) de Broglie clock frequency of the scattered electron as a function of (a) scattering angle of the recoiled electron (θ) using Eq. (4), (12), and (14) respectively for an incident photon of wavelength $\lambda = 0.707831$ Å.

The calculated de Broglie wavelength of the recoiled electron in columns 2 and 3 of Table I, increases with the increasing angle of the scattered electron since the velocity of the recoiled electron is maximum at $\theta = 0$ and minimum at $\theta = \pi/2$. The value of the de Broglie wavelength in column 3 (also see Fig. 2 (a)) are lower than those of column 2 for a given θ indicative of relativistic effects. The asymptotic behavior of de Broglie wavelength (Fig. 2 (a)) close to $\theta = \pi/2$ is the result of assuming free electron at rest before collision. It is clear that under the conditions considered in Table I the relativistic effect on the de Broglie wavelength of the recoiled electron is not very significant since the maximum kinetic energy of the scattered electron under the assumed conditions (1.1238 keV) is small compared to its rest mass energy (511 keV). However at higher energies such as 0.1-1 MeV of incident photon, the relativistic effects cannot be ignored. The minimum de Broglie wavelength obtained in Table I (365.6 pm) is comparable to the wavelength of X-rays. Hence when the scattering medium is a crystalline solid the recoiled electrons under suitable conditions can undergo diffraction. de Broglie wavelength of recoiled electron has significance in Compton scattering experiments. A Compton resonance process has been reported where the recoiled electrons in Compton scattering resonate with the Si (111) crystal [16]. The de Broglie wavelength in a direction normal to the lattice plane is considered [16] for obtaining such resonance condition.

The de Broglie wave frequency of the scattered electron obtained by neglecting the rest mass of the electron (Eq. (10)) although consistent with literature [17] leads to an underestimation of the de Broglie wave frequency (see column 4 in Table I), since $\nu_{dB} = \nu_c$. For the relativistic case (column 5, Table I), the de Broglie wave frequency of the scattered electron decreases with respect to θ (see Fig. 2 (b)). However the change is not very significant for the input parameters considered here ($(V/c)_{max}$=0.0662). The de Broglie's clock frequencies (ν_{cl}) of the scattered electron obtained from Eq. (14) are shown in column 6 of Table I. The clock frequency increases with increase of θ (see Fig. 2 (c)) again indicative of relativistic effects. The measurement of the internal clock frequency of the electron (ν_C) using channeling of 80 MeV electron beam in silicon crystal has been reported [11]. The maximum kinetic energy of the scattered electron (1.1238 keV) under the conditions considered in Table I is highly insufficient for such channeling experiments. However the differential Klien-Nishina cross section $d_e\sigma/d\Omega_\theta$ as a function of θ indicates that at high incident photon energies (> 10 MeV) the recoiled electrons are very strongly forwarded directed along $\theta = 0$ and hence the possibilities of channeling of recoiled electrons at high incident photon energies (≥ 80 MeV) in crystalline scattering medium can to be explored.

The formulations mentioned in Eq. (2) and (14) can be useful to determine the de Broglie wavelength and frequency of the recoiled electrons while considering a guided photon [18] and when considering the effect of refractive index of the scattering medium on Compton scattering [19]. In addition these formulations can be extended to bound electron and when including the spin of the electron.

Teachers while evaluating students in the university examination must be receptive to new ideas or interpretations that can arise from student's answers/solutions. In an examination situation the student is forced to give an answer in a limited time. According to the experience of the first author of this paper who is involved in teaching physics and evaluating students, creativity of a student may be unknowingly triggered by the pressure to score good grades.

3. Conclusions

The de Broglie wavelength, wave and clock frequency of the scattered electron in the Compton effect were obtained in terms of the wavelength and frequency of the incident and the scattered photon respectively. The numerical values of these parameters for wavelength of incident photon that was used in the original experiment of Compton as a function of the scattering angle of the recoiled electrons indicate that (i) the relativistic effects for de Broglie wavelength is insignificant (ii) the minimum value of the de Broglie wavelength of the scattered electron calculated is in the range of wavelength of X-rays and hence under suitable conditions these recoiled electrons can undergo diffraction with the crystalline scattering medium (iii) the de Broglie wave frequency obtained by neglecting the rest mass of the electron leads to underestimation of its value. The relevance of the de Broglie wavelength and clock frequency of the recoiled electrons in Compton scattering experiments and future extensions of the obtained mathematical formulations were briefly highlighted.

Acknowlegements

The author (Dr.Vinay Venugopal) thanks VIT University, Chennai Campus, Chennai for providing a stimulating environment for innovative teaching and research infrastructure, and Dr. A. A. Samuel, Pro-Vice Chancellor for his encouragement. This work was presented as an oral contribution at the World Conference on Physics Education (WCPE)-2012, Istanbul, Turkey; made possible with the financial assistance from VIT University, Indian National Science Academy (INSA), New Delhi, and IUPAP travel grant.

References :

[1] A. Beiser, S. Mahajan, and S. R. Choudhury, *Concepts of Modern Physics* (Sixth Edition, Special Indian Edition, Delhi: Tata McGraw-Hill 2009).
[2] P. A. Tipler & R. A. Llewellyn, *Modern Physics* (Sixth Edition San Francisco: W. H. Freeman & Co. 2012).
[3] C. Eckart, Phys. Rev. **24**, 591 (1924).
[4] E. Schrödinger, Ann. Phys. **28,** 257-264 (1927).
[5] C. V. Raman, Indian J. Phys. **3**, 357 (1928).
[6] J. Strnad, Eur. J. Phys. **7**, 217 (1986).
[7] C. C. Su, Preprint physics/050621v3 (2006).
[8] See for example, R. Kidd, J. Ardini & A. Anton, Am. J. Phys., **53**, 641 (1985).
[9] L. de Broglie, C.R. Acad. Sci, **177**, 507 (1923).
[10] M. Rivas, Preprint physics/0809.3635 (2008).
[11] P. Catillon, N. Cue, M. J. Gaillard, R.Genre, M. Gouanère, R. G. Kirsch, J. –C. Poizat, J. Remillieux, L. Roussel, M. Spighel, Found. Phys. **38**, 659-664 (2008).
[12] D. Hestenes, Found. Phys. **40**, 1-54 (2010).
[13] G. R. Osche, Annales de la Fondation Louis de Broglie, **36**, 61-71 (2011).
[14] A. H. Compton, Phys. Rev. **21**, 483-502 (1923).
[15] See http://physics.nist.gov/cuu/Constants/
[16] K. D. Gupta, Phys. Rev. Lett., **13**, 338-340 (1964).
[17] R. Heyrovská, Preprint physics/040104 (2004).
[18] G. R. Osche, Physics Essays, **21**, 260-265 (2008).
[19] S. G. Chefranov, Preprint physics/1205.3774 (2012).

On Bound State of Cooper Pairs in Superconductors

Debnarayan Jana

Department of Physics
University of Calcutta
92 A PC Road, Kolkata- 700009, India.
djphy@caluniv.ac.in

(Submitted Sep 2012)

Abstract

In this pedological article, we discuss about the boundedness of two electrons above the Fermi sphere in three dimensions interacting via an attractive interaction. These two electrons having equal opposite moneta and spin are known as Cooper pairs. According to the spin content of this composite particle, they behave approximately as Bosons and the pairs are essential ingredient to theory of superconductivity. We compute the bound state of energy of the pair when the electrons interact via (i) constant potential (ii) potential separable in two coordinates and (iii) delta function potential above the spherical Fermi surface. With the help of the eigenstate, we proceed further to compute the average radius of the Cooper pair and discuss its implication in theory of superconductivity. We also generalize the form of binding energy of Cooper pair in case of ellipsoidal Fermi surface.

Keywords: Cooper Pairs, Bound State, Fermi Surface, Superconductivity

1.Introduction

In nature, all particles can be classified into Bosons and Fermions according to their statistics. Electrons, protons and neutrons are all Fermions. An atom, which contains all three can also be treated as single (composite) particle. Whether the composite is Bosonic or Fermionic depends on the total number of its constituents. For example, He^4 contains two electrons, two protons and two neutrons and thus, it is a Boson. But, the isotope He^3 is a Fermion. Fermions obey the Pauli exclusion principle while there is no such restriction on Bosons. As a result, a collection of Bosons behaves quite differently from a collection of Fermions. A good example is the dramatic difference between a superconductor and an ordinary metal. The electrical conductivity in ordinary metals can be understood in terms of the properties of Fermions (i.e. electrons); in contrast, superconductivity in terms of Cooper pairs which are Boson-like. Moreover, the response of spinless Bosons in an external magnetic field is diamagnetic [1] while the Fermions show paramagnetic behaviour.

Cooper pair consists of two electrons with equal and opposite momenta and spins. In other words, they have zero total momentum. At first instance, one might wonder that they will violate Heisenberg uncertainty relation. It is worthy to remember that the zero expectation value of momentum does not mean a zero uncertainty of momentum. In fact, there is indeed substantial amount of fluctuation of momentum of these pairs which makes the non-zero uncertainty of the quantity. Moreover, we will explicitly use the uncertainty relation to estimate the size of the Cooper pair. Note that the electrons are fermions having spin $\frac{1}{2}$ (in units of $\hbar$) while Cooper pairs are composite Bosons as its total spin is integer (0 or 1). The wave functions are symmetric under particle interchange and they are allowed to be in the same state. The tendency for all the Cooper pairs in a system is to condense into the same ground state as they are Bosons. This fact is qualitatively responsible for the peculiar properties of the superconductivity. In normal superconductors, Cooper pairs have zero spin while for He^3 system, the spin of the pair is one (in units of $\hbar$). Here, the pairs are formed by two He^3 atoms. The pairs in He^3 is responsible for the superfluidity phenomenon at low enough temperature. In case of normal superconductors, the spins are paired in such a way that the magnetic moment of the electrons cancels and the contribution of the pair to the magnetic properties will in general be a diamagnetic one. This is reflected by the Meissner effect [2, 3] seen in the superconductor.

If these two electrons interact via an attractive potential, above the filled Fermi surface, will they form a bound state? This problem was attacked [4] by L. N. Cooper in 1956 and after his name, this is known as Cooper pairing problem. This problem is a simple two particle problem in quantum mechanics that can reduced effectively to a single particle because of the nature of potential function. However, in contrast to normal hydrogen atom problem, there is another statistical interaction through the Pauli exclusion principle to the filled Fermi sea. By considering this statistical interaction, Cooper showed exactly that these electrons do form a stable bound state and the *normal* Fermi surface becomes unstable. Later on Bardeen, Cooper and Schrieffer [5] used these collection of Cooper pairs to form the stable bound state of the superconductor and showed a gap between the metal and superconducting one. This theory is known as BCS one. The experimental foundations of the so called BCS theory has been beautifully demonstrated by Ginsberg [6]. At the same time, Anderson

[7] also developed another theory based on Pseudo-spin analogy to obtain a finite gap at finite non-zero temperature between metal and superconductor. A lucid account of this feature [8] has been sketched for the graduate students. Apart from normal superconductors, BCS theory is also applicable to other fermionic system such as He^3. Immediately after the tremendous success of BCS theory, Bohr, Mottelson and Pines [9] used the BCS pairing analogy to explain the large gaps in the spectra of even-even nuclei. Thus, BCS theory has been widely used to describe superconductivity in condensed matter and nuclear systems. The nuclear BCS theory has been reviewed recently by Broglia [10]. Besides, the spin pairing of the electrons essentially influences the paramagnetic susceptibility. In fact, the paramagnetic susceptibility should decrease towards zero with temperature $T \to 0$. The concept of Cooper pairs have been used in discussing the neutrino emission in relation with neutron stars [20]. The anisotropic d-wave Cooper pair wave functions in high T_c superconductors has been considered [21] to explain the spin fluctuations in cuprates.

The question next comes to our mind is: what is the reason of studying these pairs? The formation of pairs is indeed surprising. Because two particles of equal charge should repel each other. But as we have mentioned only the attraction between the electrons will force them to bind them. Therefore, naturally one is eager to know the origin of attraction between these electrons to form Cooper pairs. It is to be noted that two free electrons in a vacuum/free space always repel each other. Thus, it is the medium which is responsible for such an attraction between them. The electrons under consideration are in a dielectric medium of the solid characterized by a dynamic dielectric constant $\epsilon(\omega)$. Under an appropriate condition on the frequency ω, the dielectric constant $\epsilon(\omega)$) can change from a positive value to a negative one making the interacation to be negative. In our modern condensed matter physics, it is the quantized unit of lattice vibration known as *phonon* which is responsible for such an effective attractive interaction between the electrons. It is worthy to mention that like photons, phonons are bosons with zero chemical potential. Unlike photons, phonons have transverse polarization and the wave vectors associated with them are restricted to Brillouin zone only.

When an electron passes through the solid, on account of its negative charge, leaves behind a deformation trail affecting the position of the ion cores . This trail is associated with an increased density of positive charge due to the ion cores, and thus has an attractive effect on a second electron. Therefore, the lattice deformation causes a weak attraction between the pair of electron. Thus, the balance between of electron-phonon interaction [2, 3, 12, 13, 14, 16] and the Coulomb interaction in a material crucially determines whether a given material is superconducting or not. Since, electrons possess both charge as well as spin, during its motion through metal, the effective Coulomb interaction is screened dynamically[2, 15] as

$$V_{\vec{q}} = \frac{4\pi e^2}{\epsilon(\vec{q},\omega) q^2} \tag{1}$$

However, with rearrangement of the electrons due to phonon interaction, the effective interaction [2, 15] reduces to

$$V_{\vec{q}}^{eff} = \frac{4\pi e^2}{q^2 + k_{TF}^2} + \frac{4\pi e^2}{q^2 + k_{TF}^2} \frac{\omega_q^2}{\omega^2 - \omega_q^2} \tag{2}$$

Here, $\vec{q} = \vec{k} - \vec{k'}, \hbar\omega = E_k - E'_k$ with the screening length $\lambda_{TF} = k_{TF}^{-1}$ depending on the density of electrons and the Fermi energy. ω_q describes the spectrum of phonons with dispersion relation $\omega_q = sq$ at long wavelength limit. In the limit $\omega < \omega_q$, the effective interaction becomes negative while in the static limit, the interaction reduces to zero. A schematic view of the formation of Cooper pairs is shown in figure 1. Thus, this interaction is of dynamic nature only. Alternatively, a simple physical intuitive level explanation has been provided for this attractive interaction between two electrons above the Fermi surface by Weisskopf [17]. In fact, it has been pointed out that the motion of Cooper pairs is similar to that of the two nucleons in a deuteron or of the two electrons in the ground state of positronium.

In this pedagogical article, we would like to discuss the many facets of Cooper pairs as follows. In the next section, we briefly explain the meaning of bound state in quantum mechanics. In section 3 , we state the Cooper pairing problem in spherical Fermi surface and solve the bound state for various interactions between the electrons. With the help of the eigenfunctions we also compute the average radius of the Cooper pair in section 4. We then generalize in section 5, the bound state problem to ellipsoidal Fermi surface. We give our conclusions in section 6.

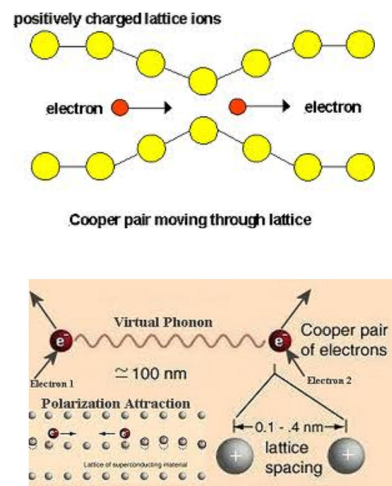


Figure 1: Formation of Cooper pairs inside the lattice.

2.Bound State Problem in Quantum Mechanics

Classically, it is known that the bound states can exist any time whenever there is a local (or global) minimum in the potential energy. However, a local minimum is insufficient to create a bound state in quantum mechanics. But, in quantum mechanics a global minimum is necessary to allow the existence of bound states. The classical bound states can exist for any value of total energy which is low enough to keep the particle in the minimum. However, this is not the case in quantum mechanics.

In physics, a bound state generally describes a system where a particle is subject to a potential such that the particle has a tendency to remain localised in one or more regions of space. The potential may be either an external potential, or may be the result of the presence of another particle.

In quantum mechanics with the conservation of number of particles, a bound state is a state (of course in Hilbert space) that corresponds to two or more particles whose interaction energy is less than the total energy of each individual particle, and therefore these particles cannot be separated unless this amount of energy is spent. The energy spectrum of a bound state is discrete, unlike the continuous spectrum of isolated particles. One may have unstable bound states with a positive interaction energy provided that there is an *energy barrier* that has to be tunnelled through in order to decay. In general, a stable bound state is said to exist in a given potential of some dimension if stationary wave functions exist (normalized in the range of the potential). The energies of these wave functions are negative. Bound state implies the *classically expected* state, so bound state energy should be smaller than $V(x \to \pm\infty)$ or $V(r \to \infty)$.

Let us illustrate some examples of the bound states. A proton and an electron can move separately and their the total center-of-mass energy is positive, and such a pair of particles can be described as an ionized atom. However, when the electron starts to orbit around the proton, the energy becomes negative, and a bound state so called the hydrogen atom is formed. Only the lowest energy bound state, the ground state is stable. The other excited states are unstable and will decay naturally to bound states with less energy by emitting a photon. A nucleus is a bound state of protons and neutrons (nucleons). A positronium *atom* is an unstable bound state of an electron and a positron. It decays into photons. The proton itself is a bound state of three quarks (two up and one down; one red, one green and one blue). However, unlike the case of the hydrogen atom, the individual quarks can never be isolated.

Any well-behaved eigenfunction ψ of a discrete spectrum should satisfy $\int |\psi|^2 \ d^3r = 1$. The integral is taken over all space. This implies immediately that $|\psi|^2$ must decrease rapidly and approaching zero at infinity. This indicates alternatively that the probabilities of infinite coordinates is rather zero. Thus, the system executes a finite motion and hence, is said to be a *bound state*. For a continuous one, the integral $\int |\psi|^2 \ d^3r$ diverges because $|\psi|^2$ does not tend to zero at infinity. In such a situation, the motion is said to be extended over the infinite space. In solving the Schroedinger equation for a given problem, there are two types of solutions for positive values as well as negative values of the energy E. The negative energy solution $(E < 0)$ is bound one while the other one is termed as scattering state or unbounded state. For harmonic oscillator, the potential

$V(x) \to \infty$ as $x \to \infty$ while for the hydrogen atom $V(r) \to 0$ as $r \to \infty$. However, in both cases, there is bound state. One has to be careful to think whether the motion is confined or not in a given potential. In this respect, there is a remarkable theorem in one dimension about the bound state.

The statement is : *In one dimensional system, there is always a bound state for any attractive potential*. Thus, the potential under consideration have the the following properties; $V(x) \to 0$ as $x \to 0$ and $\int_{-\infty}^{\infty} V(x)\, dx < 0$. Under this criteria, we are looking for a variational estimation of the Hamiltonian

$$H = \frac{p^2}{2m} + V(x) \tag{3}$$

for wave functions

$$\psi(x) = N \exp(-\alpha^2 x^2) \tag{4}$$

$$\psi(x) = N/\cosh(\alpha x) \tag{5}$$

Both these wave functions are normalizable and go to zero as $x \to \pm\infty$. Dimensional analysis [18, 19] can be used to estimate the contribution of the bound state energy eigen values of bound state problem and other related problems. A simple dimensional analysis yields that $N^2 = C\alpha$ with C being a dimensionless quantity and $[\alpha] = M^0 L^{-1} T^0$. It is easy to notice that values of α control the character of the wave function. For example, in the limit of the small values of α, the wave function is very much spread over the space and the state is regarded as a weakly bound. With the help of the wave function, we can now estimate the potential and the kinetic energy of the above Hamiltonian. For small value of α,

$$\begin{aligned} \langle V \rangle &= \int_{-\infty}^{\infty} |\psi|^2\, V(x)\, dx \\ &\approx |\psi(0)|^2 \int_{-\infty}^{\infty} V(x)\, dx \\ &= -CW\alpha \end{aligned} \tag{6}$$

where $W = \int_{-\infty}^{\infty} V(x)\, dx > 0$. Again, on dimensional ground, the kinetic energy can be estimated as $\left\langle \frac{p^2}{2m} \right\rangle = Const. \left(\frac{\alpha^2 \hbar^2}{2m} \right) = D\alpha^2$. Thus,

$$\langle H \rangle = D\alpha^2 - CW\alpha \tag{7}$$

No matter how small V is, we can have $\langle H \rangle < 0$ for small value α. The minimum value of $\langle H \rangle$ is $-\frac{C^2 W^2}{4D}$ for $\alpha = \frac{CW}{2D}$. Essentially, the magnitude of the bound state depends on the shape rather than its strength. The variational proof of the existence of at least one bound state in 1d is firmly rooted to one dimension as shown explicitly on equation (3)-(7) and cannot be generalized to 2d or 3d. In fact, for three dimensional attractive potential problem given by

$$V(r) = -V_0, \quad r \le a; \quad V(r) = 0 \quad r > a \tag{8}$$

there exists a bound state only when the strength of the potential V_0 is greater than the critical strength $V_c = \left(\frac{\hbar^2 \pi^2}{8ma^2} \right)$. In fact, for higher dimensions, the condition $\int V(x)\, d^d x < 0$ does not essentially support [22, 23] always a bound state.

3.Bound state of single Cooper pair

The ground state of a non-interacting Fermi gas of electron in a potential well corresponds to the situation where all the electron states with wave vector $\vec{k}$ within the Fermi sphere $[E_F^0 = \frac{\hbar^2 k_F^2}{2m}$ at T=0K] are filled and all states with $E > E_F^0$ are unoccupied. We consider a simple model [2, 3, 4] of two electrons added to just above the filled Fermi surface characterized by the Fermi energy E_F^0. A weak attractive interaction $V(\vec{r_1}, \vec{r_2})$ between these two electrons is switched on resulting from the phonon exchange as discussed in the introduction section. All other electrons in the Fermi sea are assumed to be non-interacting, and, on account of the Pauli exclusion principle, they exclude a further occupation of states with $|k| < k_F$. The added two electrons do feel a statistical interaction through Pauli exclusion principle with those inside the filled Fermi sphere. Hence, the most simple bound state energy eigenvalue equation reads as

$$\begin{aligned} H\Psi_0(\vec{r}_1, \vec{r}_2) &= E\Psi_0(\vec{r}_1, \vec{r}_2) \\ H &= \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + V(\vec{r_1}, \vec{r}_2) \quad (9) \end{aligned}$$

Now, if the interaction $V(\vec{r_1}, \vec{r_2})$ depends only on the magnitude of the relative coordinate $\vec{r} = \vec{r_1} - \vec{r}_2$, then the lowest energy state will correspond to the zero momentum of centre of mass. This is true because we have a translational invariant system $([H, P] = 0; \vec{P} = \vec{p_1} + \vec{p}_2)$. This fact also can be viewed from the phonon mechanism. Due to phonon exchange the two additional electrons continually exchange their wave vector, where by, however, momentum must be conserved.

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_1' + \vec{k}_2' = \vec{K} \qquad (10)$$

We also assume that the interaction is short-ranged and in k-space is restricted to a shell with an energy thickness of $\hbar\omega_D$ (ω_D= Debye frequency) above E_F^0. The strength of the attractive interaction is maximum for $\vec{K}$=0.

It is therefore sufficient in what follows to consider the case $\vec{k}_1$=$-\vec{k}_2$=$\vec{k}$, i.e. electron pairs with equal and opposite wave vectors. This suggest the orbital wave function to be

$$\Psi_0(\vec{r_1}, \vec{r_2}) = \sum_{\vec{k}} g_{\vec{k}} e^{i\vec{k}.\vec{r_1}} e^{-i\vec{k}.\vec{r_2}} \qquad (11)$$

It should be noted here that zero momentum of the pair does not mean that the uncertainty in momentum (Δp) of the pair is zero.

Taking into account the antisymmetry [2, 24] of the total wave function with respect to exchange of the two electrons, Ψ_0 is converted either to a sum of products of $\cos(\vec{k}.(\vec{r}_1 - \vec{r}_2))$ with the antisymmetric singlet spin function ($\alpha_1\beta_2$ - $\beta_1\alpha_2$) or to a sum of products of $\sin(\vec{k}.(\vec{r}_1 - \vec{r}_2))$ with one of the symmetric triplet spin function $\alpha_1\alpha_2$, $(\alpha_1\beta_2 + \beta_1\alpha_2)$,$\beta_1\beta_2$, where α= up spin state, β= down spin state. However, because of an attractive interaction, we expect the singlet coupling to have lower energy because the cosinusoidal dependence of its orbital wave function on $(\vec{r}_1 - \vec{r}_2)$ gives a larger probability amplitude for the electrons to be near each other. Hence, we consider a two electrons singlet wave function of the form

$$\begin{aligned}\Psi_0(\vec{r}_1,\vec{r}_2) &= [\sum_{\vec{k}>\vec{k}_F} g_{\vec{k}} \cos \vec{k}.(\vec{r}_1-\vec{r}_2)] \\ &\times \frac{1}{\sqrt{2}}(\alpha_1\beta_2-\beta_1\alpha_2) \qquad (12)\end{aligned}$$

This kind of pairing of electrons is known as S-wave pairing wave as g_k will depend on the magnitude of the wave vector $\vec{k}$. The summation is confined to pairs with $\vec{k}$=$\vec{k}_1$=$-\vec{k}_2$, which, because of the interaction is restricted to the region $\hbar\omega_D$, must obey the condition

$$E_F^0 < \frac{\hbar^2 k^2}{2m} < E_F^0 + \hbar\omega_D$$

The quantity $|g_k|^2$ is the probability of finding one electron in state $\vec{k}$ and the other in $-\vec{k}$, that is, the electron pair in ($\vec{k}$,$-\vec{k}$). Due to Pauli exclusion principle and above condition, we have

$$g_k{=}0 \quad \text{for} \left\{ \begin{array}{l} k < k_F \\ k > \sqrt{2m(E_F^0+\hbar\omega_D)/\hbar^2} \end{array} \right.$$

With the help of the above Hamiltonian depicted in equation (9), the eigen value equation can be rewritten in terms of the coefficients g_k as

$$(E-2\epsilon_{\vec{k}})g_{\vec{k}} = \sum_{\vec{k}'} g_{\vec{k}'} V_{\vec{k}\vec{k}'} \qquad (13)$$

In this expression, the $\epsilon_{\vec{k}}$ are unperturbed plane wave energies and $V_{\vec{k}\vec{k}'}$ are the matrix element of the interacting potential obtained through Fourier transformation of $V(\vec{r})$ is given by

$$V_{\vec{k}\vec{k}'} = \Omega^{-1} \int V(\vec{r}) e^{i(\vec{k}'-\vec{k}).\vec{r}} d\vec{r} \qquad (14)$$

where $\vec{r}$ is the distance between the two electrons and Ω is the normalization volume. This $V_{\vec{k}\vec{k}'}$ characterizes the strength of the potential for scattering a pair of electrons with momenta ($\vec{k}'$, $-\vec{k}'$) to momenta ($\vec{k}$, $-\vec{k}$). If a set of $g_{\vec{k}}$ satisfying equation (13) with $E < 2E_F^0$ can be found, then a bound pair exists. Since it is hard to analyse this situation for general $V_{\vec{k}\vec{k}'}$, below we consider three forms of the interaction for which we can solve the equation (13) exactly.

3.1 Constant potential in k-space

Cooper [4] introduces the approximation that all $V_{\vec{k}\vec{k}'}$ =-V for $\vec{k}$ states out to a cut off energy $\hbar\omega_D$ away from E_F, and that $V_{\vec{k}\vec{k}'}$=0 beyond $\hbar\omega_D$. The right hand side of equation (13) is a constant, independent of $\vec{k}$, and we have

$$g_{\vec{k}} = V \frac{\sum_{\vec{k}'} g_{\vec{k}'}}{2\epsilon_{\vec{k}} - E} \qquad (15)$$

Summing both sides and cancelling $\sum g_{\vec{k}}$ we obtain,

$$\frac{1}{V} = \sum_{\vec{k}} (2\epsilon_{\vec{k}} - E)^{-1} \qquad (16)$$

Note that a constant V in the reciprocal space means a spatially varying in real space. In figure 2, we show the schematic variation of the varying potential in real space.

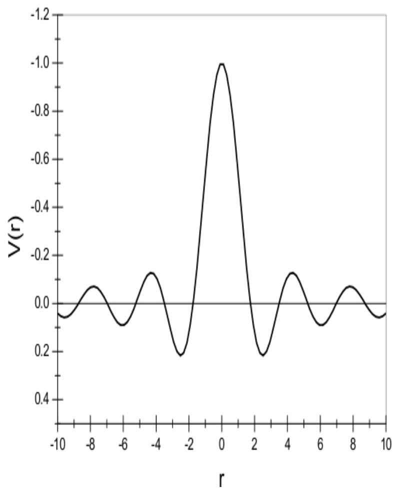


Figure 2: Schematic view of potential in real space.

When we replace the summation by an integration, with $N(\epsilon)$ denoting the density of states at the Fermi level for electrons, then the above equation reduces to

$$\frac{1}{V} = \int_{E_F^0}^{E_F^0+\hbar\omega_D} \frac{N(\epsilon)d\epsilon}{(2\epsilon - E)} \tag{17}$$

Since $N(\epsilon)$ in three dimensions is proportional to $\sqrt{\epsilon}$ and $\hbar\omega_D \ll E_F$, then we can assume that $N(\epsilon)$ does not change appreciably over the energy interval and can be replaced by the density of states at the Fermi energy ($N(E_F^0)$). Thus, we get a very simple equation connecting the strength of interaction as

$$\frac{1}{V} = N(E_F^0) \int_{E_F^0}^{E_F^0+\hbar\omega_D} \frac{d\epsilon}{2\epsilon - E} \tag{18}$$

$$1 = \frac{1}{2} N(E_F^0) V \ln \frac{2E_F^0 - E + 2\hbar\omega_D}{2E_F^0 - E} \tag{19}$$

or,

$$2E_F^0 - E = 2\hbar\omega_D \frac{e^{-2/N(E_F^0)V}}{1 - e^{-2/N(E_F^0)V}} \tag{20}$$

Now ($\hbar\omega_D << E_F^0$) and weak-coupling approximation is ($N(E_F^0)V << 1$), then the solution can be written as

$$E = 2E_F^0 - 2\hbar\omega_D e^{-2/N(E_F^0)V} \tag{21}$$

There thus exists two electron bound state, whose energy is lower than that of the fully occupied Fermi sea ($T = 0K$) by an amount $E - 2E_F^0 < 0$. In reality the instability leads to the formation of a new lower energy ground state.

It is interesting to note that from equation (21) that the strength of the interaction is inversely proportional to the density of electrons at the Fermi surface. Moreover, the binding energy is finite for any arbitrary value of the interaction potential. This brings out the open question of the impossibility of forming the binding state in three dimensions. This can be however argued in the following way as illustrated in [11]. The particles under consideration are not isolated particles but quasi-particles related with the filled Fermi sphere. This eventually leads to the reduction of three dimensional problem ($\int d^3k = N(E_F^0) \int dE$) to two dimensional ($\int d^2k = m \int dE$) one. Any attractive interaction in two dimensions will be sufficient enough for the formation of bound state. In fact this also indicates the key role played by the filled Fermi sea in the formation of Cooper pair. Physically speaking, in one or two dimensions, the motion of the particle is restricted to a straight line or a plane. In such a situation, any order of magnitude is enough to produce a bound state.

An important conclusion [12] can be drawn from equation (18). The equation (18) has a solution for an arbitrarily weak potential if and only if $N(E_F^0) \neq 0$. Note that in 3d, $N(\epsilon) \sim \sqrt{\epsilon}$ and hence $N(\epsilon) = 0$ at $\epsilon = 0$. Thus, in the above integral, if the lower cut-off were zero instead of E_F^0, then there would not be a solution for any arbitrary weak coupling V. This points out the important key role played by Pauli exclusion principle.

Suppose we split the Fermi sphere by a magnetic field so that $E_{F\uparrow} \neq E_{F\downarrow}$. In this situation, for a pair with $\vec{K} = 0$, the minimum excitation energy relative to free particles can be computed as $\Delta E_F = E_{F\uparrow} - E_{F\downarrow}$. Hence, the gap connecting equation reads as

$$\frac{1}{V} = N(E_F^0) \int_{E_F^0}^{E_F^0 + \hbar\omega_D} \frac{d\epsilon}{2\epsilon + \Delta E_F - \tilde{E}} \tag{22}$$

Here, $|\tilde{E}| = |E| - \Delta E_F$. If ΔE_F is greater than the binding energy of the state for $|\Delta E_F| = 0$, no binding state is possible. Suppose $\Delta E_F = 0$, but we would like to have a finite center of mass momentum $\vec{K}$ relative to Fermi sea. In that situation,it is clear that if the minimum value of ϵ_k is of the order $\hbar v_F K$, bound state solution disappears when $|\vec{K}| > \frac{E}{\hbar v_F}$. In that sense, it is really interesting to note that Cooper treated the two electrons as really special and the rest as only blocking states in Fermi sea.

If we measure the binding energy from the Fermi energy, it is evident from the equation (21) that the magnitude of the binding energy is given by

$$|E_B| = 2\hbar\omega_D e^{-2/N(E_F^0)V} \tag{23}$$

Moreover, as an order of magnitude estimation, one can approximate $|E_B|$ to be order of $k_B T_c$. In that situation, we can get a simple relation between microscopic interaction and the observed critical temperature T_c in the weak coupling limit as

$$T_c = 2\theta_D \exp\left[-\frac{2}{N(E_F^0)V}\right] \tag{24}$$

Here θ_D is the corresponding Debye temperature of the phonon frequency. This equation (23) can be compared with the many body calculation involving many Cooper pairs in BCS theory[5]

$$T_c = 1.14\theta_D \exp\left[-\frac{1}{N(E_F^0)V}\right] \tag{25}$$

Again $\theta_D \propto \frac{1}{\sqrt{M}}$, M being the mass of the ion and $N(E_F^0)V$ is assumed to be independent of the mass of the ion, then we notice from the above equation (23) that $T_c \propto \frac{1}{\sqrt{M}}$. This dependence on the ion's mass is known as isotope effect [2, 24] in superconductivity a milestone in realizing the importance of role of lattice or ion. Furthermore this equation (23) can be utilized to compute the various parameters for the microscopic theory such as binding energy, size of the Cooper pairs.

Let us pause for a moment to estimate the order of magnitude of this energy gap. For Hg, assuming the transition temperature $T_c = 4K$, we find that the gap is 0.001 eV. Note that the typical Fermi energy or the average energy of free electron states is of the order 3-10 eV and the typical thermal energy of a particle at room temperature (300 K) is roughly 0.025 eV. The binding energy of an electron in the ground state of hydrogen atom is 13.6 eV which is almost 10,000 times that of the energy gap seen for Hg. Thus, the minute magnitude of this energy gap can mislead one to use perturbation theory. However, the previous analysis by Cooper has shown clearly that it is indeed a non-perturbative effect. We will elaborate this point later.

What we have achieved till now is the following: If one starts from a degenerate free electron gas and switch on the attractive interaction V, then one notices that the electrons above the Fermi sea pair themselves and go below the Fermi surface to form a bound state. Thus, the normal state becomes unstable. Remarkably, this instability continues whatever may be the magnitude of the interaction strong or weak. Only thing required is the attractiveness of the interaction. However, there exists a contrasting nature from other two particle states coupled by some attractive finite range potential. They may not form always a bound state the boundedness of the state depends on some critical value of the interaction. For repulsive potential, we will have no bound state as the energy is always greater than the Fermi energy to form continuum state. This can be visualized [24] as follows. Consider the integral in equation (17) as

$$F(E) = V \int \frac{N(\epsilon)\ d\epsilon}{-E+2\epsilon};\ \epsilon = \frac{\hbar^2 k^2}{2m} - E_F \quad (26)$$

For any general two body problem, we have $k_F = 0$ but $N(\epsilon) \sim \sqrt{\epsilon}$ in three dimensions. Then, it is easy to notice that the above integral converges for $E = 0$. For small V, $F(0) < 1$ and for $E < 0$, one can convince that $F(E) < F(0) < 1$. Hence, $F(E) - 1$ is not realized for bound state. Following the same logic, we can see that for Copper pairing problem that $N(\epsilon)$ is constant over the energy interval and $F(0) \to \infty$ and there exists always a bound state value of $E < 0$ such that $F(E) = 1$.

It is also evident for constant $V_{kk'}$, $g_{\vec{k}}$ depends on the magnitude of the wave vector. This implies immediately that the spatial part of the wave function is symmetric in nature $(\psi(\vec{r}_1, \vec{r}_2) = \psi(\vec{r}_2, \vec{r}_1))$. Hence, according to Pauli exclusion principle, the spin part of the wave function must be antisymmetric. Therefore, the spin part must have the following antisymmetric form

$$\phi_{spin} = \frac{1}{\sqrt{2}} (\alpha_1 \beta_2 - \beta_1 \alpha_2) \quad (27)$$

where α and β are the spin up and spin down eigenstates of spin 1/2 Pauli spin matrices. The total spin operator S_{tot} and the z component of the spin operator S_z acting on the above eigenstate ϕ_{spin} gives identically zero eigenvalues only. This is consistent with the argument given in the section for writing down the variational form of the wave function for the two particles.

With the help of antisymmetric spin wave function, we can now construct the full wave function of two electrons as

$$\begin{aligned} &\psi(\vec{r}_1, \sigma_1 : \vec{r}_2, \sigma_2) \\ &= \frac{1}{\sqrt{2}} (\alpha_1 \beta_2 - \beta_1 \alpha_2) \end{aligned}$$

$$\times \sum_k g_{\vec{k}} e^{i\vec{k}\cdot(\vec{k}_1-\vec{r}_2)} \quad (28)$$

with $g_{-\vec{k}} = g_{\vec{k}}$. This wave function, however can be further simplified as

$$\begin{aligned}
&\psi(\vec{r}_1,\sigma_1 : \vec{r}_2,\sigma_2) \\
= &\frac{1}{\sqrt{2}}\sum_k g_{\vec{k}}\Big(\alpha_1\beta_2\ e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)} \\
&-\beta_1\alpha_2\ e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}\Big) \\
= &\frac{1}{\sqrt{2}}\sum_k g_{\vec{k}}\Big((\vec{k}\uparrow)_1(-\vec{k}\downarrow)_2 \\
&-(\vec{k}\uparrow)_2(-\vec{k}\downarrow)_1\Big) \\
= &\sum_k g_{\vec{k}} c^\dagger_{\vec{k}\uparrow} c^\dagger_{-\vec{k}\downarrow}|vac> \quad (29)
\end{aligned}$$

In the last step, we have used the formalism of second quantization with fermionic creation and annihilation operators. A quick inspection to the form of wave function reveals that

$$\begin{aligned}
\psi(\vec{r}) &= \mathcal{N}\int_{FS} d^3k\ e^{i\vec{k}\cdot\vec{r}} \\
&= \mathcal{N}\ \frac{\sin(k_F r) - k_F r\cos(k_F r)}{r^3} \quad (30)
\end{aligned}$$

Here, $\mathcal{N}$ is the normalization constant. In Figure 3, we show the variation of the wave function with radial distance for two different values of the Fermi wave vector. We notice the drastic difference of the behaviour of the wave function for large Fermi wave vector at small values of distance. It is clear that the wave function is spatially symmetric and the radial probability density $P(r)$ varies as r^{-4} as $r \to \infty$. This indicates immediately that the mean square radius $< r^2 >= 4\pi\int P(r)r^2dr$ diverges.

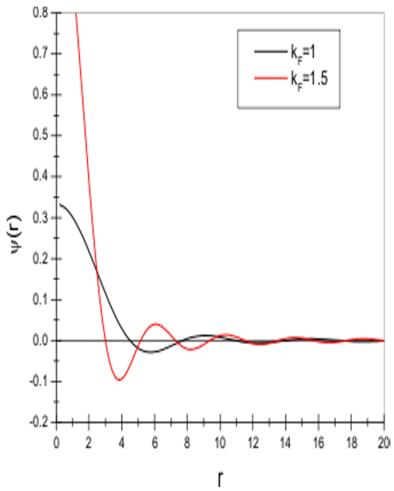


Figure 3: Schematic view of the wave function for two different values of the Fermi wave vector.

The variation of the probability density can be visualized in another way. Using the value $g_{\vec{k}}$, we can rewrite the wave function as

$$\begin{aligned}
\psi(r) &= \frac{1}{r}\int_{k_F}^{k_c} dk\ \frac{k\sin(kr)}{2\epsilon_k + |E|} \\
&= -\frac{1}{r}\frac{d}{dr}\left\{\cos(k_F r)\int_0^{\epsilon_c}\frac{\cos(\frac{\epsilon r}{\hbar v_F})\ d\epsilon}{\epsilon + |E|/2}\right\}
\end{aligned} \quad (31)$$

Here we have used $k - k_F \approx \frac{\epsilon_k}{\hbar v_F}$. If we represent the integral as $J(r)$, then it is noticed that the wave function is nothing but that of two free scattering particles ($\frac{1}{r}\sin(k_F r)$ times the integral $J(r)$. A careful look reveals that $J(r)$ is roughly constant for $r < \frac{\hbar v_F}{|E|}$ and beyond that it falls of as $\frac{1}{r}$. Therefore, the wave function varies as $\frac{1}{r^2}$ in the limit $r \to \infty$. This fact thus justifies the statement about the radial probability density in a natural way.

Now one may ask the following question: Is this spatial symmetric wave function unique for any form of interaction $V_{kk'}$? In fact, the nature of the wave function depends strongly on the nature of the interaction. If the interaction depends on the angle between $\vec{k}$ and $\vec{k}'$ (such as $\lambda(\vec{k}\cdot\vec{k}')$), then several bound states [24] may arise. Moreover, spatially anisotropic solutions leading to complicated spin dependences rather than the simplistic as suggested above may be found. However, it has been noticed that in most of the normal superconductors do not show a strong angular dependence of $V_{kk'}$ on $\vec{k}$ and $\vec{k}'$.

3.2 Potential separable in two coordinates

Note that in the original Cooper pair problem, the interaction potential $V(\vec{r}_1,\vec{r}_2) = V(|\vec{r}_1-\vec{r}_2|) = V(r)$. That's why the potential $V(\vec{r}_1,\vec{r}_2)$ is separable in relative coordinate r only. There is no part of the centre of mass coordinate $\vec{R}$. If the potential could depend on the individual coordinates rather than the distance between the particles, then we cannot write $V(\vec{r}_1,\vec{r}_2)$ as $f(r)g(R)$. As in the case of constant potential, here also we define the matrix element $V_{\vec{k}\vec{k}'}$ as

$$V_{\vec{k}\vec{k}'} =< \vec{k},-\vec{k}|V|\vec{k}',-\vec{k}' > \qquad (32)$$

There is no general solution to equation (13) unless $V_{\vec{k}\vec{k}'}$ is assumed to be separable as functions of $\vec{k}$ and $\vec{k}'$. So now we choose the separable potential [22] given by $V_{\vec{k}\vec{k}'} = -\lambda\omega_{\vec{k}}\omega^*_{\vec{k}'}$. We consider the potential V in relative coordinate is spherically symmetric and hence, $\Psi(\vec{r}_1,\vec{r}_2)$ is an eigenfunction of angular momentum with angular momentum quantum numbers l and m_1.

$V_{\vec{k}\vec{k}'}$ can be expanded into partial wave components

$$V_l(|\vec{k}|,|\vec{k}'|) = \lambda_l\omega^l_{\vec{k}}\omega^{*l}_{\vec{k}'} \qquad (33)$$

Thus, the eigen value equation in this case for each value of l of $V_l(|\vec{k}|,|\vec{k}'|)$ reads as

$$(B_{lm_1} - 2\epsilon_{\vec{k}})g_{\vec{k}} = -\lambda_l\omega^l_{\vec{k}}\sum_{\vec{k}'}\omega^{*l}_{\vec{k}'}g_{\vec{k}'} \qquad (34)$$

with

$$g_{\vec{k}} = -\frac{\lambda_l\omega^l_{\vec{k}}C}{(B_{lm_1} - 2\epsilon_{\vec{k}})} \qquad (35)$$

Where the constant C is defined as

$$C = \sum_{\vec{k}'}\omega^{*l}_{\vec{k}'}g_{\vec{k}'} \qquad (36)$$

Repeating the same arguments as done in the previous section, we obtain the desired equation

$$\begin{aligned} 1 &= -\lambda_l\sum_{\vec{k}}|\omega^l_{\vec{k}}|^2\frac{1}{(B_{lm_1} - 2\epsilon_{\vec{k}})} \\ &= -\lambda_l\Phi(B_{lm_1}) \qquad (37) \end{aligned}$$

determining the energy eigenvalues B_{lm_1}. Before we proceed for the energy eigen value, we would like to study the analytic structure [25] of the equation (32). The poles of $\Phi(B_{lm_1})$ occurs at $B_{lm_1} = 2\epsilon_{\vec{k}}$. As $B_{lm_1} \to 2\epsilon_{\vec{k}}$ from below, $\Phi(B_{lm_1}) \to -\infty$ while just above $B_{lm_1} \to 2\epsilon^+_{\vec{k}}$, $\Phi(B_{lm_1}) \to +ve$. For all the values of $B_{lm_1} \ll 2E^0_F$, $\Phi(B_{lm_1}) = -ve$. Therefore, a bound state forms when

$\Phi(B_{lm_1})$ crosses $-\frac{1}{\lambda_l}$. Thus, the intersection of $\Phi(B_{lm_1})$ with the straight line $-\frac{1}{\lambda_l}$ determines the bound state solutions for the Cooper pair in such a solution.

For the simple case $\omega^l_{\vec{k}} = 1$ for $0 < \epsilon_{\vec{k}} < \hbar\omega_D$ and $\omega^l_{\vec{k}} = 0$ for otherwise, and $\lambda_l < 0$, the binding energy $|B_{lm_1}|$ of the pair in the split off state is given by

$$\frac{1}{\lambda_l} = \frac{N(E_F^0)}{2}\log[\frac{|B_{lm_1}| + 2\hbar\omega_D}{|B_{lm_1}|}]$$
$$|B_{lm_1}| = \frac{2\hbar\omega_D}{\exp(2/N(E_F^0)\lambda_l) - 1} \quad (38)$$

We have assumed the density of states $N(\epsilon)$ is slowly varying in the interval $0 < \epsilon_{\vec{k}} < \hbar\omega_D$ and have approximated it by $N(E_F^0)$, the density of single electron states of one spin orientation, evaluated at the Fermi surface. From above equation one has in the weak coupling limit,

$$|B_{lm_1}| = 2\hbar\omega_D \exp(-2/N(E_F^0)\lambda_l) \quad (39)$$

Thus, like constant potential case, we notice in this situation also that the binding energy is an extremely sensitive function of the coupling strength for weak coupling; however, a bound state exists for weak coupling so long as the potential is attractive near the Fermi surface. One more remark is in order. In contrast to the previous constant potential case, instead of single bound state, here we get many bound states as indicated in the figure 4. The bound state does not exist for repulsive interaction between the electrons above the filled Fermi surface. It is interesting to note that the form of the binding energy is not analytic at $V = 0$ or $\lambda = 0$. In other words, the energy cannot be expanded in powers of V or λ, Hence, the result cannot be obtained by perturbation theory. This points out the importance of variational principle as adopted here. This is one of very few occasions in condensed matter physics, where the final result cannot be obtained from the perspectives of perturbation theory. The failure of perturbation theory can be traced back in the frame work of change of different symmetry associated with the system [26].

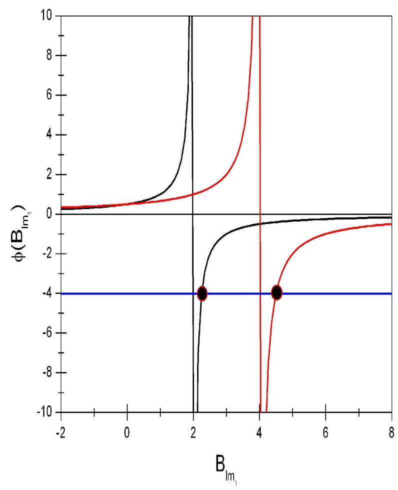


Figure 4: Schematic view of bound states for potential separable in two coordinates.

We can also generalize the above analysis to

$$V_{\vec{k}\vec{k}'} = -V_0\delta(\vec{k} - \vec{k}') \quad (40)$$

to obtain exactly the energy eigen value. So, we see that for all three above cases we get bound state. Now it is almost impossible to get a general analytic solution from any $V_{\vec{k}\vec{k}'}$. The exact analytic solution exists only for three forms of $V_{\vec{k}\vec{k}'}$ as discussed above. Of course any linear combination of the three forms of $V_{\vec{k}\vec{k}'}$ will also form a bound state and can be found exactly. It has been shown by Randeria et al [27] that in two spatial dimensions, the many body ground state of a dilute gas of fermions interacting through an arbitrary pair potential is unstable to s-wave pairing. This work has a remarkable implications in high-T_c superconductivity because of the effective two dimensions of the materials used.

3.3 Binding Energy and Center of Mass Momentum

Dimensionally, one may argue that the binding energy depends on the center of mass momentum corordinate $\vec{P}$ as P^2. This is reasonably true because the wave function contains a part of the order $\exp\left(\frac{i\vec{P}\cdot\vec{R}}{\hbar}\right)$. However, we will demonstrate below that the presence of the Fermi surface will force the binding energy to depend only linearly [25] on the magnitude of the center of mass momentum $\vec{P}$.

The equation (16) in such a case is modified as

$$1 = -V \sum_k \frac{1}{E - \epsilon_{\vec{k}+\frac{\vec{P}}{2}} - \epsilon_{-\vec{k}+\frac{\vec{P}}{2}}} \tag{41}$$

Now, for small $\vec{P}$, neglecting the terms of the order P^2, we can rewrite the above equation in an integral form

$$1 = -V \int_{E_F^0+\frac{v_F P}{2}}^{E_F^0+\frac{v_F P}{2}+\hbar\omega_D} \frac{N(\epsilon)\, d\epsilon}{E - 2\epsilon} \tag{42}$$

to obain the final expression of the binding energy in terms of the Fermi velocity v_F and other relevant parameters

$$E = 2E_F^0 + Pv_F - \frac{2\hbar\omega_D}{\exp\left(\frac{2}{N(E_F^0)V}\right) - 1} \tag{43}$$

Thus, we notice that the inclusion of the center of mass energy in the expression of the binding energy for the relative motion reduces its order of magnitude and can even break the pair. If we measure the energy from the Fermi surface, we can set E_F^0 to zero; then the appropriate value of the magnitude of center of mass momentum for which the breaking of Cooper pair can be found as

$$Pv_F = \frac{2\hbar\omega_D}{\exp\left(\frac{2}{N(E_F^0)V}\right) - 1} \sim k_B T_c \tag{44}$$

If we now divide this center of mass momentum by Planck's constant, we get an inverse of length scale $[\frac{P}{\hbar} \sim \frac{k_B T_c}{\hbar v_F} = 10^4\ cm^{-1}]$. This length scale ξ is known as Pippard coherence length [2, 13, 24] and the estimated order of magnitude turns out as $\xi \sim 10^{-4}\ cm = 1000\mathring{A}$. Interestingly, ξ, the coherence length or the average size of the Cooper pair is seen to be larger than the inter-atomic distance between the particles. Such a large coherence length is the signature of the electron-phonon interaction.

4. Average Size of Cooper pairs

In the previous section, we have used simple dimensional argument to estimate the size of the Cooper pair. This has been done from the energy gap at the Fermi energy. That's the key point for using this dimensional analysis. This can be also be dome via uncertainty principle [2]. In this section, we would like to compute the size of the pair from the Cooper pair wave function through the expectation value [13, 24].

Cooper pair wave function can be written as

$$\psi(\vec{r}_1, \vec{r}_2) = \sum_{\vec{k}} g_{\vec{k}} e^{i\vec{k}.\vec{r}} \tag{45}$$

where $\vec{r} = \vec{r}_1 - \vec{r}_2$, therefore

$$|\psi(\vec{r}_1, \vec{r}_2)|^2 = \sum_{\vec{k}} \sum_{\vec{k}'} g_{\vec{k}} g_{\vec{k}'} e^{i(\vec{k}-\vec{k}').\vec{r}} \tag{46}$$

Now the expectation value of the Cooper pair radius squared is given by

$$< r^2 >= \frac{\int |\psi(\vec{r}_1, \vec{r}_2)|^2 r^2 d\vec{r}}{\int |\psi(\vec{r}_1, \vec{r}_2)|^2 d\vec{r}} \tag{47}$$

putting the value of equation(31), we get

$$< r^2 >= \frac{\sum_{\vec{k}} \sum_{\vec{k}'} \int r^2 g_{\vec{k}} g_{\vec{k}'} e^{i(\vec{k}-\vec{k}').\vec{r}} d\vec{r}}{\sum_{\vec{k}} \sum_{\vec{k}'} \int g_{\vec{k}} g_{\vec{k}'} e^{i(\vec{k}-\vec{k}').\vec{r}} d\vec{r}} \tag{48}$$

or,

$$< r^2 >= \frac{\sum_{\vec{k}} \sum_{\vec{k}'} (\nabla_{\vec{k}}.\nabla_{\vec{k}'} \int e^{i(\vec{k}-\vec{k}').\vec{r}} d\vec{r}) g_{\vec{k}} g_{\vec{k}'}}{\sum_{\vec{k}} \sum_{\vec{k}'} \delta(\vec{k}-\vec{k}') g_{\vec{k}} g_{\vec{k}'}} \tag{49}$$

therefore,

$$< r^2 >= \frac{\sum_{\vec{k}} \sum_{\vec{k}'} \nabla_{\vec{k}}.\nabla_{\vec{k}'} \delta(\vec{k}-\vec{k}') g_{\vec{k}} g_{\vec{k}'}}{\sum_{\vec{k}} \sum_{\vec{k}'} \delta(\vec{k}-\vec{k}') g_{\vec{k}} g_{\vec{k}'}} \tag{50}$$

or, we can write

$$< r^2 >= \frac{\sum_{\vec{k}} |\nabla_{\vec{k}} g_{\vec{k}}|^2}{\sum_{\vec{k}} |g_{\vec{k}}|^2} \tag{51}$$

Now, from previous section we know that

$$g_{\vec{k}} = \frac{C}{2\epsilon_{\vec{k}} - E} \tag{52}$$

where C be the constant. Therefore,

$$g_{\vec{k}} = \frac{C}{2(\frac{\hbar^2 k^2}{2m} - E_F - w/2)} \tag{53}$$

where $w = -2\hbar\omega_D e^{-2/N(0)V}$ is the binding energy of the Cooper pair. Now,

$$\nabla_{\vec{k}} g_{\vec{k}} = -\frac{C\frac{\hbar^2 k}{m}}{2(\frac{\hbar^2 k^2}{2m} - E_F - w/2)^2} \tag{54}$$

therefore,

$$|\nabla_{\vec{k}} g_{\vec{k}}|^2 = \frac{C^2 \frac{\hbar^4 k^2}{m^2}}{4(\frac{\hbar^2 k^2}{2m} - E_F - w/2)^4} \tag{55}$$

and

$$|g_{\vec{k}}|^2 = \frac{C^2}{4(\frac{\hbar^2 k^2}{2m} - E_F - w/2)^2} \tag{56}$$

Substituting this value of $|g_k|^2$ in equation (46), we get

$$< r^2 >= \frac{2\hbar^2}{m} \frac{\int_{E_F}^{E_F+\hbar\omega_D} \frac{N(\epsilon)\epsilon d\epsilon}{(\frac{\hbar^2 k^2}{2m} - E_F - w/2)^4}}{\int_{E_F}^{E_F+\hbar\omega_D} \frac{N(\epsilon) d\epsilon}{(\frac{\hbar^2 k^2}{2m} - E_F - w/2)^2}} \tag{57}$$

One can approximate those integrals as

$$\int_{E_F}^{E_F+\hbar\omega_D} \frac{N(\epsilon)\epsilon d\epsilon}{(\frac{\hbar^2 k^2}{2m} - E_F - w/2)^4} \sim N(E_F^0)E_F \int_{E_F}^{\infty} \frac{d\epsilon}{(\frac{\hbar^2 k^2}{2m} - E_F - w/2)^4} \tag{58}$$

or,

$$\int_{E_F}^{E_F+\hbar\omega_D} \frac{N(\epsilon)\epsilon d\epsilon}{(\frac{\hbar^2 k^2}{2m} - E_F - w/2)^4} = \frac{-8N(E_F^0)E_F}{3w^3} \tag{59}$$

similarly,

$$\int_{E_F}^{E_F+\hbar\omega_D} \frac{N(\epsilon)d\epsilon}{(\frac{\hbar^2 k^2}{2m} - E_F - w/2)^2} = \frac{-2N(E_F^0)}{w} \tag{60}$$

Incorporating these values of the integrals, we obtain finally,

$$< r^2 >= \frac{4\hbar^2 v_F^2}{3w^2} \tag{61}$$

where v_F is the Fermi velocity given by $v_F = \frac{\hbar k_F}{m} = \frac{\hbar}{m}(3\pi^2 n)^{1/3}$ with n being the density of electrons.

Now the order of binding energy is 10^{-4} eV. So we get,

$$< r^2 >^{1/2}= 10^{-4} cm \tag{62}$$

So the extension of Cooper pair is much greater than mean interatomic separation. So, such a high density of such electron pairs create a new lower energy ground state which is known as BCS ground state.

Below we estimate the size of the Cooper pairs in Table I for different systems assuming the binding energy $w \sim k_B T_c$. Note that

Table 1: Size of Cooper pair

Metal	T_c	$v_F(\times 10^6 m/s)$	Size (μm)
Al	1.2	2.03	14.92
Cd	0.56	1.62	25.51
Ga	1.09	1.92	15.54
Zn	0.9	1.83	17.93
Sn	3.72	1.90	4.50
Pb	7.2	1.83	2.24
Nb	9.26	1.37	1.30
Hg	4.15	1.58	3.36
In	3.4	1.74	4.51

as $V_0 \to 0$, $w \to 0$ rendering the average radius to diverge. Kadin [28] has beautifully illustrated the real space physical picture for the Copper pair consistent with BCS theory.

5.Cooper pair above the ellipsoidal Fermi surface

Suppose we put two electrons above the ellipsoidal Fermi surface rather than the original spherical Fermi surface as done by Cooper himself. Will they still form a bound state like the spherical one? We know that the ellipsoidal Fermi surface can be represented by the equation

$$E = \frac{\hbar^2}{2m_x^*}k_x^2 + \frac{\hbar^2}{2m_y^*}k_y^2 + \frac{\hbar^2}{2m_z^*}k_z^2 \tag{63}$$

This equation can be rearranged in the following form

$$1 = \frac{\hbar^2}{2m_x^* E}k_x^2 + \frac{\hbar^2}{2m_y^* E}k_y^2 + \frac{\hbar^2}{2m_z^* E}k_z^2 \tag{64}$$

The three axis of the ellipsoid can be read of as

$$a = \frac{\sqrt{2m_x^* E}}{\hbar}$$
$$b = \frac{\sqrt{2m_y^* E}}{\hbar}$$
$$c = \frac{\sqrt{2m_z^* E}}{\hbar} \qquad (65)$$

Now, the total number of electronic state be

$$n = \frac{\frac{4}{3}\pi abc}{(\frac{2\pi}{L})^3} \qquad (66)$$

Putting the value of a, b, c into equation (61), the value of n is given by

$$n = \frac{\frac{4}{3}\pi\sqrt{8m_x^* m_y^* m_z^*}}{\hbar^3(\frac{2\pi}{L})^3}E^{3/2} \qquad (67)$$

This, the density of state is

$$N(E) = \frac{dn}{dE} = C\sqrt{m_x^* m_y^* m_z^*}E^{1/2} \qquad (68)$$

where C be the constant. Thus, like the spherical case, here $N(E)$ is also proportional to $\sqrt{E}$. This can be understood from the fact that by defining $\tilde{k}_x = \frac{k_x}{\sqrt{m_x^*/m}}$, $\tilde{k}_y = \frac{k_y}{\sqrt{m_y^*/m}}$ $\tilde{k}_z = \frac{k_z}{\sqrt{m_z^*/m}}$, the ellipsoidal Fermi surface defined in equation (58) can be transformed to spherical one. So it is clear that, the form of binding energy of Cooper pair does not change for ellipsoidal Fermi surface. Only the magnitude of the effective mass is modified in such a situation as $m_d = (m_x^* m_y^* m_z^*)^{1/3}$. Thus, the equations (20) or (33) remain valid with suitable replacement of the density of states at the Fermi surface as shown above. Although the density of states will not change shape, however, it is to be noted that the Fermi velocity at different k-points on the Fermi surface will be different. This in turn will indicate the electron-phonon coupling will necessarily depend on k and hence, the gap calculated above becomes anisotropic instead of isotropic.

6. Conclusions

To conclude, we have discussed the energetics of a single Cooper pair which eventually forms an important ingredient for the development of BCS theory which is nothing but the condensate of interacting many Cooper pairs. The meaning of bound state along with the inapplicability of perturbation theory have been addressed here. The original bound state of Cooper pairs in spherical Fermi surface has been generalized to ellipsoidal Fermi surface with various forms of the interacting potentials. The average radii of the Cooper pairs for many superconducting materials have been computed and it has been argued that they are quite larger than the typical average distance between the quantum particles (here electrons).

Acknowledgements

The author would like to acknowledge many post-graduate students for asking several interesting questions related to this subject. In particular, I would like to thank Mr. Sanjib

Maity for completing a project work on this topic. I would also like to thank the anonymous referee for many critical comments and suggestions to improve the quality of the paper.

References

[1] B. Simon, *Phys. Rev. Lett.*, **36** , (1976). See for a review, D. Jana, in *Astrophysics and Condensed Matter*, Editor Thomas G. Hardwell, pp 71-104 (Nova Science Publishers, NY, 2008).

[2] M. Tinkham, *Introduction to Superconductivity*, (Dover Publications, 2004).

[3] T. V. Ramakrishnan and C. N. R. Rao, *Superconductivity Today*, (Wiley Eastern Limited, 1992).

[4] L. N. Cooper, *Phys. Rev.*, **104**, 1189 (1956). *Am. J. Phys.*, **28**, 91 (1960).

[5] J. Bardeen, L. N. Cooper and J. R. Scrieffer, *Phys. Rev*, **108**, 1175 (1957).

[6] D. M. Ginsberg, *Am. J. Phys.*, **30**, 433 (1962).

[7] P. W. Anderson, *Phys. Rev.*, **112**, 1900 (1958).

[8] D. Jana, *Physics Teacher*, **48**, 39 (2006).

[9] A. Bohr, B. R. Mottelson and D. Pines, *Phys. Rev.*, **110**, 936 (1958).

[10] R. A. Broglia, *More is different: 50 years of nuclear BCS*, arXiv:1206.1523v1.

[11] A. A. Abrikosov, *Fundamentals of the theory of metals*, (North-Holland, Amsterdam, 1988).

[12] N. W. Ashcroft and N. D. Mermin, *Solid State Physics*, (Holt, Rinehart, Winston, 1976).

[13] J. R. Schrieffer, *Theory of Superconductivity*, (Benjamin-Cummings, Menlo Park, 1964).

[14] H. Ibach and H. Lüth, *Solid State Physics*, (Springer, 3rd edition, 2003).

[15] J. B. Ketterson and S. N. Song, *Superconductivty*, (Cambridge University Press, Cambridge, England, 1998).

[16] S. V. Vettoor, *Resonance*, **September**, 41 (2003).

[17] V. F. Weisskopf, *Contemp. Phys.*, **22**, 375 (1981).

[18] D. Jana, *Phys. Edu*, **25**, 35 (2008).

[19] D. Jana, *Dimensional Analysis: Modern Perspectives* (Lambert Academic Publishing, Germany, 2011).

[20] D. Page, J. M. Lattimer, M. Prakash and A. W. Steiner, *Astrophys. Journal*, **707**, 1131 (2009).

[21] R. D. Mcdonald, N. Harrison and J. Singletron, *J. Phys: Condens. Matter*, **21**, 012201 (2009).

[22] W. F. Buell and A. Shadwick, *Am. J. Phys.*, **63**, 256 (1995).

[23] B. Simon, *Ann. Phys.*, **97**, 279 (1976).

[24] P. G. de Gennes, *Superconductivity of metals and alloys* (Westview Press, USA, 1999).

[25] Phillip Phillips, *Advanced Solid State Physics*, (Westview Press, USA, 2004).

[26] D. Jana, *Physics Teacher*, **44**, 21 (2002).

[27] M. Randeria, J.-M. Duan, L.-Y. Shieh, *Phys. Rev. Lett.*, **62**, 981 (1989).

[28] A. M. Kadin, *Spatial Structure of the Cooper Pair.* cond-mat/0510279v1

Physics through Teaching Lab XXIII

LabVIEW Based Weighing Machine

D. Hanumeshkumar[1], P.Jyothi, C.Nagaraja and P.S.S.Sushama

Department of Instrumentation
Sri Krishnadevaraya University
Anantapur 515055, India.
[1]Corresponding author hanumeshkumar69@gmail.com

(Re-*Submitted* 18-01-2013)

Abstract

LabVIEW is used in the development of dedicated instruments for laboratory and industrial applications. This paper presents the implementation of Lab VIEW for weights measurement using load cell. Virtual instruments are presently used in scientific and industrial applications. Aim of this paper is to build a simple weighing machine implementing the Lab VIEW. The hardware, software details are presented in this paper. The results are satisfactory. This paper helps the students to understand and build a basic instrumentation system.

Key words: LabVIEW, Weights measurement.

Introduction

Weight is generally used as a measurement in many situations as a measure of quantity. It is commonly used for batching out quantities. Many types of methods are available for weight measurement, one of the methods to measure the weight by using a load cell. It is a passive transducer which can convert a force (strain) into electrical signal. It has a strain gauge as a sensing element. Strain gauges are two types; one is semiconductor strain gauge and anther one metallic strain gauge. In the metallic type gauge is resistance varies linearly with strain. In this Experiment, the metallic type strain gauges are used.

A load cell usually consists of four strain gauges in a Wheatstone bridge configuration (Full Bridge). This load cell has a cantilever, one end of which is fixed firmly to a rigid support and the other end, where the unknown force is applied, is free. At the free end of the cantilever beam, applied force cause bending moment, develops the strain on the strain gauges, which is proportional to the applied force [1]. When the cantilever beam is stretched, the resistance of the strain gauges is increased due to which, its resistance changes in a definite manner. The force is applied through the different weights. In this experiment a load cell having linear response is used. The Wheatstone bridge consisting of four resistive arms with a DC excitation source acts as sensing element in the load cell. Measurements can be carried out either by balancing the bridge or by determining the magnitude of imbalance [4].

Instrumentation:

The weight measurement achieved by using labVIEW is shown in fig 1. The block diagram consists of the following blocks 1. DC Excitation Source, 2. Load cell, 3. Signal conditioner, 4. NI DAQ 6009, 5. Personal Computer.

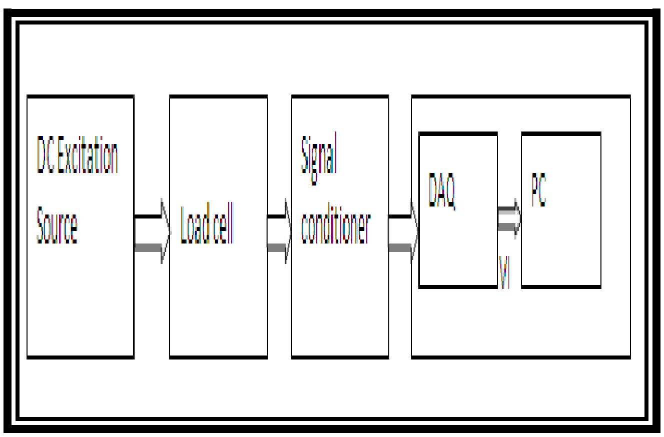


Fig. 1 Block diagram for measurement weights by load cell.

Basically load cell is a passive transducer. Excitation is needed for passive transducers because these transducers do not generate their own voltage or current. The excitation source may be an alternating current or D.C voltage source. The D.C system is comparatively simple and generally used for resistance transducers such as strain gauge. In load cell, the strain gauges are in Wheatstone bridge configuration (Full Bridge). It is the most commonly used Bridge for measurement of resistance. There are two ways in which a Wheatstone bridge can be used. They are Null Type Bridge and Deflection Type Bridge. For measurement of rapidly changing input quantities, the Deflection type is used. The output of Deflection Type Bridge is the result of imbalance caused by the strain on the strain gauge. The output of the bridge is directly proportional to the change in the input quantities; the output of the bridge is very low. Hence it needs a signal conditioning element. The signal conditioning element consists of an instrumentation amplifier and scaling amplifier. The instrumentation amplifier has high input impedance. This bridge is called the Voltage Sensitive Bridge [1]. The bridge output is applied to the differential inputs of the Instrumentation amplifier. The output of an instrumentation amplifier is applied to the scaling amplifier. The scaling amplifier is configured in non-inverting configuration with adjustable gain.

The output of the scaling amplifier is applied to the Data acquisition device **NI DAQ 6009** [6] from National Instruments. It has eight analog input channels and two 12 bit DAC channels, The ADC can be configured for either 12bit/14bit, the ADC can be used for single ended input or Differential input mode [7]. In this experiment the output of the scaling amplifier is read from A0 of the ADC of the NI DAQ 6009. The data is processed and the weight is displayed in Gms.

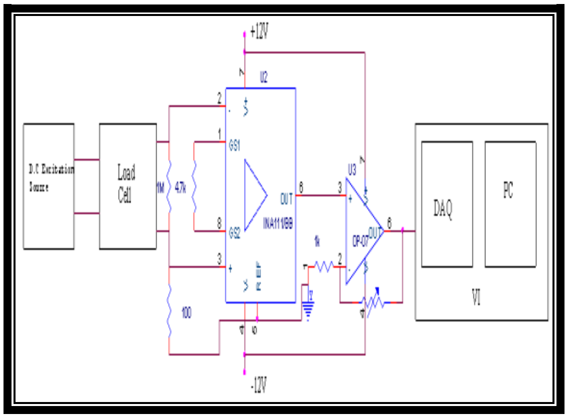


Fig2.Schematic Diagram of weights measurement

Fig. 2 shows the schematic diagram of weights measurement using lab VIEW. The external D.C excitation source is connected to two arms of the load cell and another two arms of load cell are applied to Load resistor of 1 MΩ. The output of the bridge is applied to the differential inputs of an instrumentation amplifier. A small resistor is connected at the non inverting input of the instrumentation amplifier to the ground. The 100Ω resistor acts a ground point eliminating grounding errors. The output of the instrumentation amplifier is fed to scaling amplifier. The scaling amplifier has adjustable gain for suitable amplification for linear measurement. The output of the scaling amplifier is applied to National Instruments DAQ card NI DAQ 6009. In load cell, the strain gauges are in Wheatstone bridge configuration. In this experiment deflection type measurement is used. Due to the small variation in the resistors, there exists a small amount of DC voltage at the output of the bridge at no weights condition. Hence to offset the bridge voltages, the offset voltage is compensated through the lab view programming. The programming on LabVIEW measures the voltages and displays the weight on the front panel of the virtual instrument.

Software Development:

The following Algorithm is used for the software development in the present application.

1. Initialization of DAQ for data acquisition.
2. Enabling the function for the Amplitude and level measurements in VI (virtual instrumentation) for measurement of DC voltage.
3. Find out the offset voltage of NI DAQ.
4. Offset Nulling: The offset voltage is subtracted from the measured voltage obtained from the amplitude measurement function. The corrected output voltage value is applied to the Formula function for the exact voltage measurement.
5. A case loop is designed for tare weights function.
6. Weights are displayed with tare and without tare.

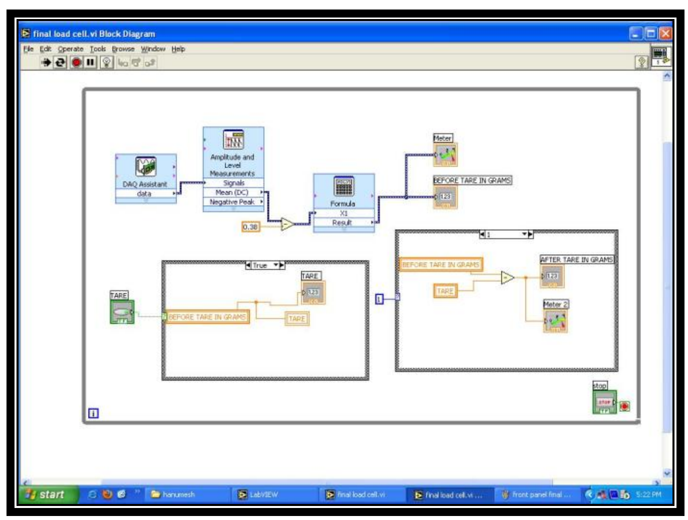


Fig.3. Block Diagram of the VI

Fig. 4.front panel of the weighing machine on VI

The experimental setup is shown, in plate 1.

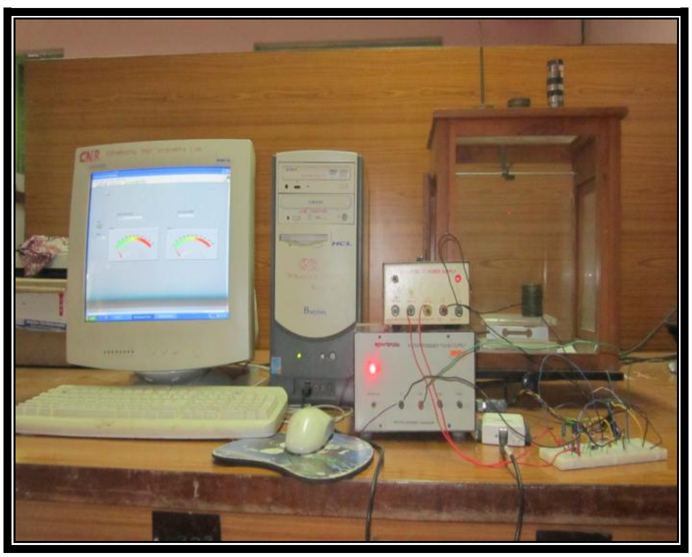


Plate1. Experimental Setup of Lab VIEW based weighing machine

Results and Conclusions:

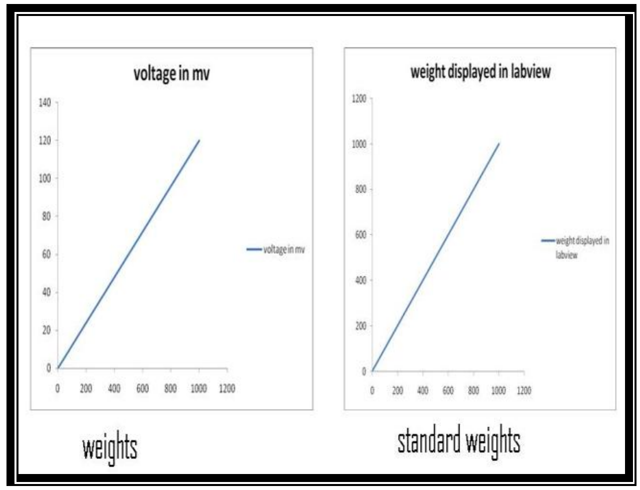


Fig. 5(a) Weight vs. Voltage, fig.5 (b) Standard weights vs. weights displayed in Lab VIEW

Weight measurements are achieved by implementing the load cell as sensor and LabVIEW as platform for the weighing machine.

The fig.5 (a) shows Weights vs. Voltage, 5(b) shows Standard Weights vs. weights displayed on LabVIEW. The weights are measured from 5 Gms to 5 Kg with this present unit. The experiment is done without any additional mechanical couplings. The results are satisfactory.

References:

[1] A Course in Electrical and Electronic Measurements and Instrumentation by A.K. SAWHNEY

[2] Op-Amps and Linear Integrated Circuits by RAMAKANT A. GAYAKWAD

[3] INA 111 Data sheet

[4] NI Strain gauge tutorial

[5] Virtual Instrumentation using Lab VIEW- Jovitha Jerome- PHI Publications.

[6] National Instruments. (1998). Lab VIEW data acquisitions basics manual. Austin, TX: Author.

[7] BheemaLingaiah T, Dr.Nagaraja C HanumeshKumar D, “Measurement of Phonocardiograph signals and determining the heart Rate using labVIEW” Research Journal of Engineering and Technology, July-September 2011, Volume:4, Issue :3, ISSN 0974-2824.

Physics Through Problem Solving XXVI: Variational Method

Ahmed Sayeed
Department of Physics
University of Pune
Pune - 411007
email: sayeed@physics.unipune.ac.in

Abstract
In this issue we shall illustrate the variational method of estimating the ground state energy of a quantum mechanical system.

We obtain the energy eigenvalues of a quantum mechanical system by solving the Schrödinger equation for the system – when we are able to do so. There are only a small number of systems for which we can exactly solve the Schrödinger equation, and in all other cases we have to resort to some kind of approximation or numerical method. One such method of approximation is the *Variational Method*. This method is very useful when the Hamiltonian of the system is known and we need to know the ground state energy of the system. The method is as follows.

Consider a one-dimensional system. We choose some 'trial' or 'guess' wave function for the ground state of the system, say $\psi(x, b)$, where b is the *variational parameter*. We obtain the expectation value of the Hamiltonian H with respect to this wave function $E(b) = \langle H \rangle$. This energy value is minimized with respect to the variational parameter b, and this minimum value (say $E_{\min}$) is the estimate of the ground state energy. There is a theorem, called the *Variational Principle*, which states that the variational estimate so obtained is an upper bound for the exact ground state energy, that is, $E_{\min} \geq E_g$, where E_g is the exact (often difficult to calculate) ground state energy.

The above method can be straightforwardly generalized for higher dimensional problems, and also for wave functions with several variational parameters. It can also be applied to find estimates for the excited state energies, but in practice it is usually reliable only for the ground state energy estimation. The most important part of this method is the choice of the trial wave function. It is essential that the trial wave function is consistent with the symmetry of the system. For example, if the potential energy is symmetric about a position, the ground state wave function must also be even about that point and must not have any nodes.

The following problem illustrates the method.

Problem

Consider the trial wave function

$$\psi(x) = \frac{A}{x^2 + b^2} \tag{1}$$

where A normalization constant and b is the (real) variational parameter. Use this trial wave function to estimate the ground state energy of a one dimensional linear harmonic oscillator. You can use the following integrals:

$$\int_{-\infty}^{\infty} \frac{1}{(b^2 + x^2)^2} dx = \frac{\pi}{2b^3} \tag{2}$$

$$\int_{-\infty}^{\infty} \frac{x^2}{(b^2 + x^2)^2} dx = \frac{\pi}{2b} \tag{3}$$

$$\int_{-\infty}^{\infty} \frac{x^2}{(b^2 + x^2)^4} dx = \frac{\pi}{16b^5} \tag{4}$$

Solution

Note that the given trial wave function is 'reasonable' – $\psi(x) \to 0$ for $x \to \pm\infty$, and it is even about the origin. Also it does not have any nodes.

We begin by first normalizing the wave function:

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1$$
$$\Rightarrow \; |A|^2 \int_{-\infty}^{\infty} \frac{1}{(b^2 + x^2)^2} dx = 1$$
$$\Rightarrow \; |A|^2 = \frac{2b^3}{\pi}$$

In the last step we have used the integral given in stint1. Thus the normalized wave function is

$$\psi(x) = \sqrt{\frac{2b^3}{\pi}} \frac{1}{x^2 + b^2} \tag{5}$$

Harmonic oscillator Hamiltonian is

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 \tag{6}$$

where x, p, m and ω have the usual meanings. We have to find

$$E(b) = \langle\psi|H|\psi\rangle = \langle\psi|T|\psi\rangle + \langle\psi|V|\psi\rangle \tag{7}$$

where $T = \frac{p^2}{2m}$ and $V = \frac{1}{2} m\omega^2 x^2$, the kinetic and potential energy operators.

$$\begin{aligned} \langle T\rangle &= \frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \left|\frac{d\psi}{dx}\right|^2 dx \\ &= \frac{\hbar^2}{2m} \frac{2b^3}{\pi} \int_{-\infty}^{\infty} \left|\frac{-2x}{(b^2 + x^2)^2}\right|^2 dx \\ &= \frac{\hbar^2}{4mb} \end{aligned} \tag{8}$$

We have used integral given in equation stint3 in the last step. Next we find $\langle V\rangle$:

$$
\begin{aligned}
\langle V \rangle &= \int_{-\infty}^{\infty} \psi^* V \psi dx \\
&= \frac{2b^3}{\pi} \cdot \frac{1}{2} m\omega^2 \cdot \int_{-\infty}^{\infty} \frac{x^2}{(b^2+x^2)^2} dx \\
&= \frac{1}{2} m\omega^2 b^2 \qquad (9)
\end{aligned}
$$

In the last step here we have used integral in equation stint1. Thus we have expectation value of energy as a function of the variational parameter:

$$
\begin{aligned}
E(b) &= \langle H \rangle \\
&= \langle T \rangle + \langle V \rangle \\
&= \frac{\hbar^2}{4mb} + \frac{1}{2} m\omega^2 b^2 \qquad (10)
\end{aligned}
$$

Now we minimize $E(b)$ with respect to b:

$$
\begin{aligned}
\left(\frac{\partial E}{\partial b}\right)_{b=b_0} &= 0 \\
\Rightarrow \left(-\frac{\hbar^2}{2mb^3} + bm\omega^2\right)_{b=b_0} &= 0 \\
\Rightarrow -\frac{\hbar^2}{2mb_0^3} + b_0 m\omega^2 &= 0 \\
\Rightarrow b_0^4 &= \frac{\hbar^2}{2m^2\omega^2} \\
\Rightarrow b_0^2 &= \frac{\hbar}{\sqrt{2} m\omega} \qquad (11)
\end{aligned}
$$

In the last step we have chosen the positive value for b_0^2, because it gives the minimum for $E(b)$, as the reader can verify by usual methods. Also, it is stated in the problem that b real. This gives us the ground state energy estimate (from equation toten) $E_{\min} = E(b_0) = \frac{\hbar\omega}{\sqrt{2}} = 0.707\hbar\omega$. This differs from the actual ground state energy of the harmonic oscillator, which as we know is $0.5\hbar\omega$, by about 40%. Note that the estimate is greater than the exact value, in agreement with the variational principle.

The above is obviously not such a good estimate. This might be improved with some other choice of the trial wave function. In fact, if we choose a Gaussian function in x, i.e., $\psi(x) = Ae^{-bx^2}$, the ground state energy estimate will be the exact the ground state energy $\frac{1}{2}\hbar\omega$ (This exercise is done in every quantum mechanics text book). This is because the trial wave function happens to be the exact ground state wave function for the harmonic oscillator .

Difficulties Faced by College Students in Introductory Physics: A Case Study

[1,2]Elmehdi, H. M, [2,3]Pistorius, S. and [1]Suleiman, B. M.

[1]Deptartment of Applied Physics, College of Sciences, University of Sharjah,
PO Box: 27272, Sharjah, UAE.

[2]CancerCare Manitoba Foundation, Winnipeg, Manitoba, Canada

[3]University of Manitoba, Winnipeg, Manitoba, Canada, R3T 2N2

hmelmehdi@sharjah.ac.ae

(*Submitted* 25-07-2012)

Abstract

In a survey conducted at the University of Sharjah (UoS) in the United Arab Emirates (UAE), 86% of respondents from among first-year students reported that Introductory Physics was the most difficult subject they were taking. This high percentage is not limited to students at the University of Sharjah. In fact, it is internationally perceived that introductory physics is the subject college students fear the most. What is alarming is that more than 80% of the students who said that Introductory Physics was difficult believed that it had no relevance to their respective fields. This is often offered as a reason for the lower than average passing percentages obtained in Introductory Physics. Some post secondary institutions in the Middle East went so far as proposing the reduction of the number of physics courses in their curricula or cutting down on the course content in response to continuous complaints from students as well as parents.

The results of our survey will be examined with the aim of subjectively discussing the factors contributing to the struggle students have with Introductory Physics. These include inherent problems, the role of mathematical presentations of physical concepts, presentation of the subject matter, the students' study habits and assessment tools. After that, we highlight the importance of physics through various examples of applications in the respective fields in which Physics plays an integral part in understanding many functions and processes.

Finally, we propose some solutions, which we believe will assist students in learning some of seemingly difficult topics, including suggestions to make physical concepts easier and more enjoyable, without compromising the quality or the quantity of the course content.

1. Introduction

During a Conference on Engineering Education in the Arabian Gulf countries, which was held in Kuwait in 1980, several papers suggested that the number of courses in physics should be reduced and that the remaining topics should be incorporated into other non-physics courses already included in the curriculum [1]. The argument offered was partially based on the poor results students obtain in physics where a number of students end up repeating the course. This in turn tends to reduce the flow of students and limit the number of students in the later courses. In addition, it was argued that many of the physics topics could easily be fitted into other engineering courses. Another reason presented was that the number of credit hours allocated to university, college and departmental requirements leave very little time for additional courses. For example, in the College of Engineering at the University of Sharjah in the UAE, there are only two physics courses, Physics II and I, included in the program; in the Medical Colleges and College of Health Sciences, only one 3-credit-hour physics course is allocated to cover basic physics concepts and fundamentals. In the case of the College of Engineering, the two courses include classical mechanics concepts and an introduction to electricity and magnetism. Optics, light, sound, thermodynamics and modern physics are not covered. In the Colleges of Medicine and Health Sciences, the physics course covers only biomechanics, fluids and heat, depriving students of topics like sound, waves, optics, atomic and nuclear physics, radiation protection and safety, and electromagnetics. In comparison to many western universities, where the engineering curricula include physics courses (for engineering colleges), which covers all the relevant physics topics, the number of physic courses offered at the University of Sharjah leave students with a minimal exposure to many of the important physics concepts. When the issue was raised at the administration and deanship levels, the reasons offered were very similar to those presented at the Kuwait conference, which was centered on the two notions that “students find physics very difficult” "and “it is not relevant to their field of study.” In some departments Introductory Physics is not a prerequisite for advanced courses, which leads to a common phenomenon where senior engineering, medical and health sciences students delay their graduation because they have not yet passed Introductory Physics.

In light of the issues and concerns raised above, our aim in this paper is to investigate the factors contributing to college freshmen concerns with introductory college physics. In addition, we will attempt to present solutions and recommendations to deal with these issues. Our study includes conducting a survey questionnaire aimed at probing students’ perceptions, accompanied by detailed analysis of the responses in light of course outcomes and previous experiences.

2. Results and Discussion

To investigate this issue, we surveyed 326 students enrolled in various departments of the Colleges of Engineering, Medicine, and Health Sciences at the University of Sharjah in the UAE. The aim of the survey was to gauge student views on these issues and in particular the reasons behind their apparent struggle with physics courses. A list of the survey questions and student responses are found in Table 1.

Questions one to three were included to identify if the students’ prior physics background influenced their performance. The results indicated that the students had a strong background in physics, with 98% of the students having taken physics in high school. In some cases, students had taken up to three physics courses during high school. 86% of these students had obtained grades of 85% or above in high school physics while 23% of the students surveyed were “repeaters”, which included students who had taken an intro-physics course at least once before. In response to question number 5, 86% of the surveyed students ranked physics as the most difficult subject. The list included other subjects such as mathematics (including calculus and algebra) and chemistry. When students were asked to list the reasons behind their belief that physics was difficult, common answers included: “I understand the concepts, but I cannot solve the problems” and “the exams are difficult”. This former response is typical of those found elsewhere and often implies the converse, which is that the students really do not understand the concepts [2]. Some students replied that physics contained a lot of math and the exams contained “indirect” questions, which they could not answer. In addition, 75% of the subjects surveyed responded that they would evade taking physics if they had a choice of taking another course instead.

While these responses may be due to several factors, what is alarming is that more than 80% of the students surveyed believed that physics had no relevance to their respective fields. The students’ apparent frustration with the subject, the low grades they tend to obtain, which hover around the mid 60s, the quality of the instructors, teaching methodology and rigorous exams may contribute to this view. When coupled with continuous complaints from students and dismal outcomes, this perspective appears to be one of the main reasons behind reducing the number of physics courses in the curricula of universities in the Gulf region [1, 3].

In a separate survey of 125 graduates in the fields of engineering, medicine and health sciences, we examined their views of the difficulty and relevance of Introductory Physics to their respective fields. The results of this survey showed that more than 80% of the professionals surveyed believed that physics was very important to engineering, irrespective of their subspecialties. For graduates and professionals in the fields of medicine and health sciences, the percentage was lower but at 72% still much higher than that of freshmen students.

While these views are in line with the literature [4], the former results of the student surveys motivated us to investigate the reasons behind the struggle of students with physic with the hope of proposing solutions to address student concerns. Our research focuses on answers to the following questions:

Is physics really a difficult subject and if so, what makes it difficult to understand? Is it possible to deliver the same content in an “easier to comprehend approach?”

To answer this question effectively, we will investigate the possible factors/reasons contributing to the struggle of students with Introductory Physics, which as a result may have influenced their responses to the questions in the survey. As we will explain in the next section, some of these reasons are inherent to any physics course (e.g. subject matter, the presentation and assessment tools used to evaluate the performance of the students). Some of these factors may be independent of university and geographic location. On the one hand, a student’s background and preconceived notions about the difficulty of physics courses may be influenced by local conditions and thus contribute to the above results. Examples include student study habits, which tend to overlook some of the learning outcomes of a physics course and students’ perception of the non-relevance of Introductory Physics to their various fields, be it in the everyday demands of their professions or in research and development. We will argue that if students realized how important physics is to their profession, they would be motivated to take the course and be very keen to understand the subject.

Why do students think physics is difficult?

Careful examination of the poor performance of students in Introductory Physics can be traced to more deep-rooted factors rather than to the difficulty of the subject matter. One of these factors is the physics that is being taught in high school curricula. Table 1 shows that almost all of the student respondents took physics in high school, the majority of whom attained very high grades. When we looked at the content of a typical

Table 1: Summary of student responses to the survey questions (n = 326. representing the Colleges of Medicine, Health Sciences and Engineering).

No	Question	Response	Percentage of Responses
1	Have you taken physics in high school?	Yes	98%
		No	2%
2	What was your physics grade in high school?	Above 85%	86%
		75-84%	14%
3	Do you have to take introductory physics as part of your curriculum?	Yes	100%
		No	0%
4	Have you taken introductory physics before?	Yes	23%
		No	77%
5	Among the courses you have taken/taking, introductory physics is ranked number ….in terms of its difficulty	# 1	86%
		# 2-4	10%
		> 4	4%
6	If you were given the choice to evade Introductory Physics, would you take that choice?	Yes	75%
		No	18%
		No response	7%
7	Do you think introductory physics is relevant to your specialty?	No	80%
		Yes	20 %
8	Why do you think introductory physics is difficult?	Problems are difficult to solve.	67%
		Difficult exams and exam format.	56%
		Difficult to understand and is indirect.	43%
		Too much Mathematics	72%
		Concepts are easy but applications are difficult.	23%

physics course in UAE high school education curricula (national or private), it was found that it contained adequate coverage of the fundamental concepts. However, when these graduates were subjected to a placement test, designed to test their physics background, only 33% of these students met the cutoff score (set at 40%) needed to enroll in Physics I [4]. These same students had an overall average of 85% for their high school physics grade (a university admissions requirement). This huge discrepancy is attributed to the inflated grades students obtain in high school physics.

We therefore reached the conclusion that whatever the reason may be, the grades obtained in high school physics courses do not reflect the student's "true" understanding of the concepts of physics. While this argument provides convincing evidence as to why students score badly in their placement tests, the students who complained about the difficulty of physics are the ones who met the cutoff score. So why do these students still think physics is difficult?

Common sense ideas and theories:

One factor, which greatly contributes to the struggle of students with physics courses and one that hinders their comprehension and grasp of physics concepts is what is commonly referred to as *common sense theories or ideas* [2, 5, 6, 7]. As reported in [2], each student entering a first course in physics possesses a system of beliefs and intuitions about physics phenomena, which s/he derived from extensive personal experience. This system functions as a *common sense theory* of the physical world, which students use to interpret their experience, including what they use and hear in their physics courses. It is considered as a major determinant of what the student learns in the course. Yet conventional physics instruction fails almost completely to take this into account.

A student begins non-physics courses, such as Philosophy, History, etc., with unaffected minds. For example, before students take their World History course, Economics, or Philosophy, they know little or nothing about these subjects. The instructor can then help the students to implant fresh knowledge upon the palimpsest of their minds [2]. The situation in introductory physics is quite different where students arrive in their first year physics course with a set of physical theories that they have tested and refined over the years of repeated experimentation [2].

Students have spent many years of their lives exploring mechanical phenomena by walking with slow speed, climbing up a hill at varying speeds, running a marathon with maximum constant speed, kicking a ball vertically upwards to see who can kick it the farthest, and riding accelerating vehicles. They also have limited experiences with electrical phenomena, acquired while using electrical circuits at home that are related to the behavior of light, lenses and mirrors. Based on their observations, students have pieced together a set of "*common sense*" ideas about the physical universe and how it works. One might think this is an advantage, giving students studying physics a head start. On the contrary, researchers have shown that these preconceived "*common sense*" ideas are incompatible with the correct physical picture. What is worse is that these erroneous ideas are robust and difficult to dislodge from students' minds, in large measure because these ideas are not addressed by conventional physics instruction.

An example of such a situation is encountered every time we cover the concept of pressure in a pipe (or an artery) that is narrowed down to a fraction of its original cross sectional area, as shown in Figure 1 below. The question directed to the students was as follows: In which region, 1 or 2, in the figure below (Fig. 1 below) would you expect the pressure on the walls of the pipe to be the greatest? More than 90 % of the students said region 2. The students based their answer on the fact that when the pipe (or artery) is narrowed, the speed of the fluid increases, and hence so does the pressure! Some students did not believe that region 1 is under greater pressure even after pointing out that since the laws of conservation of mass tell us that velocity is greater in region 2 (i.e. going from a wide area to a narrow area the velocity increases), the fluid is accelerating in that direction. Since acceleration requires an unbalanced net force, which in this case is

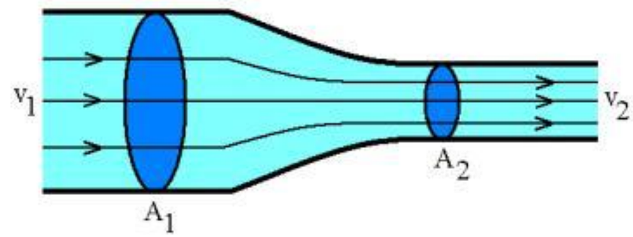


Figure 1: As the fluid flows from region 1 to region 2, its velocity increases and the pressure drops.

supplied by the pressure on the fluid by the walls, therefore the pressure in region 1 must be greater than the pressure in region 2 to accelerate the fluid.

A similar example was reported by McDermott and Shaffer [5], who conducted the following two experiments. In the first experiment, a battery was connected to two identical light bulbs in series. In the second experiment, the battery was connected to a single bulb, which is identical to the two bulbs in the first part. The authors asked the students in an introductory physics course to compare the brightness of the single bulb to that of the two bulbs in the first part. Only 10-15% of the students (some who were exposed to circuits) gave the right answer that the two bulbs in the first part are equally bright and the bulb in the second part is brighter. A number of wrong answers include, "bulb A will be the brightest because it will use all the current", or "all bulbs will be equally bright because they are exposed to the battery", which supplies each bulb with same current. Neither of these ideas were learned in the class nor were they dismissed from the minds of the students in the introductory course. The surprising result is that McDermott and Shaffer found that the result was independent from whether the question was posed before or after the introduction of electric circuits. These examples show that "*common sense*" ideas tend to hinder students' understanding of material and at times confuse them. It is therefore, the instructor's job to identify these misconceptions and try to correct them during lesson delivery.

These common sense ideas and misconceptions are found to interfere with the student's approach to answering questions and solving problems. A prime example of such a situation is encountered during the explanation of the motion of an object moving vertically upwards at constant speed [8]. A hot-balloonist is moving up at a constant speed of 5 m/s, drops an object at a height of 40 meters above the ground. Students were asked to find the time the object takes to reach the ground, 88% of these students reported the wrong answer; 2.9 sec rather than the correct answer of 3.4 sec. Students wrongfully assumed the initial speed of the object to be zero, because of their false interpretation of word "dropped", which students interpreted to mean an initial speed of zero!

Investigations of this sort show that it is not enough to merely teach students the "right" way to think and approach the solution of physics problems. Challenges facing instructors involve, in addition to introducing the concepts, identifying possible student misconceptions or misinterpretations that confront them head-on and helping the students to "unlearn" them at the same time that they are learning the correct physics. Failure to do this will inevitably leave the students with their erroneous "common sense" ideas intact. Sadly, no contemporary textbook being used in colleges today attempts to assist instructors in identifying and correcting some of these misconceptions.

Mathematical expressions: do they help or hinder students' understanding?

We now turn our attention to the reasons that are inherent in the subject matter. The most important factor, especially when it comes to applications of physics concepts, is the fact that most of these concepts are represented by mathematical expressions. As shown in Table 1, this has been cited by more than 70% of the students as a major obstacle in understanding the meaning of many physics concepts, which in turn contribute to their frustration with physics. It is a fact that physics concepts are expressed in terms of mathematical formulae and equations that contain many variables and symbols, some of which need substitutions and using extra steps that involve other concepts. Therefore, proper interpretation and understanding of each of these symbols needed to solve problems is an integral part of the solution. It is actually the first step in the solution. Oftentimes, students find themselves looking for information, which is available to them in the wording of the problem. Examples of such problems are those encountered in projectile motion, where the phrase "at the top of the trajectory", means that the vertical component of the velocity is zero. A ball is fired "horizontally", means that the initial vertical component of the velocity is zero [8].

A second concern of students is their inability to apply mathematical expressions or to properly choose the correct/right formula when attempting to solve problems. As noted above, the most reported complaint by the students in our survey was "I understand the concepts, but I do not know which formula to use when solving problems"; and "I understand the concepts but not the mathematical formulas". The mathematical expressions representing physical quantities are not well digested by students. To illustrate this, we gave students the following two expressions, which describe the same exact concept, and asked students to state the difference between the two expressions.

$$W_{net} = \Delta KE \qquad (1)$$

$$W_{NC} = \Delta KE + \Delta PE \qquad (2)$$

Where W_{net} is the work done by the net force acting on the object and ΔKE is the change is kinetic energy (energy due to change in speed). W_{NC} in equation (2) is the work done by non-conservative forces acting on the object (e.g. friction) and ΔPE is the change potential energy. The latter represents the change in the energy of the object due to change in its elevation with respect to the surface of the earth. This is commonly referred to as the work done by the force of gravity.

More than 75% of the students said that these two expressions were different and they described different concepts; failing to realize that these expressions are exactly the same! Students did not recall that the work net is simply the sum of the work done by conservative and non-conservative forces.

$$W_{net} = W_C + W_{NC} = \Delta KE \qquad (3)$$

Since the force due to gravity (the weight of the object) is a conservative force equal to change in potential energy, i.e. $W_C = -\Delta PE$. By substituting for W_C, equation (3) becomes,

$$W_{NC} = \Delta KE + \Delta PE \qquad (4)$$

Hence the equations (1) and (2) are exactly the same. It should be noted that students were asked this question after they covered the topic of conservation of energy in class.

Another example, which illustrates students' inability to translate the meaning of mathematical expressions, is that of Newton's second law of motion [5]:

$$\sum \vec{F} = m\vec{a}$$

Where $\Sigma\vec{F}$ is the net force acting on the mass *m*, and *a* is the acceleration of the object. It is common for students to interpret this equation to mean that the product of mass and its acceleration is itself a force. In other words, they fail to interpret or realize that a mathematical equality between two quantities does not imply the two quantities are conceptually distinct. As a result, students do not appreciate the fact that acceleration is a consequence of the presence of a net force. Hence, students fail to see the difference between $\Sigma\vec{F} = 0$ and $\Sigma\vec{F} \neq 0$, the latter of which indicates that the forces acting on the mass are unbalanced, that cause the object to move with a uniform acceleration, which means that its velocity changes at a constant rate. It should be noted that in a homework problem, students solved the questions if given all the information, even though the concept was not fully understood by the students.

Another important factor that contributes to students' inability to translate mathematical expressions is the fact that students tend to focus on mathematical definitions rather than on the physics meaning of the mathematical expressions. An example of such phenomena, is demonstrated by the following example, see Fig. 2, where we asked students which of the two children does more work against gravity in moving the box a height *h* above the ground (neglecting air resistance and frictional affects). Almost all students said the boy on the inclined plane does more work than the other boy, while they both do the same exact amount of work. In arriving at this answer, students used the definition of work, which states $W=F\Delta d\cos\theta$, where F is the applied force, Δd is the distance travelled and θ is the angle between F and Δd. Hence since the distance, Δd, is longer for the boy along the inclined plane, then he did more work.

Similar examples have been reported by Lawson and McDermott [4]. The conclusion here is that students based their answer on the mathematical expression rather than on the concept; failing to properly interpret the formulae describing the situation at hand.

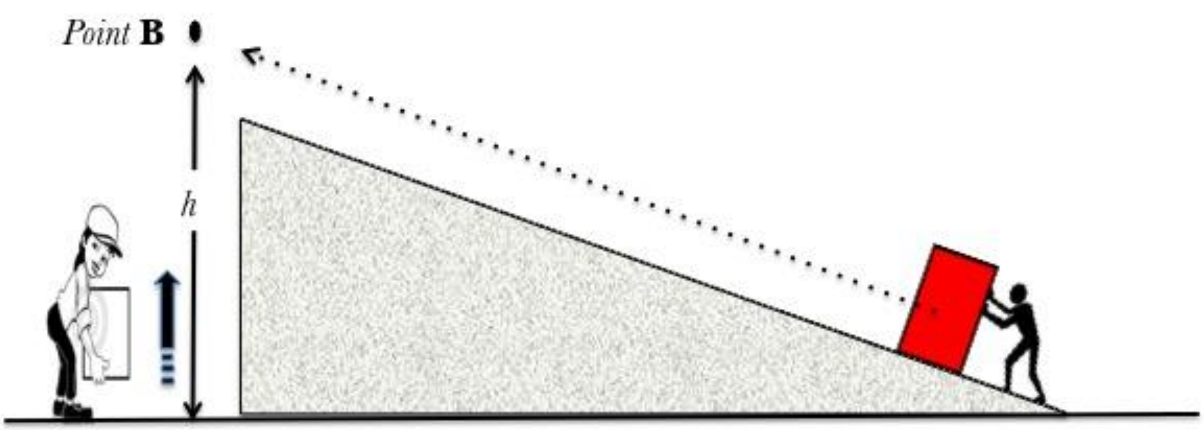


Figure 2: In absence of air resistance and friction, the work done by the boy and the girl against gravity is exactly the same.

The above discussion highlights the fact that introducing basic physics concepts using mathematical expressions contributes to students' frustration with Introductory Physics. A student may fully understand free fall motion, yet s/he gets the wrong answer to the hot-balloonist problem. Similarly, with the work question, students fail to apply the expression representing to the two situations. In fact, solving problems in physics is in itself a skill that can only be learnt in physics courses. Through problem solving skills, students are able to breakdown physics situations into a set of symbols that are represented by mathematical formulae that can be either measured or calculated. This skill is gained and refined by solving as many problems as possible. Suffice to say that the onus is on physics educators to meet these challenges and come up with a strategy by which to incorporate these important components into physics teaching in order to achieve our objectives and learning outcomes.

Student study habits:

The second factor we investigated is that of student study habits and how students prepare for quizzes and exams. Research shows that students begin the semester pursuing a conceptual understanding of the subject matter but quickly become overwhelmed by the base of the course and revert to *rote* learning in order to catch up with the instructor to solve homework and prepare for quizzes and exams. Rote learning is a learning technique, which avoids understanding of a subject and instead focuses on memorization. The major practice involved in rote learning is learning by repetition. The idea is that one will be able to quickly recall the meaning of the material the more one repeats it. While this technique may work in other subjects like history, biology and various other courses, it is not an effective method for studying physics. In preparing for a physics exam, students often rely on the end-of-chapter problems, which if not approached with the understanding of all concepts involved in their solutions may not serve as a good tool for preparing for exams. Students are advised that studying physics should be done in two stages. First, they have to understand the concepts, followed by the second stage during which they practice applying these concepts to solve various examples in real-life situations. Students tend to rely on using rote learning to “memorize” the solution to old exam problems or end-of-chapter problems with the hope that they will encounter a similar problem in the exam. Such students are found to have little grasp of the concepts covered by these problems [8]. To investigate the validity of this conclusion, two problems were given to a pool of 60 students to solve as an in-class quiz. The problems were selected from end-of-chapter questions and old exam problems. Students scores were satisfactory with more than half of the class obtaining 75% or higher. Later, these students were asked conceptual questions, which were covered by the two problems and very few students were able to answer these questions. The majority of the students were able to calculate the amount of heat flow in a metal of a specific heat capacity as a result of temperature increase, yet many of them failed to distinguish between the concept of heat and temperature.

These examples clearly demonstrate that the majority of students do not study or prepare properly for physics quizzes and exams. The consequence of this is that students find themselves either getting the grades, yet they do not fully understand the concepts, or understanding the concepts which they cannot apply it to real life situations. For a student to strike a balance, s/he must do well in both stages mentioned at the beginning of this paragraph.

Teaching methodology

The third factor, which may have a great impact on students’ perception of introductory physics, is teaching methodology. It was found that the method by which concepts are delivered influences students’ comprehension. Students’ definition of a good physics instructor was found to depend on factors that go beyond s/his understanding of the concepts. Suleiman and Elmehdi [9] investigated the effects of new teaching approaches that rely on technology such Course Managing Systems (CMS) [10], implementation of the Internet, and employing new approaches, such as Guided Critical Thinking [11]. They found that such approaches have greatly improved the performance of students. Classical approaches, where the instructor fills the board with long derivations and diagrams that are difficult to follow, should be replaced by well-organized and colorful multimedia slides that deliver the same message but in a more unique and attractive way. Animations are also a great tool to show students "dynamic" applications of physics concepts. The CMS course page may also be used to post supplementary material and communicate with students. In addition, student grades can also be posted on the CMS course page, which allows the students to follow and assess their progress their progress throughout the semester.

Other teaching approaches include student study groups, which allow students to learn from each other and discuss solutions. This method was found to be very effective, especially for students who tend to be very shy and do not participate or answer questions during the class time.

Assessment tools

The last factor that contributes to the students' frustration and the subsequent struggle with Introductory Physics is that of assessment tools, which are mainly comprised of problem-based quizzes, tests and exams. A typical physics assessment test is made up of long answer questions and multiple choice questions, most of which are based on solving problems designed to test student understanding of physics concepts covered in class. Judging from the student responses (Table 1), 56% of the students thought that exams and exam formats are the main reason for the difficulty of physics. The problem is more pronounced for students in Medicine and Health Sciences, where students are used to essay type questions, which rely on memorization and recall. The solution to this problem may take more than a short paragraph to discuss. There are attempts to deal with this issue, especially with the presentation of the questions. For example, Suleiman and Elmehdi [9] suggested that the application of the "Guided Critical Thinking" approach to help students breakdown the problem was successful in raising their grades by 32%. The approach is based on breaking down questions into several sub questions that serve to guide the student to the solution. Whatever the solution may be, we have to understand that physics assessment tools need to be revised. Among the suggestions [9] discussed are research-based assignments, presentations and essay questions, which may give students the freedom to go beyond having to worry about finding the numerical solution to a specific problem.

Realizing the importance and relevance of physics:

The last factor to be discussed in this paper is to investigate student motivation and the encouragement they receive to take Physics courses, because of the importance and relevance of physics to their field of study. As indicated in Table (1), 80% of the students did not think physics was relevant to their field of study. Such a high percentage is shared by many administrative personnel who are unaware of the importance of physics to these disciplines, prompting several authors to write articles on the relevance of physics to non-major disciplines such as medicine, health sciences and engineering. For example, Ahmed [1] details the role of physics in engineering education. The author argued that engineers should take at least four physics courses, covering areas such as classical mechanics; optics, sound and heat; electricity and magnetism; and modern physics. He listed some of the high caliber universities in the USA and Europe that have included this many physics courses into their curriculum. The author adds that these courses should be supplemented by laboratory sessions to strengthen and facilitate the student understanding of physics concepts.

Even though there is a unanimous agreement among educators on the importance and relevance of Introductory Physics to non-physics majors, such as engineering, students fail to see its relevance. To highlight the importance of physics and its relevance to the engineering profession, instructors are encouraged to review two of the famous architectural disasters that took place in the world in the past few decades. These include the Collapse of the Hyatt Regency Hotel in Kansas City, 1981 [12, 13], and the Collapse of the Bridge in Washington State, 1940 [13,14]. After careful examination of the reasons behind these tragic accidents, it becomes apparent that negligence of basic physics phenomena was behind these disasters [12,13,14]. In both cases, the results of the investigation revealed that induced oscillations approached the natural frequencies of both structures causing resonance (the process by which the frequency on an object matches its natural frequency causing a dramatic increase in amplitude) [13]. The resulting large amplitude oscillations went beyond the range of the restoring force, causing a spectacular collapse. There are innumerable other examples of how the lack of physics understanding has led to costly and unfortunate consequences [13]. Additional examples of engineering disasters were also reported by Bartlett [14,15] and Ross [12]. These examples clearly demonstrate how important basic physics is in engineering education.

In medicine and health sciences, Suleiman [16] provided a comprehensive review of the relevance and role of physics concepts in these disciplines. He argued that most biological systems and processes, such as muscle motion, blood flow, vision and hearing, etc., can be better understood using basic physics concepts such as forces, torque, fluid flow, light and optics, waves, etc. In a similar article, Varmus (1999) [17] highlights the impact of physics on biology and medicine. He presented a table of 25 basic physics concepts, which are directly correlated to medical applications, including those involved in diagnostic and therapeutic applications. He continues to add that the recent advances in molecular genetics could not have been achieved without the analytical as well as computational imaging tools of the physicist, saying that medical and health scientists should not only be exposed to such tools, but learn to use them.

A third review is that of Mackay and Santillan (2005) [18], who highlighted the role and importance of physics to the medical field throughout the 18th and 19th centuries. They reviewed the advances in science and technology related to medicine and health sciences, including imaging modalities, radiotherapy and ultrasound.

3. Conclusion

The factors attributing to the struggle of students with Introductory Physics include misconceptions and common sense knowledge, which students bring with them to the class; weak physics background; difficulty in properly interpreting mathematical expressions that describe physics concepts and situations; and study habits, which tend to be most suitable for subjects such as history, biology etc., where students are required to memorize and list a set of functions and steps. In addition, teaching methodology and assessment tools are found to contribute to the frustration and struggle that students have with Introductory Physics. It is suggested that introducing approaches that employ IT tools such as CMS tend to improve student performance. As a motivating factor for students, the relevance of physics to the professions of engineering, medicine and health sciences should be highlighted not only to students, but also to program administrators and curriculum designers. Even though professionals in the field recognize the importance of physics in engineering education, students should be reminded of this fact during their early college years.

With a common understanding of the importance and relevance of physics in the disciplines of engineering, medicine and health sciences, curriculum designers should make sure that students take an adequate number of physics courses, which provide full coverage of basic physic concepts and are consistent with what is being taught at internationally. To help students overcome their difficulty with physics, collective effort from instructors and administrators is needed with the aim of delivering high quality education to students enrolled in various university programs.

Acknowledgements

The authors would like to extend their thanks and appreciation to the College of Sciences, University of Sharjah, for supporting this research and to the lab supervisors and research assistants who helped with analyzing the survey.

References:

[1] Ahmed, M. (1984). Role of Physics in Saudi Engineering Education, Phys. Educ. Vol 19:120-124.

[2] Freedman Roger A. (1996). Challenges in Teaching and Learning Introductory Physics, http://www.physics.ucsb.edu/~airboy/challenge.html. Accessed: 19 November 2007.

[3] Jamjoom, M. (1980). Conference on Engineering Education in the Gulf Countries.

[4] Elemhdi, H. (2006). Placement Tests and Remedial Physics in Post-Secondary Institutions. Presented in the 27th Conference on Admission and Registration in the Arab Universities, Sharjah, UAE, March (2006).

[5] McDermott and Shaffer (19920. Research as a Guide for Curriculum Development: An Example from Introductory Electricity, Part I: Investigation of Student Understanding. Am J. Phys. **60**: 996

[6] McDermott, L.C. (1991). What we Teach and What is Learned – Closing the Gap. American Journal of Physics, 59(4), 301–315.

[7] Van Heuvelen, A. (1991). Learning to Think Like a Physicist: a Review of Research-based Instructional Strategy, Am. J. Phys. **59**: 891.

[8] Al-Douri, A. J. Private Communication

[9] Ladera, C. L. (2009). Evaluation in physics teaching: make it an opportunity for further learning Lat. Am. J. Phys. Educ. Vol. 3, No. 3, Sept. 2009

[9] Suleiman, B. and Elmehdi, HM. (2209) Guided Critical Thinking: a New Approach in Physics Exams. The Union of the Arab Universities Journal. Issue 5, April (2009).

[10] Feld, Jacob and Carper, Kenneth, C*onstruction Failure* (1997), 2nd Ed., John Wiley & Sons, New York, N.Y.

[11] Martín-Blas, T. and Serrano-Fernández, A (2009). The role of new technologies in the learning process: Moodle as a teaching tool in Physics. Computers & Education 52:35–44.

[12] Poveda, IL, Robert (2012). Development of Critical Thinking, Mediated by Problem Solving in Mechanical Physics. Presented at The 3rd International Conference on Education, Training and Informatics: ICETI 2012. http://www.iiis.org/CDs2012/CD2012IMC/ICETI_2012/PapersPdf/EB897NW.pdf .

[11] Levy, Matthys and Salvadori, Mario (1992). Why *Buildings Fall Down: How Structures Fail*. W. W. Norton, New York, N.Y.

[12] Ross, S. et al. (1984). “Tacoma Narrows, 1940.” *Construction Disasters*. McGraw-Hill Book Co, New York, NY.

[13] Delatte, Norbert (1997). "Failure Case Studies and Ethics in Engineering Mechanics Courses" *Journal of Professional Issues in Engineering Education and Practice*, July 1997.

[14] Bartlett, A.A. (1981). Are We Overlooking Something, Am. J. Phys 49:1105.

[15] Bartlett, A.A. 1981. More Physics for Architecture, Am. J. Phys 34:9.

[16] Suleiman B, Int. J. Sci. res. Vol 16(2006) pp. 499-505.

[17] Varmus, H. (1999). The Impact of Physics on Biology and Medicine. http://physicsworld.com/cws/article/print/956

[18] Mackay, M. C. and Santillan, M. (2005). Mathematics, Biology and Physics: Interactions and Interdependence. Notices of the AMS, Vol 52:8, 832-840.

Book Review:

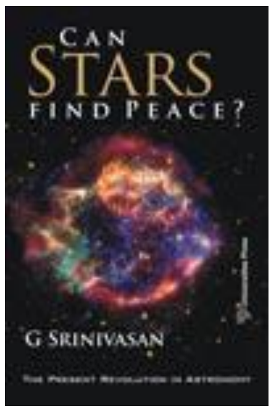
What are the STARS? xiii + 246 Can STARS find peace? Xiii + 254

both by **G. Srinivasan**, University Press, Hyderabad (2011)

Reviwed by:

Jayant V Narlikar

Inter University Centre for Astronomy and Astrophysics,
University of Pune campus, Ganeshkhind Pune 411007



Stars, their origin, structure, evolution and end, today constitute the most successful part of astrophysics. The media, however, tell us that glamour lies elsewhere, in cosmology, with dark matter, dark energy, God's particle, etc. the main actors. It is a pity that stellar physics, the really solid part of modern astrophysics gets neglected in the presence of the physics of the origin and evolution of the universe. Against this background the two books listed above are most welcome, especially so as they come from an author with excellent teaching experience. For, someone like **G. Srinivasan** who has taught the subject to graduate students can very well express the textual information in a format that the student reader will appreciate. The reader will also find the encouraging words of Lord Martin Rees who has written a foreword in the beginning of each volume.

The two books together are meant to cover the different aspects and stages of a star's life. The **first text** deals with Sun-like (main sequence) stars whereas the second one deals with red giants and the more dramatic aspects like exploding stars, pulsars, black holes,etc. The chapters and coverage of the first book are as follows.

Against the backdrop of the International Year of Astronomy, the year 2009 AD, the author gives a brief description of the present revolution in astronomy. It is a broad brush picture of what one talks of today in astronomical research literature. This account of course includes cosmology and the belief that the universe started with a big bang. The author's glorification of spacetime singularity as an exact result, however, reminds this reviewer of the Aesop's fable of the fox who lost his tail and sought to glorify this defect.

Indeed, it might have been more relevant to stick to the stars and start with the Chapter 1. Here the physics behind spectra of stars is very well explained. On this observed evidence the astronomer bases any theory of the stars. It is surprising that in this discussion the Hertzsprung Russell diagram has been bypassed! Although the HR diagram is referred to in the second volume, with the accolade "Perhaps the most important diagram in stellar astronomy is what is known as the Hertzsprung Russell diagram (H-R diagram)" the question arises as to why its arrival on the scene is delayed to the second volume. This is like delaying the stage entry of the Prince of Denmark in the Shakespearean play *Hamlet.* Chapters 2 and 3 are devoted to Eddington's classic work modeling the typical main sequence star. The author has explained the temperature drop as one move from the core to the outer surface of the star and the role of opacity. It is good to see a derivation of the Eddington limit representing the tussle between the forces of gravitation and radiation pressure. First considered in the context of stars this result has been applied also in high energy astrophysics.

Chapter 5 deals with the crucial question of solar energy and here the author's teaching ability is obvious. Having spent a good bit of time explaining the nuclear fusion process which led to Bethe's work on the solar model, he asks the desired question: the model works well but how do we test its validity? And he goes on to give a lucid outline of the solar neutrino experiments. This is followed by a chapter on *Sounds of the Sun* which show how the surface phenomena observed for the Sun can be linked to interior features of the solar model. Helioseismology is thus briefly described as well as the *GONG* programme that links solar observatories round the globe into a programme of uninterrupted observation of the Sun. The Solar and Heliospheric Observatory (*SOHO*) with its multiple uses is also described. The standard model of particle physics is finally discussed and the phenomena of neutrino oscillations are identified in the solar neutrino detectors.

The **second book** appears to allocate a disproportionately large share to white dwarfs and neutron stars. This is presumably because the author may have wanted to emphasize the pioneering work of S. Chandrasekhar in the area of stars containing degenerate matter. There are 157 pages devoted to such stars whereas the rest including red giants and supernovae get only 88 pages. The result of this skew distribution is that some of the important topics are missed out. Thus a reader brought up on this book will be unaware of Fred Hoyle's brilliant solution of making carbon through a *resonant* nuclear reaction involving a triple alpha fusion. Nor will the pioneering work of Burbidge, Burbidge, Fowler and Hoyle (commonly referred to as B^2FH) on stellar nucleosynthesis ever enter the reading horizons of the readers of this book. Attempts to locate these topics through the index failed; although in the process it became clear that the index also left out many important names and topics. Further, this reviewer could have added some texts to the list given for 'suggested reading' at the end.

These are shortcomings that may be easily corrected provided the author and the publisher consider them important enough. Even as it is, these books will bring the subject of stars to the attention of the typical physics undergraduate who should not miss the chance of reading an exciting account.